Computer Algebra Independent Integration Tests

Summer 2023 edition

3-Logarithms/56-3.1.2-d-x-^m-a+b-log-c-x^n-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [193]. This is test number [56].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (193)	0.00 (0)
Mathematica	100.00 (193)	0.00 (0)
Fricas	63.73 (123)	36.27 (70)
Maple	62.69 (121)	37.31 (72)
Maxima	54.92 (106)	45.08 (87)
Giac	52.85 (102)	47.15 (91)
Sympy	40.93 (79)	59.07 (114)
Mupad	31.09 (60)	68.91 (133)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

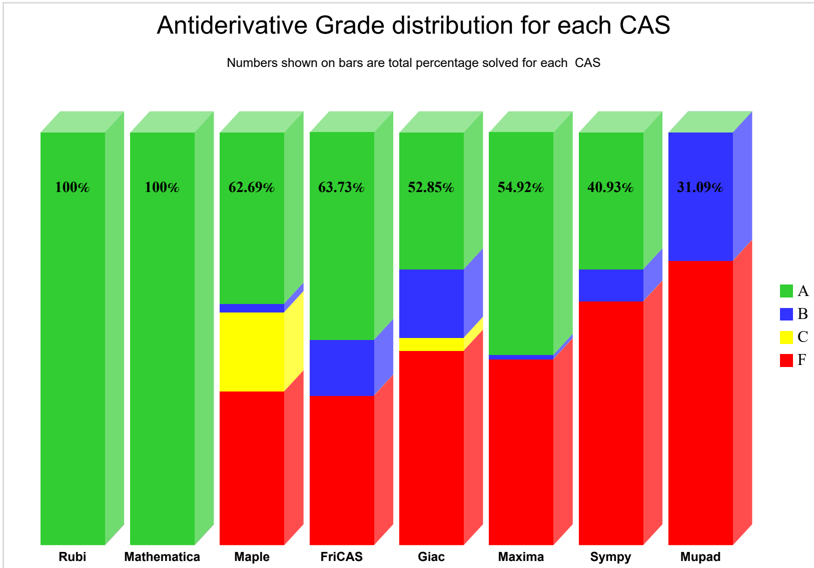
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

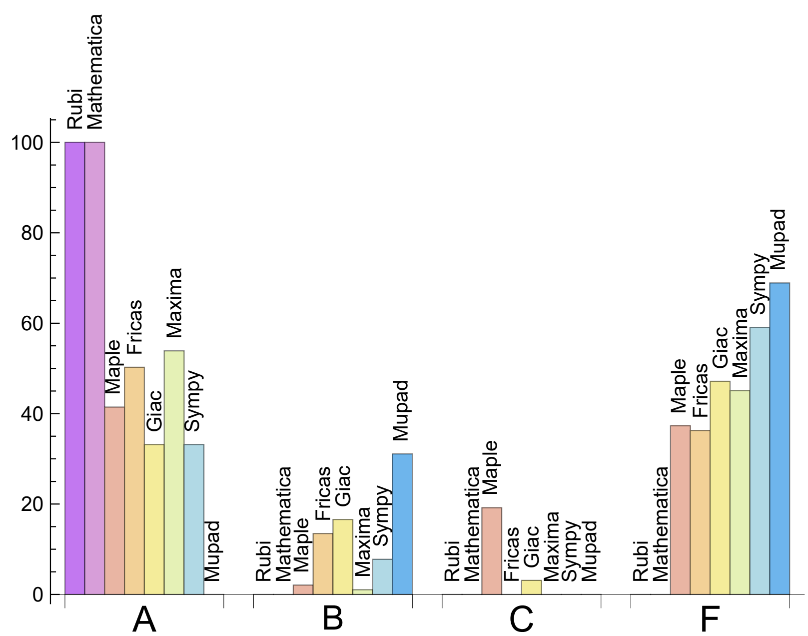
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	100.000	0.000	0.000	0.000
Maxima	53.886	1.036	0.000	45.078
Fricas	50.259	13.472	0.000	36.269
Maple	41.451	2.073	19.171	37.306
Giac	33.161	16.580	3.109	47.150
Sympy	33.161	7.772	0.000	59.067
Mupad	0.000	31.088	0.000	68.912

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	70	55.71	0.00	44.29
Maple	72	100.00	0.00	0.00
Maxima	87	90.80	0.00	9.20
Giac	91	100.00	0.00	0.00
Sympy	114	98.25	1.75	0.00
Mupad	133	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.03
Mathematica	0.05
Maxima	0.19
Maple	0.19
Mupad	0.27
Fricas	0.30
Giac	0.34
Sympy	1.30

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	33.55	0.98	22.00	0.92
Maxima	42.14	1.00	26.00	0.92
Mathematica	52.97	0.95	54.00	1.00
Rubi	56.56	1.00	58.00	1.00
Fricas	70.10	1.35	37.00	1.11
Sympy	74.39	1.53	39.00	1.17
Giac	132.12	2.20	37.00	1.08
Maple	148.26	2.36	34.00	1.13

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

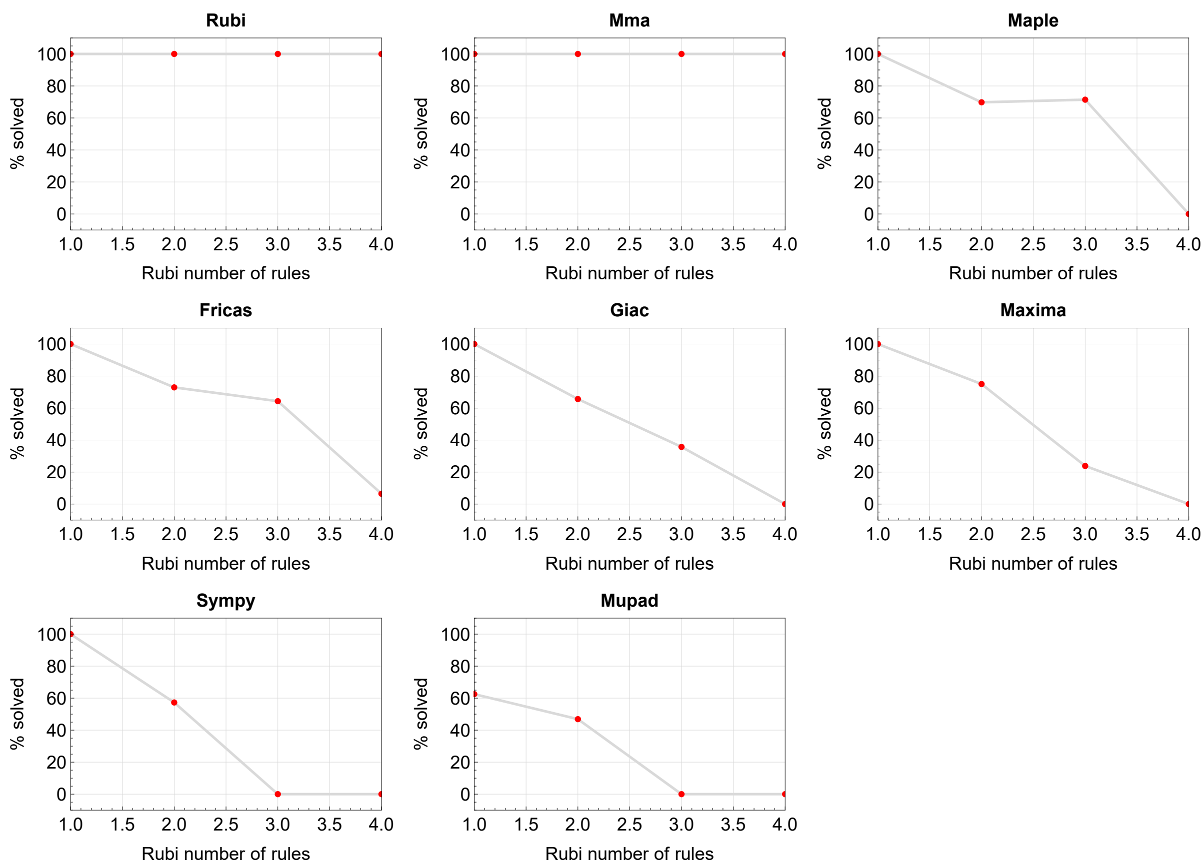


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

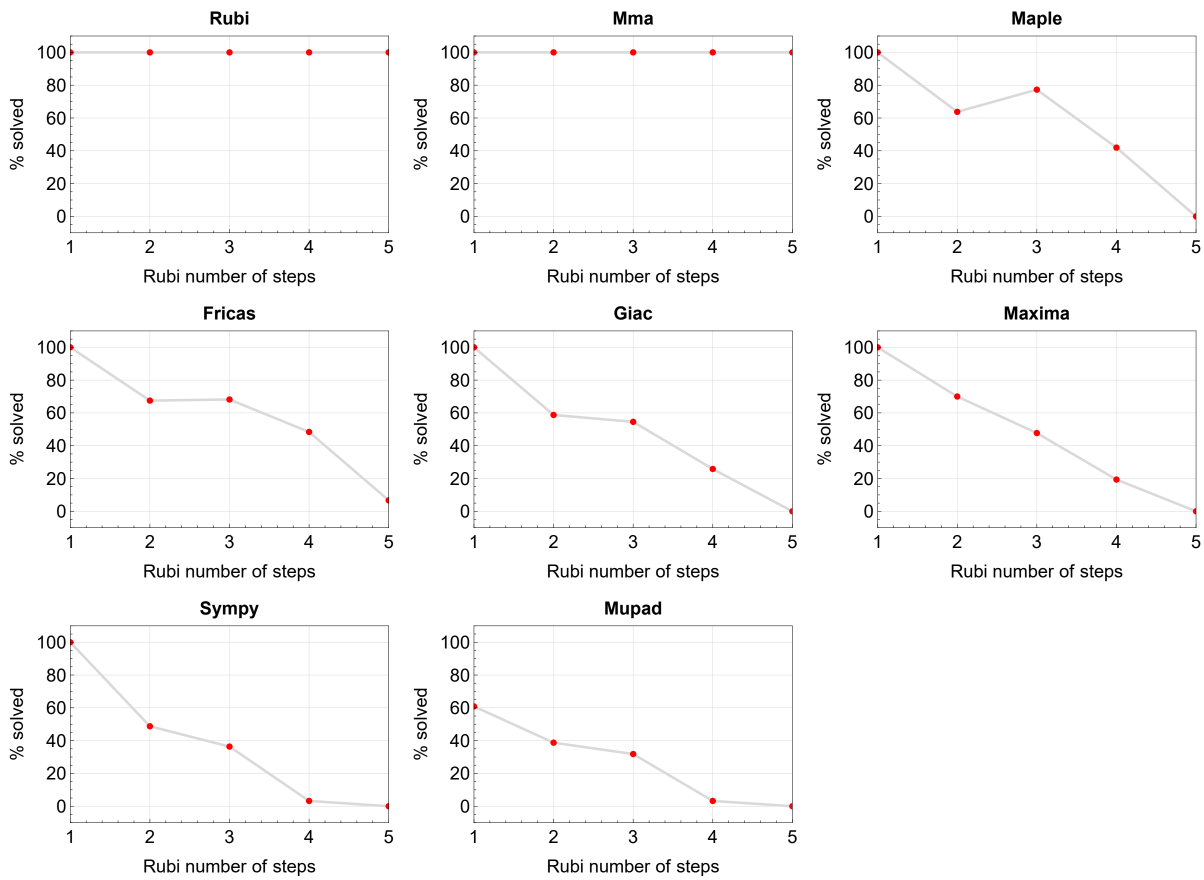


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

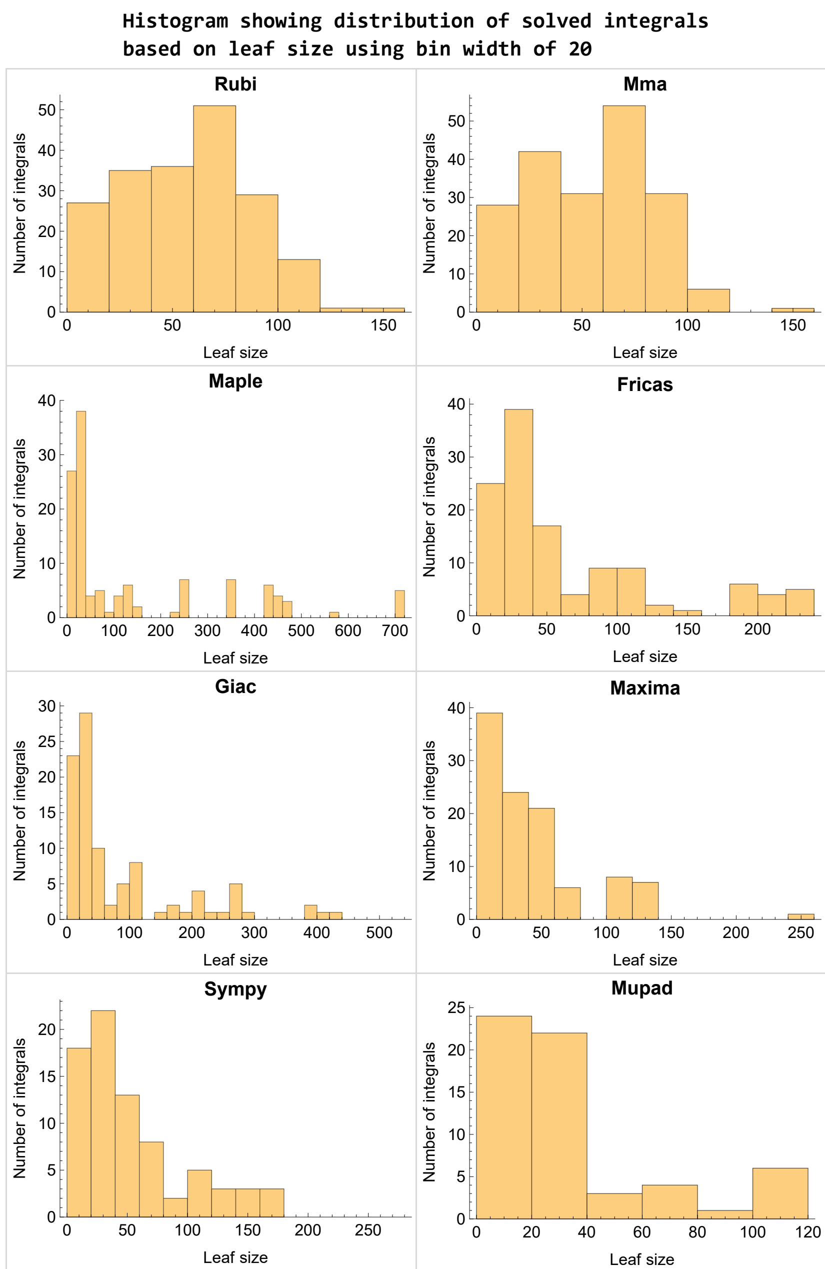


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

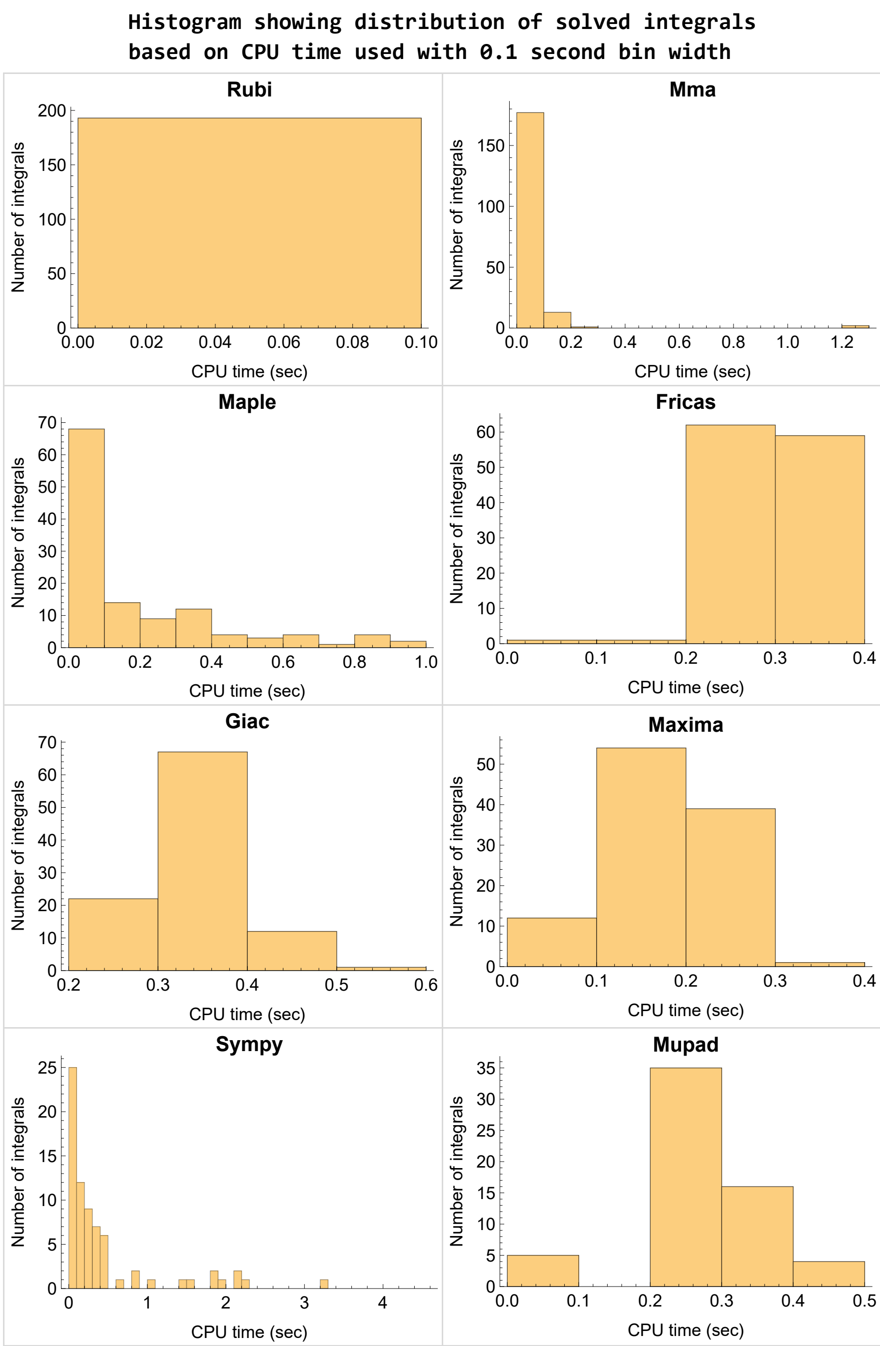


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

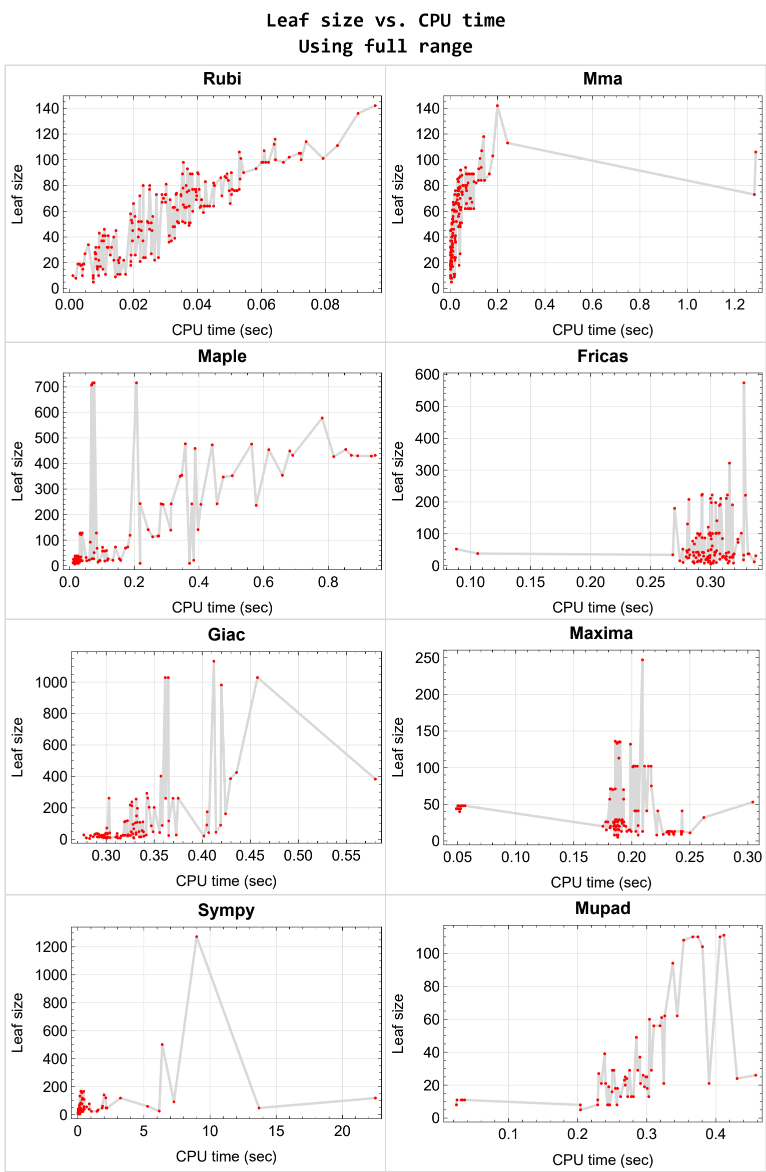


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 99, 100, 107, 108, 109, 110, 111, 112}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

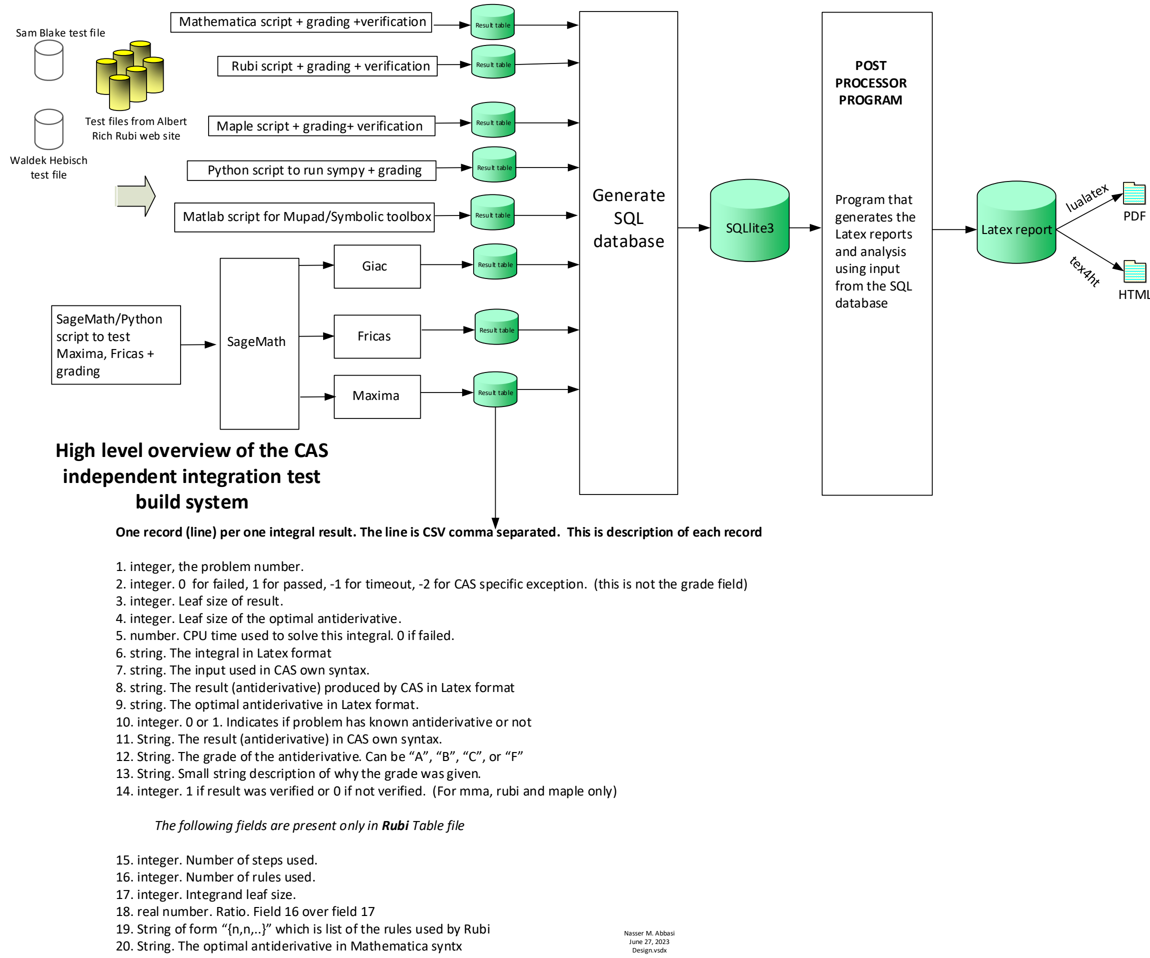
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 60, 61, 62, 63, 64, 69, 77, 85, 92, 98, 118, 125, 132, 139, 146, 149, 152, 156, 157, 158, 171, 179, 187 }

B grade { 57, 59, 150, 151 }

C grade { 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 99, 100, 107, 108, 109, 110, 111, 112 }

F normal fail { 101, 102, 103, 104, 105, 106, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 55, 56, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 118, 132, 139, 149, 152, 153, 154, 156, 157, 158, 159, 160, 161, 170, 171, 178, 179, 187 }

B grade { 50, 51, 52, 54, 57, 58, 59, 60, 61, 62, 63, 64, 81, 82, 83, 84, 85, 86, 87, 88, 95, 125, 146, 150, 151, 155 }

C grade { }

F normal fail { 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 162, 163, 164, 165, 166, 167, 168, 169, 172, 173, 174, 175, 176, 177, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

F(-1) timedout fail { }

F(-2) exception fail { 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 77, 85, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 118, 125, 132, 139, 146, 151, 152, 156, 157, 158, 171, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190 }

B grade { 149, 150 }

C grade { }

F normal fail { 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 175, 183, 192, 193 }

F(-1) timedout fail { }

F(-2) exception fail { 167, 168, 169, 170, 172, 173, 174, 191 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 55, 56, 65, 66, 67, 68, 77, 85, 92, 93, 98, 105, 118, 132, 139, 146, 157, 158, 171, 179, 187 }

B grade { 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 69, 73, 74, 75, 76, 81, 82, 83, 84, 94, 99, 100, 111, 125, 149, 150, 151, 152, 156 }

C grade { 89, 90, 91, 95, 96, 97 }

F normal fail { 27, 28, 34, 35, 41, 42, 70, 71, 72, 78, 79, 80, 86, 87, 88, 101, 102, 103, 104, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25, 26, 32, 33, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 77, 85, 118, 125, 132, 139, 146, 171, 179, 187 }

C grade { }

F normal fail { }

F(-1) timedout fail { 22, 23, 24, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 41, 42, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 25, 26, 32, 33, 39, 40, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 55, 56, 60, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 118, 125, 132, 139, 146, 149, 156, 157, 158, 171, 179, 187 }

B grade { 47, 54, 57, 58, 59, 61, 62, 63, 64, 69, 77, 85, 150, 151, 152 }

C grade { }

F normal fail { 22, 23, 24, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 41, 42, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 188, 189, 191, 192 }

F(-1) timedout fail { 190, 193 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	15	11
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.79	0.58
time (sec)	N/A	0.004	0.002	0.023	0.197	0.290	0.042	0.296	0.229

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	15	11
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.79	0.58
time (sec)	N/A	0.004	0.002	0.016	0.186	0.302	0.036	0.293	0.032

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	14	15	11
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.74	0.79	0.58
time (sec)	N/A	0.003	0.002	0.014	0.189	0.274	0.036	0.291	0.033

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	16	10	7	16	8
N.S.	1	1.00	1.00	1.10	1.60	1.00	0.70	1.60	0.80
time (sec)	N/A	0.001	0.002	0.011	0.188	0.305	0.041	0.287	0.024

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.004	0.002	0.027	0.185	0.314	0.036	0.312	0.257

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	13	15	11	10	15	11
N.S.	1	1.00	1.00	0.87	1.00	0.73	0.67	1.00	0.73
time (sec)	N/A	0.004	0.002	0.018	0.198	0.298	0.041	0.335	0.036

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	13	15	13	17	15	11
N.S.	1	1.00	1.00	0.68	0.79	0.68	0.89	0.79	0.58
time (sec)	N/A	0.004	0.002	0.019	0.178	0.292	0.045	0.336	0.025

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	21	26	26	26	21
N.S.	1	1.00	1.00	0.84	0.66	0.81	0.81	0.81	0.66
time (sec)	N/A	0.013	0.003	0.021	0.187	0.318	0.046	0.317	0.240

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	21	26	29	26	21
N.S.	1	1.00	1.00	0.84	0.66	0.81	0.91	0.81	0.66
time (sec)	N/A	0.012	0.002	0.019	0.186	0.310	0.046	0.320	0.234

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	21	26	26	26	21
N.S.	1	1.00	1.00	0.84	0.66	0.81	0.81	0.81	0.66
time (sec)	N/A	0.008	0.002	0.017	0.188	0.297	0.044	0.323	0.291

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	16	19	19	19	16
N.S.	1	1.00	1.00	1.05	0.84	1.00	1.00	1.00	0.84
time (sec)	N/A	0.003	0.002	0.014	0.185	0.313	0.044	0.299	0.250

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.009	0.002	0.371	0.205	0.286	0.036	0.329	0.246

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	19	19	20	26	19
N.S.	1	1.00	1.00	0.81	0.73	0.73	0.77	1.00	0.73
time (sec)	N/A	0.013	0.002	0.020	0.186	0.296	0.050	0.297	0.297

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	21	21	29	26	21
N.S.	1	1.00	1.00	0.66	0.66	0.66	0.91	0.81	0.66
time (sec)	N/A	0.012	0.002	0.025	0.205	0.297	0.061	0.295	0.325

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	29	37	42	37	29
N.S.	1	1.00	1.00	0.84	0.64	0.82	0.93	0.82	0.64
time (sec)	N/A	0.022	0.003	0.026	0.192	0.332	0.056	0.299	0.307

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	29	37	41	37	29
N.S.	1	1.00	1.00	0.84	0.64	0.82	0.91	0.82	0.64
time (sec)	N/A	0.020	0.003	0.021	0.187	0.331	0.054	0.301	0.250

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	38	29	37	42	37	29
N.S.	1	1.00	1.00	0.84	0.64	0.82	0.93	0.82	0.64
time (sec)	N/A	0.015	0.002	0.017	0.186	0.320	0.053	0.311	0.275

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	24	28	29	28	24
N.S.	1	1.00	1.00	1.04	0.86	1.00	1.04	1.00	0.86
time (sec)	N/A	0.008	0.002	0.017	0.185	0.299	0.049	0.277	0.271

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.007	0.002	0.218	0.221	0.302	0.036	0.315	0.245

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	29	27	27	31	37	27
N.S.	1	1.00	1.00	0.78	0.73	0.73	0.84	1.00	0.73
time (sec)	N/A	0.023	0.002	0.026	0.191	0.317	0.061	0.300	0.230

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	29	29	29	44	37	29
N.S.	1	1.00	1.00	0.64	0.64	0.64	0.98	0.82	0.64
time (sec)	N/A	0.026	0.002	0.025	0.189	0.313	0.075	0.291	0.252

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	12	0	11	0
N.S.	1	1.00	1.00	1.27	1.00	1.09	0.00	1.00	0.00
time (sec)	N/A	0.015	0.018	0.031	0.250	0.336	0.000	0.329	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	12	0	11	0
N.S.	1	1.00	1.00	1.27	1.00	1.09	0.00	1.00	0.00
time (sec)	N/A	0.016	0.018	0.016	0.243	0.286	0.000	0.298	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	12	0	11	0
N.S.	1	1.00	1.00	1.27	1.00	1.09	0.00	1.00	0.00
time (sec)	N/A	0.011	0.017	0.014	0.236	0.289	0.000	0.304	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	14	9	8	5	9	8
N.S.	1	1.00	1.00	1.75	1.12	1.00	0.62	1.12	1.00
time (sec)	N/A	0.002	0.007	0.016	0.232	0.301	0.162	0.279	0.204

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.007	0.005	0.016	0.188	0.300	0.038	0.283	0.204

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	10	0	0	0
N.S.	1	1.00	1.00	1.11	1.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.014	0.018	0.019	0.227	0.277	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	12	0	0	0
N.S.	1	1.00	1.00	1.27	1.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.018	0.020	0.022	0.232	0.285	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	13	33	0	24	0
N.S.	1	1.00	1.00	1.08	0.54	1.38	0.00	1.00	0.00
time (sec)	N/A	0.023	0.019	0.027	0.237	0.290	0.000	0.284	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	13	33	0	24	0
N.S.	1	1.00	1.00	1.08	0.54	1.38	0.00	1.00	0.00
time (sec)	N/A	0.028	0.019	0.024	0.232	0.303	0.000	0.294	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	13	33	0	24	0
N.S.	1	1.00	1.00	1.08	0.54	1.38	0.00	1.00	0.00
time (sec)	N/A	0.016	0.018	0.023	0.232	0.291	0.000	0.325	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	12	25	12	19	18
N.S.	1	1.00	1.00	1.33	0.67	1.39	0.67	1.06	1.00
time (sec)	N/A	0.004	0.007	0.020	0.230	0.288	0.167	0.298	0.254

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	8	8	7	8	8
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.007	0.002	0.018	0.188	0.295	0.032	0.301	0.243

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	9	28	0	0	0
N.S.	1	1.00	1.00	0.95	0.41	1.27	0.00	0.00	0.00
time (sec)	N/A	0.027	0.021	0.029	0.243	0.310	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	26	13	33	0	0	0
N.S.	1	1.00	1.00	1.08	0.54	1.38	0.00	0.00	0.00
time (sec)	N/A	0.024	0.022	0.027	0.242	0.327	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	13	47	0	35	0
N.S.	1	1.00	1.00	0.92	0.35	1.27	0.00	0.95	0.00
time (sec)	N/A	0.032	0.021	0.029	0.244	0.305	0.000	0.289	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	34	13	47	0	35	0
N.S.	1	1.00	1.00	0.83	0.32	1.15	0.00	0.85	0.00
time (sec)	N/A	0.031	0.022	0.028	0.234	0.280	0.000	0.328	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	13	47	0	35	0
N.S.	1	1.00	1.00	0.92	0.35	1.27	0.00	0.95	0.00
time (sec)	N/A	0.020	0.006	0.023	0.231	0.298	0.000	0.301	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	13	34	26	29	29
N.S.	1	1.00	1.00	0.88	0.38	1.00	0.76	0.85	0.85
time (sec)	N/A	0.006	0.010	0.021	0.233	0.268	0.191	0.323	0.287

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80
time (sec)	N/A	0.007	0.002	0.021	0.187	0.319	0.037	0.300	0.229

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	28	9	34	0	0	0
N.S.	1	1.00	1.00	0.72	0.23	0.87	0.00	0.00	0.00
time (sec)	N/A	0.033	0.020	0.030	0.236	0.319	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	13	41	0	0	0
N.S.	1	1.00	1.00	0.94	0.36	1.14	0.00	0.00	0.00
time (sec)	N/A	0.031	0.023	0.035	0.231	0.294	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	32	27	26	30	27	31	25
N.S.	1	1.00	1.19	1.00	0.96	1.11	1.00	1.15	0.93
time (sec)	N/A	0.008	0.003	0.119	0.193	0.311	0.233	0.292	0.299

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	32	27	26	30	27	31	25
N.S.	1	1.00	1.19	1.00	0.96	1.11	1.00	1.15	0.93
time (sec)	N/A	0.008	0.002	0.069	0.179	0.294	0.161	0.288	0.300

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	32	27	26	30	27	31	25
N.S.	1	1.00	1.19	1.00	0.96	1.11	1.00	1.15	0.93
time (sec)	N/A	0.005	0.002	0.075	0.178	0.299	0.119	0.331	0.269

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	22	15	20	18
N.S.	1	1.00	1.00	1.06	1.00	1.22	0.83	1.11	1.00
time (sec)	N/A	0.003	0.002	0.039	0.181	0.313	0.085	0.339	0.258

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	20	20	18	34	19	19
N.S.	1	1.00	0.95	0.91	0.91	0.82	1.55	0.86	0.86
time (sec)	N/A	0.009	0.002	0.094	0.175	0.327	1.536	0.311	0.245

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	19	26	19	19	24	23
N.S.	1	1.00	1.13	0.83	1.13	0.83	0.83	1.04	1.00
time (sec)	N/A	0.008	0.002	0.053	0.184	0.296	0.117	0.365	0.268

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	32	22	26	23	29	27	26
N.S.	1	1.00	1.19	0.81	0.96	0.85	1.07	1.00	0.96
time (sec)	N/A	0.008	0.002	0.061	0.186	0.312	0.222	0.342	0.295

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	43	73	71	102	78	111	61
N.S.	1	1.00	0.83	1.40	1.37	1.96	1.50	2.13	1.17
time (sec)	N/A	0.035	0.011	0.180	0.186	0.325	0.322	0.338	0.321

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	46	73	71	103	85	111	62
N.S.	1	1.00	0.88	1.40	1.37	1.98	1.63	2.13	1.19
time (sec)	N/A	0.024	0.015	0.142	0.181	0.299	0.225	0.319	0.326

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	72	70	102	76	108	60
N.S.	1	1.00	0.79	1.38	1.35	1.96	1.46	2.08	1.15
time (sec)	N/A	0.022	0.010	0.102	0.193	0.306	0.167	0.335	0.304

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	57	57	85	65	88	49
N.S.	1	1.00	0.77	1.33	1.33	1.98	1.51	2.05	1.14
time (sec)	N/A	0.009	0.007	0.102	0.181	0.310	0.126	0.358	0.285

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	51	60	56	37
N.S.	1	1.00	1.00	0.95	0.91	2.32	2.73	2.55	1.68
time (sec)	N/A	0.017	0.004	0.158	0.184	0.300	5.282	0.337	0.290

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	57	70	77	66	86	56
N.S.	1	1.00	0.76	1.24	1.52	1.67	1.43	1.87	1.22
time (sec)	N/A	0.025	0.010	0.109	0.184	0.285	0.127	0.347	0.319

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	59	71	83	78	90	62
N.S.	1	1.00	0.79	1.13	1.37	1.60	1.50	1.73	1.19
time (sec)	N/A	0.022	0.010	0.114	0.182	0.323	0.235	0.419	0.344

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	66	141	135	222	167	262	110
N.S.	1	1.00	0.86	1.83	1.75	2.88	2.17	3.40	1.43
time (sec)	N/A	0.039	0.023	0.397	0.189	0.314	0.450	0.303	0.374

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	67	139	134	224	156	256	108
N.S.	1	1.00	0.87	1.81	1.74	2.91	2.03	3.32	1.40
time (sec)	N/A	0.041	0.012	0.313	0.187	0.293	0.320	0.331	0.353

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	141	135	222	167	262	110
N.S.	1	1.00	0.78	1.83	1.75	2.88	2.17	3.40	1.43
time (sec)	N/A	0.025	0.022	0.243	0.189	0.301	0.237	0.343	0.367

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	119	113	198	133	219	94
N.S.	1	1.00	0.76	1.80	1.71	3.00	2.02	3.32	1.42
time (sec)	N/A	0.020	0.007	0.187	0.189	0.304	0.172	0.325	0.338

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	100	92	114	56
N.S.	1	1.00	1.00	0.95	0.91	4.55	4.18	5.18	2.55
time (sec)	N/A	0.014	0.004	0.384	0.182	0.291	7.271	0.320	0.310

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	52	113	133	180	134	197	104
N.S.	1	1.00	0.75	1.64	1.93	2.61	1.94	2.86	1.51
time (sec)	N/A	0.041	0.014	0.257	0.187	0.270	0.170	0.332	0.381

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	116	135	189	168	203	111
N.S.	1	1.00	0.78	1.51	1.75	2.45	2.18	2.64	1.44
time (sec)	N/A	0.038	0.017	0.276	0.190	0.307	0.251	0.350	0.412

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	116	136	191	158	204	110
N.S.	1	1.00	0.78	1.51	1.77	2.48	2.05	2.65	1.43
time (sec)	N/A	0.039	0.018	0.273	0.186	0.318	0.309	0.345	0.406

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	51	51	242	0	42	0	48	0
N.S.	1	1.00	1.00	4.75	0.00	0.82	0.00	0.94	0.00
time (sec)	N/A	0.036	0.047	0.378	0.000	0.288	0.000	0.349	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	51	51	242	0	42	0	48	0
N.S.	1	1.00	1.00	4.75	0.00	0.82	0.00	0.94	0.00
time (sec)	N/A	0.038	0.043	0.314	0.000	0.315	0.000	0.326	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	51	51	242	0	42	0	48	0
N.S.	1	1.00	1.00	4.75	0.00	0.82	0.00	0.94	0.00
time (sec)	N/A	0.034	0.046	0.283	0.000	0.285	0.000	0.339	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	48	48	48	240	0	39	0	42	0
N.S.	1	1.00	1.00	5.00	0.00	0.81	0.00	0.88	0.00
time (sec)	N/A	0.032	0.035	0.289	0.000	0.313	0.000	0.407	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	19	31	45	18
N.S.	1	1.00	1.00	1.06	1.00	1.06	1.72	2.50	1.00
time (sec)	N/A	0.019	0.038	0.091	0.181	0.292	0.363	0.414	0.301

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	48	48	48	236	0	41	0	0	0
N.S.	1	1.00	1.00	4.92	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.033	0.042	0.577	0.000	0.304	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	51	51	240	0	42	0	0	0
N.S.	1	1.00	1.00	4.71	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.037	0.042	0.406	0.000	0.293	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	51	51	51	242	0	42	0	0	0
N.S.	1	1.00	1.00	4.75	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.034	0.043	0.456	0.000	0.290	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	76	76	70	354	0	101	0	261	0
N.S.	1	1.00	0.92	4.66	0.00	1.33	0.00	3.43	0.00
time (sec)	N/A	0.052	0.086	0.347	0.000	0.301	0.000	0.375	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	76	76	70	354	0	101	0	261	0
N.S.	1	1.00	0.92	4.66	0.00	1.33	0.00	3.43	0.00
time (sec)	N/A	0.050	0.084	0.347	0.000	0.304	0.000	0.370	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	76	76	70	350	0	101	0	261	0
N.S.	1	1.00	0.92	4.61	0.00	1.33	0.00	3.43	0.00
time (sec)	N/A	0.037	0.084	0.342	0.000	0.289	0.000	0.362	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	70	70	66	350	0	95	0	238	0
N.S.	1	1.00	0.94	5.00	0.00	1.36	0.00	3.40	0.00
time (sec)	N/A	0.029	0.069	0.342	0.000	0.290	0.000	0.327	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	25	39	21	20
N.S.	1	1.00	1.00	1.05	1.00	1.25	1.95	1.05	1.00
time (sec)	N/A	0.015	0.005	0.117	0.189	0.299	0.867	0.322	0.269

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	73	73	76	347	0	88	0	0	0
N.S.	1	1.00	1.04	4.75	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.051	0.069	0.475	0.000	0.293	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	76	76	80	352	0	102	0	0	0
N.S.	1	1.00	1.05	4.63	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.052	0.066	0.503	0.000	0.307	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	76	76	80	354	0	102	0	0	0
N.S.	1	1.00	1.05	4.66	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.052	0.070	0.658	0.000	0.305	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	101	101	89	473	0	211	0	1029	0
N.S.	1	1.00	0.88	4.68	0.00	2.09	0.00	10.19	0.00
time (sec)	N/A	0.079	0.095	0.441	0.000	0.308	0.000	0.365	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	105	89	477	0	211	0	1029	0
N.S.	1	1.00	0.85	4.54	0.00	2.01	0.00	9.80	0.00
time (sec)	N/A	0.072	0.098	0.358	0.000	0.300	0.000	0.361	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	101	101	89	476	0	211	0	1029	0
N.S.	1	1.00	0.88	4.71	0.00	2.09	0.00	10.19	0.00
time (sec)	N/A	0.054	0.092	0.563	0.000	0.313	0.000	0.458	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	98	98	82	459	0	198	0	982	0
N.S.	1	1.00	0.84	4.68	0.00	2.02	0.00	10.02	0.00
time (sec)	N/A	0.036	0.076	0.388	0.000	0.300	0.000	0.420	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	62	61	21	39
N.S.	1	1.00	1.00	0.95	0.91	2.82	2.77	0.95	1.77
time (sec)	N/A	0.015	0.004	0.134	0.195	0.301	1.874	0.316	0.239

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	102	102	94	449	0	192	0	0	0
N.S.	1	1.00	0.92	4.40	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.069	0.065	0.681	0.000	0.308	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	100	100	89	454	0	221	0	0	0
N.S.	1	1.00	0.89	4.54	0.00	2.21	0.00	0.00	0.00
time (sec)	N/A	0.072	0.084	0.616	0.000	0.329	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	105	105	89	455	0	221	0	0	0
N.S.	1	1.00	0.85	4.33	0.00	2.10	0.00	0.00	0.00
time (sec)	N/A	0.072	0.080	0.854	0.000	0.292	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	41	41	29	128	41	50	48	117	0
N.S.	1	1.00	0.71	3.12	1.00	1.22	1.17	2.85	0.00
time (sec)	N/A	0.012	0.012	0.083	0.243	0.282	13.699	0.365	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	41	41	29	128	41	42	48	108	0
N.S.	1	1.00	0.71	3.12	1.00	1.02	1.17	2.63	0.00
time (sec)	N/A	0.010	0.009	0.033	0.213	0.280	1.846	0.332	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	41	41	29	124	41	32	48	105	0
N.S.	1	1.00	0.71	3.02	1.00	0.78	1.17	2.56	0.00
time (sec)	N/A	0.010	0.006	0.032	0.222	0.286	0.202	0.330	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	24	36	41	25	42	41	0
N.S.	1	1.00	0.65	0.97	1.11	0.68	1.14	1.11	0.00
time (sec)	N/A	0.010	0.006	0.027	0.205	0.282	0.235	0.304	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	37	37	24	122	41	28	44	43	0
N.S.	1	1.00	0.65	3.30	1.11	0.76	1.19	1.16	0.00
time (sec)	N/A	0.011	0.007	0.036	0.204	0.279	0.453	0.356	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	41	41	29	128	41	32	49	67	0
N.S.	1	1.00	0.71	3.12	1.00	0.78	1.20	1.63	0.00
time (sec)	N/A	0.011	0.007	0.039	0.203	0.299	2.140	0.329	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	A	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	73	73	61	716	102	141	119	425	0
N.S.	1	1.00	0.84	9.81	1.40	1.93	1.63	5.82	0.00
time (sec)	N/A	0.030	0.016	0.207	0.211	0.305	22.490	0.436	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	73	73	61	716	102	121	119	386	0
N.S.	1	1.00	0.84	9.81	1.40	1.66	1.63	5.29	0.00
time (sec)	N/A	0.033	0.014	0.076	0.214	0.298	3.229	0.429	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	C	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	73	73	61	710	102	99	119	383	0
N.S.	1	1.00	0.84	9.73	1.40	1.36	1.63	5.25	0.00
time (sec)	N/A	0.027	0.012	0.069	0.217	0.299	0.362	0.580	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	92	102	87	109	118	0
N.S.	1	1.00	0.81	1.37	1.52	1.30	1.63	1.76	0.00
time (sec)	N/A	0.029	0.011	0.064	0.203	0.295	0.331	0.322	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	67	67	54	707	101	87	110	149	0
N.S.	1	1.00	0.81	10.55	1.51	1.30	1.64	2.22	0.00
time (sec)	N/A	0.030	0.010	0.068	0.201	0.288	0.479	0.332	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	73	73	61	716	102	94	121	213	0
N.S.	1	1.00	0.84	9.81	1.40	1.29	1.66	2.92	0.00
time (sec)	N/A	0.030	0.011	0.072	0.205	0.300	2.126	0.327	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.074	0.000	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	64	62	0	0	0	0	49	0
N.S.	1	0.96	0.93	0.00	0.00	0.00	0.00	0.73	0.00
time (sec)	N/A	0.044	0.080	0.000	0.000	0.000	0.000	0.334	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	98	98	84	432	0	0	0	0	0
N.S.	1	1.00	0.86	4.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.145	0.945	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	98	98	84	432	0	0	0	0	0
N.S.	1	1.00	0.86	4.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.131	0.690	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	98	98	84	430	0	0	0	0	0
N.S.	1	1.00	0.86	4.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.119	0.891	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	98	98	83	427	0	0	0	0	0
N.S.	1	1.00	0.85	4.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.101	0.817	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	101	98	93	429	0	0	0	293	0
N.S.	1	0.97	0.92	4.25	0.00	0.00	0.00	2.90	0.00
time (sec)	N/A	0.062	0.121	0.933	0.000	0.000	0.000	0.342	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	98	98	94	432	0	0	0	0	0
N.S.	1	1.00	0.96	4.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.131	0.871	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	85	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	61	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	0.017	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.025	0.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.025	0.000	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.012	0.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	14	29	14	13
N.S.	1	1.00	1.00	0.82	0.76	0.82	1.71	0.82	0.76
time (sec)	N/A	0.009	0.003	0.088	0.196	0.315	0.412	0.301	0.262

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	65	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	76	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	79	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.022	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	34	24	72	13
N.S.	1	1.00	1.00	0.82	0.76	2.00	1.41	4.24	0.76
time (sec)	N/A	0.010	0.004	0.020	0.199	0.283	1.462	0.301	0.303

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	79	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	90	90	88	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.022	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.010	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.014	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	14	22	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.93	1.47	0.93	0.87
time (sec)	N/A	0.010	0.003	0.021	0.203	0.299	0.380	0.295	0.281

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	52	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	60	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.027	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	73	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.032	0.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	78	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	78	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.034	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	69	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	24	24	14	13
N.S.	1	1.00	1.00	0.93	0.87	1.60	1.60	0.93	0.87
time (sec)	N/A	0.009	0.003	0.015	0.209	0.281	1.039	0.299	0.278

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	0.038	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	66	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.043	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	92	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	92	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	83	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	37	26	14	13
N.S.	1	1.00	1.00	0.82	0.76	2.18	1.53	0.82	0.76
time (sec)	N/A	0.009	0.003	0.022	0.191	0.308	6.157	0.310	0.272

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	70	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	0.042	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	78	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.058	0.042	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	18	102	26	15	175	0
N.S.	1	1.00	0.81	0.86	4.86	1.24	0.71	8.33	0.00
time (sec)	N/A	0.019	0.010	0.109	0.201	0.299	0.191	0.405	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	76	578	247	574	1273	1133	0
N.S.	1	1.00	0.66	4.98	2.13	4.95	10.97	9.77	0.00
time (sec)	N/A	0.064	0.032	0.781	0.209	0.328	8.987	0.412	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	76	243	132	208	502	402	0
N.S.	1	1.00	0.94	3.00	1.63	2.57	6.20	4.96	0.00
time (sec)	N/A	0.030	0.022	0.218	0.199	0.282	6.383	0.357	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	32	68	57	52	141	95	0
N.S.	1	1.00	0.70	1.48	1.24	1.13	3.07	2.07	0.00
time (sec)	N/A	0.011	0.010	0.085	0.193	0.277	1.990	0.328	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	67	0	0	68	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.050	0.092	0.000	0.000	0.286	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	89	0	0	131	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.064	0.165	0.000	0.000	0.281	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	113	0	0	322	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	2.27	0.00	0.00	0.00
time (sec)	N/A	0.096	0.242	0.000	0.000	0.316	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	40	69	75	73	78	162	0
N.S.	1	1.00	0.54	0.93	1.01	0.99	1.05	2.19	0.00
time (sec)	N/A	0.033	0.007	0.173	0.217	0.323	0.872	0.424	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	30	51	53	42	58	91	0
N.S.	1	1.00	0.57	0.96	1.00	0.79	1.09	1.72	0.00
time (sec)	N/A	0.019	0.006	0.076	0.304	0.303	0.468	0.405	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	20	32	32	20	34	42	0
N.S.	1	1.00	0.62	1.00	1.00	0.62	1.06	1.31	0.00
time (sec)	N/A	0.009	0.005	0.051	0.262	0.308	0.256	0.331	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	27	0	0	20	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.026	0.042	0.000	0.000	0.298	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	50	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.037	0.040	0.000	0.000	0.300	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	61	0	0	84	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	1.09	0.00	0.00	0.00
time (sec)	N/A	0.053	0.044	0.000	0.000	0.300	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	101	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	86	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.010	0.000	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	1.282	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	106	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	1.288	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	107	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.133	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.074	0.000	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	52	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.025	0.058	0.000	0.000	0.088	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	26	35	56	27	26
N.S.	1	1.00	1.00	1.04	1.00	1.35	2.15	1.04	1.00
time (sec)	N/A	0.021	0.009	0.154	0.188	0.290	0.672	0.373	0.458

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.067	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	86	82	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.109	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	44	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.70	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	0.044	0.000	0.052	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	44	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.70	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.038	0.000	0.051	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	44	38	0	0	0
N.S.	1	1.00	1.00	0.00	0.79	0.68	0.00	0.00	0.00
time (sec)	N/A	0.026	0.035	0.000	0.049	0.106	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	21	26	39	21	21
N.S.	1	1.00	1.00	1.05	1.00	1.24	1.86	1.00	1.00
time (sec)	N/A	0.022	0.007	0.035	0.193	0.316	0.491	0.402	0.390

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	48	0	40	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.77	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.043	0.000	0.052	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	44	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.70	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.042	0.000	0.049	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	44	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.70	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	0.046	0.000	0.052	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	103	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.180	0.000	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	48	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.055	0.000	0.053	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	48	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.040	0.049	0.000	0.053	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	48	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.66	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.019	0.000	0.050	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	24	31	48	25	24
N.S.	1	1.00	1.00	0.96	0.92	1.19	1.85	0.96	0.92
time (sec)	N/A	0.019	0.007	0.012	0.192	0.337	2.211	0.318	0.431

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	48	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.66	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.051	0.000	0.051	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	48	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.64	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	0.051	0.000	0.056	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	48	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.60	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.053	0.000	0.055	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	118	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.141	0.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	142	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.199	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [117] had the largest ratio of [.4000000000000000022]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	8	0.125
2	A	1	1	1.00	8	0.125
3	A	1	1	1.00	6	0.167
4	A	1	1	1.00	4	0.250
5	A	1	1	1.00	8	0.125
6	A	1	1	1.00	8	0.125
7	A	1	1	1.00	8	0.125
8	A	2	2	1.00	10	0.200
9	A	2	2	1.00	10	0.200
10	A	2	2	1.00	8	0.250
11	A	2	2	1.00	6	0.333
12	A	2	2	1.00	10	0.200
13	A	2	2	1.00	10	0.200
14	A	2	2	1.00	10	0.200
15	A	3	2	1.00	10	0.200
16	A	3	2	1.00	10	0.200
17	A	3	2	1.00	8	0.250
18	A	3	2	1.00	6	0.333
19	A	2	2	1.00	10	0.200
20	A	3	2	1.00	10	0.200
21	A	3	2	1.00	10	0.200
22	A	2	2	1.00	10	0.200
23	A	2	2	1.00	10	0.200
24	A	2	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	1	1	1.00	6	0.167
26	A	2	2	1.00	10	0.200
27	A	2	2	1.00	10	0.200
28	A	2	2	1.00	10	0.200
29	A	3	3	1.00	10	0.300
30	A	3	3	1.00	10	0.300
31	A	3	3	1.00	8	0.375
32	A	2	2	1.00	6	0.333
33	A	2	2	1.00	10	0.200
34	A	3	3	1.00	10	0.300
35	A	3	3	1.00	10	0.300
36	A	4	3	1.00	10	0.300
37	A	4	3	1.00	10	0.300
38	A	4	3	1.00	8	0.375
39	A	3	2	1.00	6	0.333
40	A	2	2	1.00	10	0.200
41	A	4	3	1.00	10	0.300
42	A	4	3	1.00	10	0.300
43	A	1	1	1.00	14	0.071
44	A	1	1	1.00	14	0.071
45	A	1	1	1.00	12	0.083
46	A	2	1	1.00	10	0.100
47	A	1	1	1.00	14	0.071
48	A	1	1	1.00	14	0.071
49	A	1	1	1.00	14	0.071
50	A	2	2	1.00	16	0.125
51	A	2	2	1.00	16	0.125
52	A	2	2	1.00	14	0.143
53	A	3	2	1.00	12	0.167
54	A	2	2	1.00	16	0.125
55	A	2	2	1.00	16	0.125
56	A	2	2	1.00	16	0.125
57	A	3	2	1.00	16	0.125
58	A	3	2	1.00	16	0.125
59	A	3	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	4	2	1.00	12	0.167
61	A	2	2	1.00	16	0.125
62	A	3	2	1.00	16	0.125
63	A	3	2	1.00	16	0.125
64	A	3	2	1.00	16	0.125
65	A	2	2	1.00	16	0.125
66	A	2	2	1.00	16	0.125
67	A	2	2	1.00	14	0.143
68	A	2	2	1.00	12	0.167
69	A	2	2	1.00	16	0.125
70	A	2	2	1.00	16	0.125
71	A	2	2	1.00	16	0.125
72	A	2	2	1.00	16	0.125
73	A	3	3	1.00	16	0.188
74	A	3	3	1.00	16	0.188
75	A	3	3	1.00	14	0.214
76	A	3	3	1.00	12	0.250
77	A	2	2	1.00	16	0.125
78	A	3	3	1.00	16	0.188
79	A	3	3	1.00	16	0.188
80	A	3	3	1.00	16	0.188
81	A	4	3	1.00	16	0.188
82	A	4	3	1.00	16	0.188
83	A	4	3	1.00	14	0.214
84	A	4	3	1.00	12	0.250
85	A	2	2	1.00	16	0.125
86	A	4	3	1.00	16	0.188
87	A	4	3	1.00	16	0.188
88	A	4	3	1.00	16	0.188
89	A	1	1	1.00	18	0.056
90	A	1	1	1.00	18	0.056
91	A	1	1	1.00	18	0.056
92	A	1	1	1.00	18	0.056
93	A	1	1	1.00	18	0.056
94	A	1	1	1.00	18	0.056

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	20	0.100
96	A	2	2	1.00	20	0.100
97	A	2	2	1.00	20	0.100
98	A	2	2	1.00	20	0.100
99	A	2	2	1.00	20	0.100
100	A	2	2	1.00	20	0.100
101	A	2	2	1.00	20	0.100
102	A	2	2	1.00	20	0.100
103	A	2	2	1.00	20	0.100
104	A	2	2	1.00	20	0.100
105	A	2	2	0.96	20	0.100
106	A	2	2	1.00	20	0.100
107	A	3	3	1.00	20	0.150
108	A	3	3	1.00	20	0.150
109	A	3	3	1.00	20	0.150
110	A	3	3	1.00	20	0.150
111	A	3	3	0.97	20	0.150
112	A	3	3	1.00	20	0.150
113	A	4	4	1.00	14	0.286
114	A	4	4	1.00	14	0.286
115	A	4	4	1.00	14	0.286
116	A	4	4	1.00	12	0.333
117	A	4	4	1.00	10	0.400
118	A	2	2	1.00	14	0.143
119	A	4	4	1.00	14	0.286
120	A	4	4	1.00	14	0.286
121	A	5	4	1.00	14	0.286
122	A	5	4	1.00	14	0.286
123	A	5	4	1.00	12	0.333
124	A	5	4	1.00	10	0.400
125	A	2	2	1.00	14	0.143
126	A	5	4	1.00	14	0.286
127	A	5	4	1.00	14	0.286
128	A	3	3	1.00	14	0.214
129	A	3	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	3	1.00	12	0.250
131	A	3	3	1.00	10	0.300
132	A	2	2	1.00	14	0.143
133	A	3	3	1.00	14	0.214
134	A	3	3	1.00	14	0.214
135	A	4	4	1.00	14	0.286
136	A	4	4	1.00	14	0.286
137	A	4	4	1.00	12	0.333
138	A	4	4	1.00	10	0.400
139	A	2	2	1.00	14	0.143
140	A	4	4	1.00	14	0.286
141	A	4	4	1.00	14	0.286
142	A	5	4	1.00	14	0.286
143	A	5	4	1.00	14	0.286
144	A	5	4	1.00	12	0.333
145	A	5	4	1.00	10	0.400
146	A	2	2	1.00	14	0.143
147	A	5	4	1.00	14	0.286
148	A	5	4	1.00	14	0.286
149	A	1	1	1.00	22	0.045
150	A	3	2	1.00	18	0.111
151	A	2	2	1.00	18	0.111
152	A	1	1	1.00	16	0.062
153	A	2	2	1.00	18	0.111
154	A	3	3	1.00	18	0.167
155	A	4	3	1.00	18	0.167
156	A	3	2	1.00	16	0.125
157	A	2	2	1.00	16	0.125
158	A	1	1	1.00	14	0.071
159	A	3	3	1.00	16	0.188
160	A	4	4	1.00	16	0.250
161	A	5	4	1.00	16	0.250
162	A	5	4	1.00	14	0.286
163	A	4	4	1.00	14	0.286
164	A	3	3	1.00	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	4	4	1.00	14	0.286
166	A	5	4	1.00	14	0.286
167	A	2	2	1.00	18	0.111
168	A	2	2	1.00	16	0.125
169	A	2	2	1.00	14	0.143
170	A	2	2	1.00	12	0.167
171	A	2	2	1.00	16	0.125
172	A	2	2	1.00	16	0.125
173	A	2	2	1.00	16	0.125
174	A	2	2	1.00	16	0.125
175	A	2	2	1.00	16	0.125
176	A	2	2	1.00	14	0.143
177	A	2	2	1.00	12	0.167
178	A	2	2	1.00	10	0.200
179	A	2	2	1.00	14	0.143
180	A	2	2	1.00	14	0.143
181	A	2	2	1.00	14	0.143
182	A	2	2	1.00	14	0.143
183	A	2	2	1.00	20	0.100
184	A	2	2	1.00	18	0.111
185	A	2	2	1.00	16	0.125
186	A	2	2	1.00	14	0.143
187	A	2	2	1.00	18	0.111
188	A	2	2	1.00	18	0.111
189	A	2	2	1.00	18	0.111
190	A	2	2	1.00	18	0.111
191	A	2	2	1.00	18	0.111
192	A	3	3	1.00	20	0.150
193	A	4	3	1.00	27	0.111

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3 \log(cx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int x^3 \log(cx)\,dx = -\frac{x^4}{16} + \frac{1}{4}x^4\log(cx)$$

```
[Out] -1/16*x^4+1/4*x^4*ln(c*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2341\}$

$$\int x^3 \log(cx)\,dx = \frac{1}{4}x^4\log(cx) - \frac{x^4}{16}$$

```
[In] Int[x^3*Log[c*x],x]
```

```
[Out] -1/16*x^4 + (x^4*Log[c*x])/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^4}{16} + \frac{1}{4}x^4\log(cx)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^3 \log(cx)\, dx = -\frac{x^4}{16} + \frac{1}{4}x^4 \log(cx)$$

```
[In] Integrate[x^3*Log[c*x],x]
[Out] -1/16*x^4 + (x^4*Log[c*x])/4
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
norman	$-\frac{x^4}{16}+\frac{x^4 \ln(xc)}{4}$	16
risch	$-\frac{x^4}{16}+\frac{x^4 \ln(xc)}{4}$	16
parallelrisch	$-\frac{x^4}{16}+\frac{x^4 \ln(xc)}{4}$	16
parts	$-\frac{x^4}{16}+\frac{x^4 \ln(xc)}{4}$	16
derivativedivides	$\frac{\frac{x^4 c^4 \ln(xc)}{4}-\frac{x^4 c^4}{16}}{c^4}$	26
default	$\frac{\frac{x^4 c^4 \ln(xc)}{4}-\frac{x^4 c^4}{16}}{c^4}$	26

```
[In] int(x^3*ln(x*c),x,method=_RETURNVERBOSE)
[Out] -1/16*x^4+1/4*x^4*ln(x*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3 \log(cx)\, dx = \frac{1}{4}\, x^4 \log\left(cx\right) - \frac{1}{16}\, x^4$$

```
[In] integrate(x^3*log(c*x),x, algorithm="fricas")
[Out] 1/4*x^4*log(c*x) - 1/16*x^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int x^3 \log(cx)\, dx = \frac{x^4 \log{(cx)}}{4} - \frac{x^4}{16}$$

```
[In] integrate(x**3*ln(c*x),x)
[Out] x**4*log(c*x)/4 - x**4/16
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3 \log(cx)\, dx = \frac{1}{4}\, x^4 \log{(cx)} - \frac{1}{16}\, x^4$$

```
[In] integrate(x^3*log(c*x),x, algorithm="maxima")
[Out] 1/4*x^4*log(c*x) - 1/16*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^3 \log(cx)\, dx = \frac{1}{4}\, x^4 \log{(cx)} - \frac{1}{16}\, x^4$$

```
[In] integrate(x^3*log(c*x),x, algorithm="giac")
[Out] 1/4*x^4*log(c*x) - 1/16*x^4
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int x^3 \log(cx)\, dx = \frac{x^4 \left(\ln{(c\, x)} - \frac{1}{4}\right)}{4}$$

```
[In] int(x^3*log(c*x),x)
[Out] (x^4*(log(c*x) - 1/4))/4
```

3.2 $\int x^2 \log(cx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int x^2 \log(cx)\,dx = -\frac{x^3}{9} + \frac{1}{3}x^3\log(cx)$$

```
[Out] -1/9*x^3+1/3*x^3*ln(c*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2341\}$

$$\int x^2 \log(cx)\,dx = \frac{1}{3}x^3\log(cx) - \frac{x^3}{9}$$

```
[In] Int[x^2*Log[c*x],x]
```

```
[Out] -1/9*x^3 + (x^3*Log[c*x])/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^3}{9} + \frac{1}{3}x^3\log(cx)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^2 \log(cx) \, dx = -\frac{x^3}{9} + \frac{1}{3} x^3 \log(cx)$$

```
[In] Integrate[x^2*Log[c*x],x]
[Out] -1/9*x^3 + (x^3*Log[c*x])/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
norman	$-\frac{x^3}{9}+\frac{x^3\ln(xc)}{3}$	16
risch	$-\frac{x^3}{9}+\frac{x^3\ln(xc)}{3}$	16
parallelrisch	$-\frac{x^3}{9}+\frac{x^3\ln(xc)}{3}$	16
parts	$-\frac{x^3}{9}+\frac{x^3\ln(xc)}{3}$	16
derivativedivides	$\frac{\frac{x^3c^3\ln(xc)}{3}-\frac{x^3c^3}{9}}{c^3}$	26
default	$\frac{\frac{x^3c^3\ln(xc)}{3}-\frac{x^3c^3}{9}}{c^3}$	26

```
[In] int(x^2*ln(x*c),x,method=_RETURNVERBOSE)
[Out] -1/9*x^3+1/3*x^3*ln(x*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^2 \log(cx) \, dx = \frac{1}{3} \, x^3 \log\left(cx\right) - \frac{1}{9} \, x^3$$

```
[In] integrate(x^2*log(c*x),x, algorithm="fricas")
[Out] 1/3*x^3*log(c*x) - 1/9*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int x^2 \log(cx)\, dx = \frac{x^3 \log{(cx)}}{3} - \frac{x^3}{9}$$

```
[In] integrate(x**2*ln(c*x),x)
[Out] x**3*log(c*x)/3 - x**3/9
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^2 \log(cx)\, dx = \frac{1}{3}\, x^3 \log{(cx)} - \frac{1}{9}\, x^3$$

```
[In] integrate(x^2*log(c*x),x, algorithm="maxima")
[Out] 1/3*x^3*log(c*x) - 1/9*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^2 \log(cx)\, dx = \frac{1}{3}\, x^3 \log{(cx)} - \frac{1}{9}\, x^3$$

```
[In] integrate(x^2*log(c*x),x, algorithm="giac")
[Out] 1/3*x^3*log(c*x) - 1/9*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int x^2 \log(cx)\, dx = \frac{x^3\,\left(\ln{(c\,x)} - \frac{1}{3}\right)}{3}$$

```
[In] int(x^2*log(c*x),x)
[Out] (x^3*(log(c*x) - 1/3))/3
```

3.3 $\int x \log(cx) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 19

$$\int x \log(cx) \, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(cx)$$

```
[Out] -1/4*x^2+1/2*x^2*ln(c*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2341}

$$\int x \log(cx) \, dx = \frac{1}{2}x^2 \log(cx) - \frac{x^2}{4}$$

```
[In] Int[x*Log[c*x],x]
```

```
[Out] -1/4*x^2 + (x^2*Log[c*x])/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(cx)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x\log(cx)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2\log(cx)$$

```
[In] Integrate[x*Log[c*x],x]
[Out] -1/4*x^2 + (x^2*Log[c*x])/2
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
norman	$-\frac{x^2}{4}+\frac{x^2\ln(xc)}{2}$	16
risch	$-\frac{x^2}{4}+\frac{x^2\ln(xc)}{2}$	16
parallelrisch	$-\frac{x^2}{4}+\frac{x^2\ln(xc)}{2}$	16
parts	$-\frac{x^2}{4}+\frac{x^2\ln(xc)}{2}$	16
derivativedivides	$\frac{\frac{x^2c^2\ln(xc)}{2}-\frac{x^2c^2}{4}}{c^2}$	26
default	$\frac{\frac{x^2c^2\ln(xc)}{2}-\frac{x^2c^2}{4}}{c^2}$	26

```
[In] int(x*ln(x*c),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2+1/2*x^2*ln(x*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\log(cx)\,dx = \frac{1}{2}\,x^2\log\left(cx\right) - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(c*x),x, algorithm="fricas")
[Out] 1/2*x^2*log(c*x) - 1/4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int x\log(cx)\,dx = \frac{x^2\log{(cx)}}{2} - \frac{x^2}{4}$$

```
[In] integrate(x*ln(c*x),x)
[Out] x**2*log(c*x)/2 - x**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\log(cx)\,dx = \frac{1}{2}\,x^2\log{(cx)} - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(c*x),x, algorithm="maxima")
[Out] 1/2*x^2*log(c*x) - 1/4*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\log(cx)\,dx = \frac{1}{2}\,x^2\log{(cx)} - \frac{1}{4}\,x^2$$

```
[In] integrate(x*log(c*x),x, algorithm="giac")
[Out] 1/2*x^2*log(c*x) - 1/4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int x\log(cx)\,dx = \frac{x^2\,\left(\ln{(c\,x)} - \frac{1}{2}\right)}{2}$$

```
[In] int(x*log(c*x),x)
[Out] (x^2*(log(c*x) - 1/2))/2
```

3.4 $\int \log(cx)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 10

$$\int \log(cx)\,dx = -x + x\log(cx)$$

```
[Out] -x+x*ln(c*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{2332\}$

$$\int \log(cx)\,dx = x\log(cx) - x$$

```
[In] Int[Log[c*x],x]
```

```
[Out] -x + x*Log[c*x]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\text{integral} = -x + x\log(cx)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(cx)\,dx = -x + x\log(cx)$$

```
[In] Integrate[Log[c*x],x]
[Out] -x + x*Log[c*x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
norman	$-x + x\ln(xc)$	11
risch	$-x + x\ln(xc)$	11
parallelrisch	$-x + x\ln(xc)$	11
parts	$-x + x\ln(xc)$	11
derivativedivides	$\frac{xc\ln(xc)-xc}{c}$	17
default	$\frac{xc\ln(xc)-xc}{c}$	17

```
[In] int(ln(x*c),x,method=_RETURNVERBOSE)
[Out] -x+x*ln(x*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \log(cx)\,dx = x\log(cx) - x$$

```
[In] integrate(log(c*x),x, algorithm="fricas")
[Out] x*log(c*x) - x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \log(cx)\,dx = x\log(cx) - x$$

```
[In] integrate(ln(c*x),x)
[Out] x*log(c*x) - x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \log(cx)\,dx = \frac{cx\log(cx) - cx}{c}$$

```
[In] integrate(log(c*x),x, algorithm="maxima")
[Out] (c*x*log(c*x) - c*x)/c
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \log(cx)\,dx = \frac{cx\log(cx) - cx}{c}$$

```
[In] integrate(log(c*x),x, algorithm="giac")
[Out] (c*x*log(c*x) - c*x)/c
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \log(cx)\,dx = x\,(\ln(c\,x) - 1)$$

```
[In] int(log(c*x),x)
[Out] x*(log(c*x) - 1)
```

3.5 $\int \frac{\log(cx)}{x}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 10

$$\int \frac{\log(cx)}{x}\,dx = \frac{1}{2}\log^2(cx)$$

```
[Out] 1/2*ln(c*x)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2338}

$$\int \frac{\log(cx)}{x}\,dx = \frac{1}{2}\log^2(cx)$$

```
[In] Int[Log[c*x]/x,x]
```

```
[Out] Log[c*x]^2/2
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\log^2(cx)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\log(cx)}{x} \, dx = \frac{1}{2} \log^2(cx)$$

```
[In] Integrate[Log[c*x]/x,x]
[Out] Log[c*x]^2/2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\ln(xc)^2}{2}$	9
default	$\frac{\ln(xc)^2}{2}$	9
norman	$\frac{\ln(xc)^2}{2}$	9
risch	$\frac{\ln(xc)^2}{2}$	9
parts	$\ln(x)\ln(xc) - \frac{\ln(x)^2}{2}$	15

```
[In] int(ln(x*c)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x*c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(cx)}{x} \, dx = \frac{1}{2} \log(cx)^2$$

```
[In] integrate(log(c*x)/x,x, algorithm="fricas")
[Out] 1/2*log(c*x)^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\log(cx)}{x} \, dx = \frac{\log(cx)^2}{2}$$

```
[In] integrate(ln(c*x)/x,x)
[Out] log(c*x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(cx)}{x} \, dx = \frac{1}{2} \log(cx)^2$$

```
[In] integrate(log(c*x)/x,x, algorithm="maxima")
[Out] 1/2*log(c*x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(cx)}{x} \, dx = \frac{1}{2} \log(cx)^2$$

```
[In] integrate(log(c*x)/x,x, algorithm="giac")
[Out] 1/2*log(c*x)^2
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log(cx)}{x} \, dx = \frac{\ln(c\,x)^2}{2}$$

```
[In] int(log(c*x)/x,x)
[Out] log(c*x)^2/2
```

3.6 $\int \frac{\log(cx)}{x^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 15

$$\int \frac{\log(cx)}{x^2}\,dx = -\frac{1}{x} - \frac{\log(cx)}{x}$$

```
[Out] -1/x-ln(c*x)/x
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341}

$$\int \frac{\log(cx)}{x^2}\,dx = -\frac{\log(cx)}{x} - \frac{1}{x}$$

```
[In] Int[Log[c*x]/x^2,x]
```

```
[Out] -x^(-1) - Log[c*x]/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{x} - \frac{\log(cx)}{x}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log(cx)}{x^2} \, dx = -\frac{1}{x} - \frac{\log(cx)}{x}$$

```
[In] Integrate[Log[c*x]/x^2,x]
[Out] -x^(-1) - Log[c*x]/x
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result	size
norman	$\frac{-1-\ln(xc)}{x}$	13
parallelrisch	$\frac{-1-\ln(xc)}{x}$	13
risch	$-\frac{1}{x} - \frac{\ln(xc)}{x}$	16
parts	$-\frac{1}{x} - \frac{\ln(xc)}{x}$	16
derivativedivides	$c\left(-\frac{\ln(xc)}{xc} - \frac{1}{xc}\right)$	24
default	$c\left(-\frac{\ln(xc)}{xc} - \frac{1}{xc}\right)$	24

```
[In] int(ln(x*c)/x^2,x,method=_RETURNVERBOSE)
[Out] (-1-ln(x*c))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\log(cx)}{x^2} \, dx = -\frac{\log(cx) + 1}{x}$$

```
[In] integrate(log(c*x)/x^2,x, algorithm="fricas")
[Out] -(log(c*x) + 1)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{\log(cx)}{x^2} \, dx = -\frac{\log{(cx)}}{x} - \frac{1}{x}$$

```
[In] integrate(ln(c*x)/x**2,x)
[Out] -log(c*x)/x - 1/x
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log(cx)}{x^2} \, dx = -\frac{\log{(cx)}}{x} - \frac{1}{x}$$

```
[In] integrate(log(c*x)/x^2,x, algorithm="maxima")
[Out] -log(c*x)/x - 1/x
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\log(cx)}{x^2} \, dx = -\frac{\log{(cx)}}{x} - \frac{1}{x}$$

```
[In] integrate(log(c*x)/x^2,x, algorithm="giac")
[Out] -log(c*x)/x - 1/x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\log(cx)}{x^2} \, dx = -\frac{\ln{(c\,x)}+1}{x}$$

```
[In] int(log(c*x)/x^2,x)
[Out] -(log(c*x) + 1)/x
```

3.7 $\int \frac{\log(cx)}{x^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int \frac{\log(cx)}{x^3} dx = -\frac{1}{4x^2} - \frac{\log(cx)}{2x^2}$$

```
[Out] -1/4/x^2-1/2*ln(c*x)/x^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2341}

$$\int \frac{\log(cx)}{x^3} dx = -\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

```
[In] Int[Log[c*x]/x^3,x]
```

```
[Out] -1/4*1/x^2 - Log[c*x]/(2*x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{1}{4x^2} - \frac{\log(cx)}{2x^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\log(cx)}{x^3}\,dx = -\frac{1}{4x^2} - \frac{\log(cx)}{2x^2}$$

```
[In] Integrate[Log[c*x]/x^3,x]
[Out] -1/4*1/x^2 - Log[c*x]/(2*x^2)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

method	result	size
norman	$\frac{-\frac{1}{4}-\frac{\ln(xc)}{2}}{x^2}$	13
parallelrisch	$\frac{-1-2\ln(xc)}{4x^2}$	14
risch	$-\frac{1}{4x^2}-\frac{\ln(xc)}{2x^2}$	16
parts	$-\frac{1}{4x^2}-\frac{\ln(xc)}{2x^2}$	16
derivativedivides	$c^2\left(-\frac{\ln(xc)}{2x^2c^2}-\frac{1}{4x^2c^2}\right)$	26
default	$c^2\left(-\frac{\ln(xc)}{2x^2c^2}-\frac{1}{4x^2c^2}\right)$	26

```
[In] int(ln(x*c)/x^3,x,method=_RETURNVERBOSE)
[Out] (-1/4-1/2*ln(x*c))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{\log(cx)}{x^3}\,dx = -\frac{2\,\log\left(cx\right)+1}{4\,x^2}$$

```
[In] integrate(log(c*x)/x^3,x, algorithm="fricas")
[Out] -1/4*(2*log(c*x) + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\log(cx)}{x^3}\,dx = -\frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

```
[In] integrate(ln(c*x)/x**3,x)
[Out] -log(c*x)/(2*x**2) - 1/(4*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\log(cx)}{x^3}\,dx = -\frac{\log(cx)}{2\,x^2} - \frac{1}{4\,x^2}$$

```
[In] integrate(log(c*x)/x^3,x, algorithm="maxima")
[Out] -1/2*log(c*x)/x^2 - 1/4/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\log(cx)}{x^3}\,dx = -\frac{\log(cx)}{2\,x^2} - \frac{1}{4\,x^2}$$

```
[In] integrate(log(c*x)/x^3,x, algorithm="giac")
[Out] -1/2*log(c*x)/x^2 - 1/4/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\log(cx)}{x^3}\,dx = -\frac{\ln(c\,x) + \frac{1}{2}}{2\,x^2}$$

```
[In] int(log(c*x)/x^3,x)
[Out] -(log(c*x) + 1/2)/(2*x^2)
```

3.8 $\int x^3 \log^2(cx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int x^3 \log^2(cx)\,dx = \frac{x^4}{32} - \frac{1}{8}x^4\log(cx) + \frac{1}{4}x^4\log^2(cx)$$

```
[Out] 1/32*x^4-1/8*x^4*ln(c*x)+1/4*x^4*ln(c*x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2342, 2341}

$$\int x^3 \log^2(cx)\,dx = \frac{1}{4}x^4\log^2(cx) - \frac{1}{8}x^4\log(cx) + \frac{x^4}{32}$$

```
[In] Int[x^3*Log[c*x]^2,x]
```

```
[Out] x^4/32 - (x^4*Log[c*x])/8 + (x^4*Log[c*x]^2)/4
```

<u>Rule 2341</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

<u>Rule 2342</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x^4\log^2(cx) - \frac{1}{2}\int x^3\log(cx)\,dx$$
$$= \frac{x^4}{32} - \frac{1}{8}x^4\log(cx) + \frac{1}{4}x^4\log^2(cx)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int x^3\log^2(cx)\,dx = \frac{x^4}{32} - \frac{1}{8}x^4\log(cx) + \frac{1}{4}x^4\log^2(cx)$$

```
[In] Integrate[x^3*Log[c*x]^2,x]
[Out] x^4/32 - (x^4*Log[c*x])/8 + (x^4*Log[c*x]^2)/4
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
norman	$\frac{x^4}{32}-\frac{x^4\ln(xc)}{8}+\frac{x^4\ln(xc)^2}{4}$	27
risch	$\frac{x^4}{32}-\frac{x^4\ln(xc)}{8}+\frac{x^4\ln(xc)^2}{4}$	27
parallelrisch	$\frac{x^4}{32}-\frac{x^4\ln(xc)}{8}+\frac{x^4\ln(xc)^2}{4}$	27
parts	$\frac{x^4\ln(xc)^2}{4}-\frac{\frac{x^4c^4\ln(xc)}{4}-\frac{x^4c^4}{16}}{2c^4}$	39
derivativedivides	$\frac{\frac{x^4c^4\ln(xc)^2}{4}-\frac{x^4c^4\ln(xc)}{8}+\frac{x^4c^4}{32}}{c^4}$	40
default	$\frac{\frac{x^4c^4\ln(xc)^2}{4}-\frac{x^4c^4\ln(xc)}{8}+\frac{x^4c^4}{32}}{c^4}$	40

```
[In] int(x^3*ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] 1/32*x^4-1/8*x^4*ln(x*c)+1/4*x^4*ln(x*c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^3 \log^2(cx) \, dx = \frac{1}{4} x^4 \log(cx)^2 - \frac{1}{8} x^4 \log(cx) + \frac{1}{32} x^4$$

```
[In] integrate(x^3*log(c*x)^2,x, algorithm="fricas")
[Out] 1/4*x^4*log(c*x)^2 - 1/8*x^4*log(c*x) + 1/32*x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^3 \log^2(cx) \, dx = \frac{x^4 \log(cx)^2}{4} - \frac{x^4 \log(cx)}{8} + \frac{x^4}{32}$$

```
[In] integrate(x**3*ln(c*x)**2,x)
[Out] x**4*log(c*x)**2/4 - x**4*log(c*x)/8 + x**4/32
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int x^3 \log^2(cx) \, dx = \frac{1}{32} \left(8 \log(cx)^2 - 4 \log(cx) + 1\right) x^4$$

```
[In] integrate(x^3*log(c*x)^2,x, algorithm="maxima")
[Out] 1/32*(8*log(c*x)^2 - 4*log(c*x) + 1)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^3 \log^2(cx) \, dx = \frac{1}{4} x^4 \log(cx)^2 - \frac{1}{8} x^4 \log(cx) + \frac{1}{32} x^4$$

```
[In] integrate(x^3*log(c*x)^2,x, algorithm="giac")
[Out] 1/4*x^4*log(c*x)^2 - 1/8*x^4*log(c*x) + 1/32*x^4
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int x^3 \log^2(cx)\, dx = \frac{x^4 \left(8 \ln\left(c\, x\right)^2 - 4\, \ln\left(c\, x\right) + 1\right)}{32}$$

```
[In] int(x^3*log(c*x)^2,x)
[Out] (x^4*(8*log(c*x)^2 - 4*log(c*x) + 1))/32
```

3.9 $\int x^2 \log^2(cx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int x^2 \log^2(cx)\,dx = \frac{2x^3}{27} - \frac{2}{9}x^3\log(cx) + \frac{1}{3}x^3\log^2(cx)$$

```
[Out] 2/27*x^3-2/9*x^3*ln(c*x)+1/3*x^3*ln(c*x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2342, 2341}

$$\int x^2 \log^2(cx)\,dx = \frac{1}{3}x^3\log^2(cx) - \frac{2}{9}x^3\log(cx) + \frac{2x^3}{27}$$

```
[In] Int[x^2*Log[c*x]^2,x]
```

```
[Out] (2*x^3)/27 - (2*x^3*Log[c*x])/9 + (x^3*Log[c*x]^2)/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x^3\log^2(cx) - \frac{2}{3}\int x^2\log(cx)\,dx$$
$$= \frac{2x^3}{27} - \frac{2}{9}x^3\log(cx) + \frac{1}{3}x^3\log^2(cx)$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int x^2\log^2(cx)\,dx = \frac{2x^3}{27} - \frac{2}{9}x^3\log(cx) + \frac{1}{3}x^3\log^2(cx)$$

```
[In] Integrate[x^2*Log[c*x]^2,x]
[Out] (2*x^3)/27 - (2*x^3*Log[c*x])/9 + (x^3*Log[c*x]^2)/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
norman	$\frac{2x^3}{27} - \frac{2x^3\ln(xc)}{9} + \frac{x^3\ln(xc)^2}{3}$	27
risch	$\frac{2x^3}{27} - \frac{2x^3\ln(xc)}{9} + \frac{x^3\ln(xc)^2}{3}$	27
parallelrisch	$\frac{2x^3}{27} - \frac{2x^3\ln(xc)}{9} + \frac{x^3\ln(xc)^2}{3}$	27
parts	$\frac{x^3\ln(xc)^2}{3} - \frac{2\left(\frac{x^3c^3\ln(xc)}{3} - \frac{x^3c^3}{9}\right)}{3c^3}$	39
derivativedivides	$\frac{\frac{x^3c^3\ln(xc)^2}{3} - \frac{2x^3c^3\ln(xc)}{9} + \frac{2x^3c^3}{27}}{c^3}$	40
default	$\frac{\frac{x^3c^3\ln(xc)^2}{3} - \frac{2x^3c^3\ln(xc)}{9} + \frac{2x^3c^3}{27}}{c^3}$	40

```
[In] int(x^2*ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] 2/27*x^3-2/9*x^3*ln(x*c)+1/3*x^3*ln(x*c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^2 \log^2(cx)\,dx = \frac{1}{3}\,x^3\log\left(cx\right)^2 - \frac{2}{9}\,x^3\log\left(cx\right) + \frac{2}{27}\,x^3$$

```
[In] integrate(x^2*log(c*x)^2,x, algorithm="fricas")
[Out] 1/3*x^3*log(c*x)^2 - 2/9*x^3*log(c*x) + 2/27*x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int x^2 \log^2(cx)\,dx = \frac{x^3\log\left(cx\right)^2}{3} - \frac{2x^3\log\left(cx\right)}{9} + \frac{2x^3}{27}$$

```
[In] integrate(x**2*ln(c*x)**2,x)
[Out] x**3*log(c*x)**2/3 - 2*x**3*log(c*x)/9 + 2*x**3/27
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int x^2 \log^2(cx)\,dx = \frac{1}{27}\left(9\,\log\left(cx\right)^2 - 6\,\log\left(cx\right) + 2\right)x^3$$

```
[In] integrate(x^2*log(c*x)^2,x, algorithm="maxima")
[Out] 1/27*(9*log(c*x)^2 - 6*log(c*x) + 2)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x^2 \log^2(cx)\,dx = \frac{1}{3}\,x^3\log\left(cx\right)^2 - \frac{2}{9}\,x^3\log\left(cx\right) + \frac{2}{27}\,x^3$$

```
[In] integrate(x^2*log(c*x)^2,x, algorithm="giac")
[Out] 1/3*x^3*log(c*x)^2 - 2/9*x^3*log(c*x) + 2/27*x^3
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int x^2 \log^2(cx)\, dx = \frac{x^3 \left(9 \ln\left(c\, x\right)^2 - 6\, \ln\left(c\, x\right) + 2\right)}{27}$$

```
[In] int(x^2*log(c*x)^2,x)
[Out] (x^3*(9*log(c*x)^2 - 6*log(c*x) + 2))/27
```

3.10 $\int x \log^2(cx) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int x \log^2(cx) \, dx = \frac{x^2}{4} - \frac{1}{2}x^2 \log(cx) + \frac{1}{2}x^2 \log^2(cx)$$

```
[Out] 1/4*x^2-1/2*x^2*ln(c*x)+1/2*x^2*ln(c*x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2342, 2341}

$$\int x \log^2(cx) \, dx = \frac{1}{2}x^2 \log^2(cx) - \frac{1}{2}x^2 \log(cx) + \frac{x^2}{4}$$

```
[In] Int[x*Log[c*x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[c*x])/2 + (x^2*Log[c*x]^2)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log^2(cx) - \int x\log(cx)\,dx \\
&= \frac{x^2}{4} - \frac{1}{2}x^2\log(cx) + \frac{1}{2}x^2\log^2(cx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int x\log^2(cx)\,dx = \frac{x^2}{4} - \frac{1}{2}x^2\log(cx) + \frac{1}{2}x^2\log^2(cx)$$

```
[In] Integrate[x*Log[c*x]^2,x]
[Out] x^2/4 - (x^2*Log[c*x])/2 + (x^2*Log[c*x]^2)/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
norman	$\frac{x^2}{4}-\frac{x^2\ln(xc)}{2}+\frac{x^2\ln(xc)^2}{2}$	27
risch	$\frac{x^2}{4}-\frac{x^2\ln(xc)}{2}+\frac{x^2\ln(xc)^2}{2}$	27
parallelrisch	$\frac{x^2}{4}-\frac{x^2\ln(xc)}{2}+\frac{x^2\ln(xc)^2}{2}$	27
parts	$\frac{x^2\ln(xc)^2}{2}-\frac{\frac{x^2c^2\ln(xc)}{2}-\frac{x^2c^2}{4}}{c^2}$	39
derivativedivides	$\frac{\frac{x^2c^2\ln(xc)^2}{2}-\frac{x^2c^2\ln(xc)}{2}+\frac{x^2c^2}{4}}{c^2}$	40
default	$\frac{\frac{x^2c^2\ln(xc)^2}{2}-\frac{x^2c^2\ln(xc)}{2}+\frac{x^2c^2}{4}}{c^2}$	40

```
[In] int(x*ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^2-1/2*x^2*ln(x*c)+1/2*x^2*ln(x*c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x \log^2(cx) \, dx = \frac{1}{2} x^2 \log(cx)^2 - \frac{1}{2} x^2 \log(cx) + \frac{1}{4} x^2$$

```
[In] integrate(x*log(c*x)^2,x, algorithm="fricas")
[Out] 1/2*x^2*log(c*x)^2 - 1/2*x^2*log(c*x) + 1/4*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x \log^2(cx) \, dx = \frac{x^2 \log(cx)^2}{2} - \frac{x^2 \log(cx)}{2} + \frac{x^2}{4}$$

```
[In] integrate(x*ln(c*x)**2,x)
[Out] x**2*log(c*x)**2/2 - x**2*log(c*x)/2 + x**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int x \log^2(cx) \, dx = \frac{1}{4} \left(2 \log(cx)^2 - 2 \log(cx) + 1\right) x^2$$

```
[In] integrate(x*log(c*x)^2,x, algorithm="maxima")
[Out] 1/4*(2*log(c*x)^2 - 2*log(c*x) + 1)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int x \log^2(cx) \, dx = \frac{1}{2} x^2 \log(cx)^2 - \frac{1}{2} x^2 \log(cx) + \frac{1}{4} x^2$$

```
[In] integrate(x*log(c*x)^2,x, algorithm="giac")
[Out] 1/2*x^2*log(c*x)^2 - 1/2*x^2*log(c*x) + 1/4*x^2
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int x \log^2(cx) \, dx = \frac{x^2 \left(2 \ln\left(c\, x\right)^2 - 2\, \ln\left(c\, x\right) + 1\right)}{4}$$

```
[In] int(x*log(c*x)^2,x)
[Out] (x^2*(2*log(c*x)^2 - 2*log(c*x) + 1))/4
```

3.11 $\int \log^2(cx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 19

$$\int \log^2(cx)\,dx = 2x - 2x\log(cx) + x\log^2(cx)$$

```
[Out] 2*x-2*x*ln(c*x)+x*ln(c*x)^2
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2333, 2332}

$$\int \log^2(cx)\,dx = x\log^2(cx) - 2x\log(cx) + 2x$$

```
[In] Int[Log[c*x]^2,x]
```

```
[Out] 2*x - 2*x*Log[c*x] + x*Log[c*x]^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= x\log^2(cx) - 2\int \log(cx)\,dx \\ &= 2x - 2x\log(cx) + x\log^2(cx) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \log^2(cx)\,dx = 2x - 2x\log(cx) + x\log^2(cx)$$

```
[In] Integrate[Log[c*x]^2,x]
[Out] 2*x - 2*x*Log[c*x] + x*Log[c*x]^2
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
norman	$2x-2x\ln(xc)+x\ln(xc)^2$	20
risch	$2x-2x\ln(xc)+x\ln(xc)^2$	20
parallelrisch	$2x-2x\ln(xc)+x\ln(xc)^2$	20
derivativedivides	$\frac{xc\ln(xc)^2-2xc\ln(xc)+2xc}{c}$	27
default	$\frac{xc\ln(xc)^2-2xc\ln(xc)+2xc}{c}$	27

```
[In] int(ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] 2*x-2*x*ln(x*c)+x*ln(x*c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \log^2(cx)\,dx = x\log(cx)^2 - 2x\log(cx) + 2x$$

```
[In] integrate(log(c*x)^2,x, algorithm="fricas")
[Out] x*log(c*x)^2 - 2*x*log(c*x) + 2*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \log^2(cx)\,dx = x\log(cx)^2 - 2x\log(cx) + 2x$$

```
[In] integrate(ln(c*x)**2,x)
[Out] x*log(c*x)**2 - 2*x*log(c*x) + 2*x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \log^2(cx)\,dx = \left(\log(cx)^2 - 2\,\log(cx) + 2\right)x$$

```
[In] integrate(log(c*x)^2,x, algorithm="maxima")
[Out] (log(c*x)^2 - 2*log(c*x) + 2)*x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \log^2(cx)\,dx = x\log(cx)^2 - 2\,x\log(cx) + 2\,x$$

```
[In] integrate(log(c*x)^2,x, algorithm="giac")
[Out] x*log(c*x)^2 - 2*x*log(c*x) + 2*x
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \log^2(cx)\,dx = x\left(\ln(c\,x)^2 - 2\,\ln(c\,x) + 2\right)$$

```
[In] int(log(c*x)^2,x)
[Out] x*(log(c*x)^2 - 2*log(c*x) + 2)
```

3.12 $\int \frac{\log^2(cx)}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\log^2(cx)}{x}\,dx = \frac{1}{3}\log^3(cx)$$

```
[Out] 1/3*ln(c*x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2339, 30}

$$\int \frac{\log^2(cx)}{x}\,dx = \frac{1}{3}\log^3(cx)$$

```
[In] Int[Log[c*x]^2/x,x]
```

```
[Out] Log[c*x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^2\,dx, x, \log(cx)\right)\\ &= \frac{1}{3}\log^3(cx)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(cx)}{x}\,dx = \frac{1}{3}\log^3(cx)$$

```
[In] Integrate[Log[c*x]^2/x,x]
[Out] Log[c*x]^3/3
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\ln(xc)^3}{3}$	9
default	$\frac{\ln(xc)^3}{3}$	9
norman	$\frac{\ln(xc)^3}{3}$	9
risch	$\frac{\ln(xc)^3}{3}$	9
parts	$\ln(xc)^2\ln(x) - \ln(xc)\ln(x)^2 + \frac{\ln(x)^3}{3}$	27

```
[In] int(ln(x*c)^2/x,x,method=_RETURNVERBOSE)
[Out] 1/3*ln(x*c)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log^2(cx)}{x}\,dx = \frac{1}{3}\,\log\left(cx\right)^3$$

```
[In] integrate(log(c*x)^2/x,x, algorithm="fricas")
[Out] 1/3*log(c*x)^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\log^2(cx)}{x}\,dx = \frac{\log\left(cx\right)^3}{3}$$

```
[In] integrate(ln(c*x)**2/x,x)
[Out] log(c*x)**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log^2(cx)}{x}\,dx = \frac{1}{3}\,\log\left(cx\right)^3$$

```
[In] integrate(log(c*x)^2/x,x, algorithm="maxima")
[Out] 1/3*log(c*x)^3
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log^2(cx)}{x}\,dx = \frac{1}{3}\,\log\left(cx\right)^3$$

```
[In] integrate(log(c*x)^2/x,x, algorithm="giac")
[Out] 1/3*log(c*x)^3
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log^2(cx)}{x}\,dx = \frac{\ln\left(c\,x\right)^3}{3}$$

```
[In] int(log(c*x)^2/x,x)
[Out] log(c*x)^3/3
```

3.13 $\int \frac{\log^2(cx)}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \frac{\log^2(cx)}{x^2}\,dx = -\frac{2}{x} - \frac{2\log(cx)}{x} - \frac{\log^2(cx)}{x}$$

```
[Out] -2/x-2*ln(c*x)/x-ln(c*x)^2/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2342, 2341}

$$\int \frac{\log^2(cx)}{x^2}\,dx = -\frac{\log^2(cx)}{x} - \frac{2\log(cx)}{x} - \frac{2}{x}$$

```
[In] Int[Log[c*x]^2/x^2,x]
```

```
[Out] -2/x - (2*Log[c*x])/x - Log[c*x]^2/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log^2(cx)}{x} + 2\int \frac{\log(cx)}{x^2}\,dx \\ &= -\frac{2}{x} - \frac{2\log(cx)}{x} - \frac{\log^2(cx)}{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(cx)}{x^2}\,dx = -\frac{2}{x} - \frac{2\log(cx)}{x} - \frac{\log^2(cx)}{x}$$

```
[In] Integrate[Log[c*x]^2/x^2,x]
[Out] -2/x - (2*Log[c*x])/x - Log[c*x]^2/x
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
norman	$\frac{-2-\ln(xc)^2-2\ln(xc)}{x}$	21
parallelrisch	$\frac{-2-\ln(xc)^2-2\ln(xc)}{x}$	21
risch	$-\frac{2}{x}-\frac{2\ln(xc)}{x}-\frac{\ln(xc)^2}{x}$	27
parts	$-\frac{\ln(xc)^2}{x}+2c\left(-\frac{\ln(xc)}{xc}-\frac{1}{xc}\right)$	37
derivativedivides	$c\left(-\frac{\ln(xc)^2}{xc}-\frac{2\ln(xc)}{xc}-\frac{2}{xc}\right)$	38
default	$c\left(-\frac{\ln(xc)^2}{xc}-\frac{2\ln(xc)}{xc}-\frac{2}{xc}\right)$	38

```
[In] int(ln(x*c)^2/x^2,x,method=_RETURNVERBOSE)
[Out] (-2-ln(x*c)^2-2*ln(x*c))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{\log^2(cx)}{x^2}\,dx = -\frac{\log(cx)^2 + 2\,\log(cx) + 2}{x}$$

```
[In] integrate(log(c*x)^2/x^2,x, algorithm="fricas")
[Out] -(log(c*x)^2 + 2*log(c*x) + 2)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{\log^2(cx)}{x^2}\,dx = -\frac{\log(cx)^2}{x} - \frac{2\log(cx)}{x} - \frac{2}{x}$$

```
[In] integrate(ln(c*x)**2/x**2,x)
[Out] -log(c*x)**2/x - 2*log(c*x)/x - 2/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{\log^2(cx)}{x^2}\,dx = -\frac{\log(cx)^2 + 2\,\log(cx) + 2}{x}$$

```
[In] integrate(log(c*x)^2/x^2,x, algorithm="maxima")
[Out] -(log(c*x)^2 + 2*log(c*x) + 2)/x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(cx)}{x^2}\,dx = -\frac{\log(cx)^2}{x} - \frac{2\,\log(cx)}{x} - \frac{2}{x}$$

```
[In] integrate(log(c*x)^2/x^2,x, algorithm="giac")
[Out] -log(c*x)^2/x - 2*log(c*x)/x - 2/x
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{\log^2(cx)}{x^2}\,dx = -\frac{\ln(c\,x)^2 + 2\,\ln(c\,x) + 2}{x}$$

```
[In] int(log(c*x)^2/x^2,x)
[Out] -(2*log(c*x) + log(c*x)^2 + 2)/x
```

3.14 $\int \frac{\log^2(cx)}{x^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int \frac{\log^2(cx)}{x^3} \, dx = -\frac{1}{4x^2} - \frac{\log(cx)}{2x^2} - \frac{\log^2(cx)}{2x^2}$$

```
[Out] -1/4/x^2-1/2*ln(c*x)/x^2-1/2*ln(c*x)^2/x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2342, 2341}

$$\int \frac{\log^2(cx)}{x^3} \, dx = -\frac{\log^2(cx)}{2x^2} - \frac{\log(cx)}{2x^2} - \frac{1}{4x^2}$$

```
[In] Int[Log[c*x]^2/x^3,x]
```

```
[Out] -1/4*1/x^2 - Log[c*x]/(2*x^2) - Log[c*x]^2/(2*x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{\log^2(cx)}{2x^2} + \int \frac{\log(cx)}{x^3}\,dx$$
$$= -\frac{1}{4x^2} - \frac{\log(cx)}{2x^2} - \frac{\log^2(cx)}{2x^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\log^2(cx)}{x^3}\,dx = -\frac{1}{4x^2} - \frac{\log(cx)}{2x^2} - \frac{\log^2(cx)}{2x^2}$$

```
[In] Integrate[Log[c*x]^2/x^3,x]
[Out] -1/4*1/x^2 - Log[c*x]/(2*x^2) - Log[c*x]^2/(2*x^2)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
norman	$\frac{-\frac{1}{4}-\frac{\ln(xc)^2}{2}-\frac{\ln(xc)}{2}}{x^2}$	21
parallelrisch	$\frac{-1-2\ln(xc)^2-2\ln(xc)}{4x^2}$	22
risch	$-\frac{1}{4x^2}-\frac{\ln(xc)}{2x^2}-\frac{\ln(xc)^2}{2x^2}$	27
parts	$-\frac{\ln(xc)^2}{2x^2}+c^2\left(-\frac{\ln(xc)}{2x^2c^2}-\frac{1}{4x^2c^2}\right)$	38
derivativedivides	$c^2\left(-\frac{\ln(xc)^2}{2x^2c^2}-\frac{\ln(xc)}{2x^2c^2}-\frac{1}{4x^2c^2}\right)$	40
default	$c^2\left(-\frac{\ln(xc)^2}{2x^2c^2}-\frac{\ln(xc)}{2x^2c^2}-\frac{1}{4x^2c^2}\right)$	40

```
[In] int(ln(x*c)^2/x^3,x,method=_RETURNVERBOSE)
[Out] (-1/4-1/2*ln(x*c)^2-1/2*ln(x*c))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{\log^2(cx)}{x^3} \, dx = -\frac{2 \log{(cx)}^2 + 2 \log{(cx)} + 1}{4 \, x^2}$$

```
[In] integrate(log(c*x)^2/x^3,x, algorithm="fricas")
[Out] -1/4*(2*log(c*x)^2 + 2*log(c*x) + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{\log^2(cx)}{x^3} \, dx = -\frac{\log{(cx)}^2}{2x^2} - \frac{\log{(cx)}}{2x^2} - \frac{1}{4x^2}$$

```
[In] integrate(ln(c*x)**2/x**3,x)
[Out] -log(c*x)**2/(2*x**2) - log(c*x)/(2*x**2) - 1/(4*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{\log^2(cx)}{x^3} \, dx = -\frac{2 \log{(cx)}^2 + 2 \log{(cx)} + 1}{4 \, x^2}$$

```
[In] integrate(log(c*x)^2/x^3,x, algorithm="maxima")
[Out] -1/4*(2*log(c*x)^2 + 2*log(c*x) + 1)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{\log^2(cx)}{x^3} \, dx = -\frac{\log{(cx)}^2}{2 \, x^2} - \frac{\log{(cx)}}{2 \, x^2} - \frac{1}{4 \, x^2}$$

```
[In] integrate(log(c*x)^2/x^3,x, algorithm="giac")
[Out] -1/2*log(c*x)^2/x^2 - 1/2*log(c*x)/x^2 - 1/4/x^2
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{\log^2(cx)}{x^3}\,dx = -\frac{\frac{\ln(c\,x)^2}{2}+\frac{\ln(c\,x)}{2}+\frac{1}{4}}{x^2}$$

[In] `int(log(c*x)^2/x^3,x)`

[Out] `-(log(c*x)/2 + log(c*x)^2/2 + 1/4)/x^2`

3.15 $\int x^3 \log^3(cx) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int x^3 \log^3(cx) \, dx = -\frac{3x^4}{128} + \frac{3}{32} x^4 \log(cx) - \frac{3}{16} x^4 \log^2(cx) + \frac{1}{4} x^4 \log^3(cx)$$

```
[Out] -3/128*x^4+3/32*x^4*ln(c*x)-3/16*x^4*ln(c*x)^2+1/4*x^4*ln(c*x)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2342, 2341}

$$\int x^3 \log^3(cx) \, dx = \frac{1}{4} x^4 \log^3(cx) - \frac{3}{16} x^4 \log^2(cx) + \frac{3}{32} x^4 \log(cx) - \frac{3x^4}{128}$$

```
[In] Int[x^3*Log[c*x]^3,x]
```

```
[Out] (-3*x^4)/128 + (3*x^4*Log[c*x])/32 - (3*x^4*Log[c*x]^2)/16 + (x^4*Log[c*x]^
3)/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^4\log^3(cx) - \frac{3}{4}\int x^3\log^2(cx)\,dx \\ &= -\frac{3}{16}x^4\log^2(cx) + \frac{1}{4}x^4\log^3(cx) + \frac{3}{8}\int x^3\log(cx)\,dx \\ &= -\frac{3x^4}{128} + \frac{3}{32}x^4\log(cx) - \frac{3}{16}x^4\log^2(cx) + \frac{1}{4}x^4\log^3(cx)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int x^3\log^3(cx)\,dx = -\frac{3x^4}{128} + \frac{3}{32}x^4\log(cx) - \frac{3}{16}x^4\log^2(cx) + \frac{1}{4}x^4\log^3(cx)$$

```
[In] Integrate[x^3*Log[c*x]^3,x]
[Out] (-3*x^4)/128 + (3*x^4*Log[c*x])/32 - (3*x^4*Log[c*x]^2)/16 + (x^4*Log[c*x]^
3)/4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
norman	$-\frac{3x^4}{128}+\frac{3x^4\ln(xc)}{32}-\frac{3x^4\ln(xc)^2}{16}+\frac{x^4\ln(xc)^3}{4}$	38
risch	$-\frac{3x^4}{128}+\frac{3x^4\ln(xc)}{32}-\frac{3x^4\ln(xc)^2}{16}+\frac{x^4\ln(xc)^3}{4}$	38
parallelrisch	$-\frac{3x^4}{128}+\frac{3x^4\ln(xc)}{32}-\frac{3x^4\ln(xc)^2}{16}+\frac{x^4\ln(xc)^3}{4}$	38
parts	$\frac{x^4\ln(xc)^3}{4}-\frac{3\left(\frac{x^4c^4\ln(xc)^2}{4}-\frac{x^4c^4\ln(xc)}{8}+\frac{x^4c^4}{32}\right)}{4c^4}$	53
derivativedivides	$\frac{\frac{x^4c^4\ln(xc)^3}{4}-\frac{3x^4c^4\ln(xc)^2}{16}+\frac{3x^4c^4\ln(xc)}{32}-\frac{3x^4c^4}{128}}{c^4}$	54
default	$\frac{\frac{x^4c^4\ln(xc)^3}{4}-\frac{3x^4c^4\ln(xc)^2}{16}+\frac{3x^4c^4\ln(xc)}{32}-\frac{3x^4c^4}{128}}{c^4}$	54

```
[In] int(x^3*ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] -3/128*x^4+3/32*x^4*ln(x*c)-3/16*x^4*ln(x*c)^2+1/4*x^4*ln(x*c)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x^3 \log^3(cx)\,dx = \frac{1}{4}\,x^4 \log\left(cx\right)^3 - \frac{3}{16}\,x^4 \log\left(cx\right)^2 + \frac{3}{32}\,x^4 \log\left(cx\right) - \frac{3}{128}\,x^4$$

```
[In] integrate(x^3*log(c*x)^3,x, algorithm="fricas")
[Out] 1/4*x^4*log(c*x)^3 - 3/16*x^4*log(c*x)^2 + 3/32*x^4*log(c*x) - 3/128*x^4
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^3 \log^3(cx)\,dx = \frac{x^4 \log\left(cx\right)^3}{4} - \frac{3x^4 \log\left(cx\right)^2}{16} + \frac{3x^4 \log\left(cx\right)}{32} - \frac{3x^4}{128}$$

```
[In] integrate(x**3*ln(c*x)**3,x)
[Out] x**4*log(c*x)**3/4 - 3*x**4*log(c*x)**2/16 + 3*x**4*log(c*x)/32 - 3*x**4/12
8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int x^3 \log^3(cx)\,dx = \frac{1}{128}\left(32\,\log\left(cx\right)^3 - 24\,\log\left(cx\right)^2 + 12\,\log\left(cx\right) - 3\right)x^4$$

```
[In] integrate(x^3*log(c*x)^3,x, algorithm="maxima")
[Out] 1/128*(32*log(c*x)^3 - 24*log(c*x)^2 + 12*log(c*x) - 3)*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x^3 \log^3(cx)\,dx = \frac{1}{4}\,x^4 \log\left(cx\right)^3 - \frac{3}{16}\,x^4 \log\left(cx\right)^2 + \frac{3}{32}\,x^4 \log\left(cx\right) - \frac{3}{128}\,x^4$$

```
[In] integrate(x^3*log(c*x)^3,x, algorithm="giac")
[Out] 1/4*x^4*log(c*x)^3 - 3/16*x^4*log(c*x)^2 + 3/32*x^4*log(c*x) - 3/128*x^4
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int x^3 \log^3(cx)\, dx = \frac{x^4 \left(32 \ln\left(c\, x\right)^3 - 24 \ln\left(c\, x\right)^2 + 12\, \ln\left(c\, x\right) - 3\right)}{128}$$

```
[In] int(x^3*log(c*x)^3,x)
[Out] (x^4*(12*log(c*x) - 24*log(c*x)^2 + 32*log(c*x)^3 - 3))/128
```

3.16 $\int x^2 \log^3(cx) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int x^2 \log^3(cx) \, dx = -\frac{2x^3}{27} + \frac{2}{9}x^3 \log(cx) - \frac{1}{3}x^3 \log^2(cx) + \frac{1}{3}x^3 \log^3(cx)$$

```
[Out] -2/27*x^3+2/9*x^3*ln(c*x)-1/3*x^3*ln(c*x)^2+1/3*x^3*ln(c*x)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2342, 2341}

$$\int x^2 \log^3(cx) \, dx = \frac{1}{3}x^3 \log^3(cx) - \frac{1}{3}x^3 \log^2(cx) + \frac{2}{9}x^3 \log(cx) - \frac{2x^3}{27}$$

```
[In] Int[x^2*Log[c*x]^3,x]
```

```
[Out] (-2*x^3)/27 + (2*x^3*Log[c*x])/9 - (x^3*Log[c*x]^2)/3 + (x^3*Log[c*x]^3)/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^3\log^3(cx) - \int x^2\log^2(cx)\,dx\\ &= -\frac{1}{3}x^3\log^2(cx) + \frac{1}{3}x^3\log^3(cx) + \frac{2}{3}\int x^2\log(cx)\,dx\\ &= -\frac{2x^3}{27} + \frac{2}{9}x^3\log(cx) - \frac{1}{3}x^3\log^2(cx) + \frac{1}{3}x^3\log^3(cx)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int x^2\log^3(cx)\,dx = -\frac{2x^3}{27} + \frac{2}{9}x^3\log(cx) - \frac{1}{3}x^3\log^2(cx) + \frac{1}{3}x^3\log^3(cx)$$

```
[In] Integrate[x^2*Log[c*x]^3,x]
[Out] (-2*x^3)/27 + (2*x^3*Log[c*x])/9 - (x^3*Log[c*x]^2)/3 + (x^3*Log[c*x]^3)/3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
norman	$-\frac{2x^3}{27}+\frac{2x^3\ln(xc)}{9}-\frac{x^3\ln(xc)^2}{3}+\frac{x^3\ln(xc)^3}{3}$	38
risch	$-\frac{2x^3}{27}+\frac{2x^3\ln(xc)}{9}-\frac{x^3\ln(xc)^2}{3}+\frac{x^3\ln(xc)^3}{3}$	38
parallelrisch	$-\frac{2x^3}{27}+\frac{2x^3\ln(xc)}{9}-\frac{x^3\ln(xc)^2}{3}+\frac{x^3\ln(xc)^3}{3}$	38
parts	$\frac{x^3\ln(xc)^3}{3}-\frac{\frac{x^3c^3\ln(xc)^2}{3}-\frac{2x^3c^3\ln(xc)}{9}+\frac{2x^3c^3}{27}}{c^3}$	53
derivativedivides	$\frac{\frac{x^3c^3\ln(xc)^3}{3}-\frac{x^3c^3\ln(xc)^2}{3}+\frac{2x^3c^3\ln(xc)}{9}-\frac{2x^3c^3}{27}}{c^3}$	54
default	$\frac{\frac{x^3c^3\ln(xc)^3}{3}-\frac{x^3c^3\ln(xc)^2}{3}+\frac{2x^3c^3\ln(xc)}{9}-\frac{2x^3c^3}{27}}{c^3}$	54

```
[In] int(x^2*ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] -2/27*x^3+2/9*x^3*ln(x*c)-1/3*x^3*ln(x*c)^2+1/3*x^3*ln(x*c)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x^2 \log^3(cx)\,dx = \frac{1}{3}\,x^3\log\left(cx\right)^3 - \frac{1}{3}\,x^3\log\left(cx\right)^2 + \frac{2}{9}\,x^3\log\left(cx\right) - \frac{2}{27}\,x^3$$

```
[In] integrate(x^2*log(c*x)^3,x, algorithm="fricas")
[Out] 1/3*x^3*log(c*x)^3 - 1/3*x^3*log(c*x)^2 + 2/9*x^3*log(c*x) - 2/27*x^3
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int x^2 \log^3(cx)\,dx = \frac{x^3\log\left(cx\right)^3}{3} - \frac{x^3\log\left(cx\right)^2}{3} + \frac{2x^3\log\left(cx\right)}{9} - \frac{2x^3}{27}$$

```
[In] integrate(x**2*ln(c*x)**3,x)
[Out] x**3*log(c*x)**3/3 - x**3*log(c*x)**2/3 + 2*x**3*log(c*x)/9 - 2*x**3/27
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int x^2 \log^3(cx)\,dx = \frac{1}{27}\left(9\,\log\left(cx\right)^3 - 9\,\log\left(cx\right)^2 + 6\,\log\left(cx\right) - 2\right)x^3$$

```
[In] integrate(x^2*log(c*x)^3,x, algorithm="maxima")
[Out] 1/27*(9*log(c*x)^3 - 9*log(c*x)^2 + 6*log(c*x) - 2)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x^2 \log^3(cx)\,dx = \frac{1}{3}\,x^3\log\left(cx\right)^3 - \frac{1}{3}\,x^3\log\left(cx\right)^2 + \frac{2}{9}\,x^3\log\left(cx\right) - \frac{2}{27}\,x^3$$

```
[In] integrate(x^2*log(c*x)^3,x, algorithm="giac")
[Out] 1/3*x^3*log(c*x)^3 - 1/3*x^3*log(c*x)^2 + 2/9*x^3*log(c*x) - 2/27*x^3
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int x^2 \log^3(cx)\,dx = \frac{x^3\left(9\ln(c\,x)^3 - 9\ln(c\,x)^2 + 6\ \ln(c\,x) - 2\right)}{27}$$

```
[In] int(x^2*log(c*x)^3,x)
[Out] (x^3*(6*log(c*x) - 9*log(c*x)^2 + 9*log(c*x)^3 - 2))/27
```

3.17 $\int x \log^3(cx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 45

$$\int x \log^3(cx)\,dx = -\frac{3x^2}{8} + \frac{3}{4}x^2\log(cx) - \frac{3}{4}x^2\log^2(cx) + \frac{1}{2}x^2\log^3(cx)$$

```
[Out] -3/8*x^2+3/4*x^2*ln(c*x)-3/4*x^2*ln(c*x)^2+1/2*x^2*ln(c*x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2342, 2341}

$$\int x \log^3(cx)\,dx = \frac{1}{2}x^2\log^3(cx) - \frac{3}{4}x^2\log^2(cx) + \frac{3}{4}x^2\log(cx) - \frac{3x^2}{8}$$

```
[In] Int[x*Log[c*x]^3,x]
```

```
[Out] (-3*x^2)/8 + (3*x^2*Log[c*x])/4 - (3*x^2*Log[c*x]^2)/4 + (x^2*Log[c*x]^3)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\log^3(cx) - \frac{3}{2}\int x\log^2(cx)\,dx\\ &= -\frac{3}{4}x^2\log^2(cx) + \frac{1}{2}x^2\log^3(cx) + \frac{3}{2}\int x\log(cx)\,dx\\ &= -\frac{3x^2}{8} + \frac{3}{4}x^2\log(cx) - \frac{3}{4}x^2\log^2(cx) + \frac{1}{2}x^2\log^3(cx)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int x\log^3(cx)\,dx = -\frac{3x^2}{8} + \frac{3}{4}x^2\log(cx) - \frac{3}{4}x^2\log^2(cx) + \frac{1}{2}x^2\log^3(cx)$$

```
[In] Integrate[x*Log[c*x]^3,x]
[Out] (-3*x^2)/8 + (3*x^2*Log[c*x])/4 - (3*x^2*Log[c*x]^2)/4 + (x^2*Log[c*x]^3)/2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

method	result	size
norman	$-\frac{3x^2}{8}+\frac{3x^2\ln(xc)}{4}-\frac{3x^2\ln(xc)^2}{4}+\frac{x^2\ln(xc)^3}{2}$	38
risch	$-\frac{3x^2}{8}+\frac{3x^2\ln(xc)}{4}-\frac{3x^2\ln(xc)^2}{4}+\frac{x^2\ln(xc)^3}{2}$	38
parallelrisch	$-\frac{3x^2}{8}+\frac{3x^2\ln(xc)}{4}-\frac{3x^2\ln(xc)^2}{4}+\frac{x^2\ln(xc)^3}{2}$	38
parts	$\frac{x^2\ln(xc)^3}{2}-\frac{3\left(\frac{x^2c^2\ln(xc)^2}{2}-\frac{x^2c^2\ln(xc)}{2}+\frac{x^2c^2}{4}\right)}{2c^2}$	53
derivativedivides	$\frac{\frac{x^2c^2\ln(xc)^3}{2}-\frac{3x^2c^2\ln(xc)^2}{4}+\frac{3x^2c^2\ln(xc)}{4}-\frac{3x^2c^2}{8}}{c^2}$	54
default	$\frac{\frac{x^2c^2\ln(xc)^3}{2}-\frac{3x^2c^2\ln(xc)^2}{4}+\frac{3x^2c^2\ln(xc)}{4}-\frac{3x^2c^2}{8}}{c^2}$	54

```
[In] int(x*ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] -3/8*x^2+3/4*x^2*ln(x*c)-3/4*x^2*ln(x*c)^2+1/2*x^2*ln(x*c)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x\log^3(cx)\,dx = \frac{1}{2}\,x^2\log\left(cx\right)^3 - \frac{3}{4}\,x^2\log\left(cx\right)^2 + \frac{3}{4}\,x^2\log\left(cx\right) - \frac{3}{8}\,x^2$$

```
[In] integrate(x*log(c*x)^3,x, algorithm="fricas")
[Out] 1/2*x^2*log(c*x)^3 - 3/4*x^2*log(c*x)^2 + 3/4*x^2*log(c*x) - 3/8*x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x\log^3(cx)\,dx = \frac{x^2\log\left(cx\right)^3}{2} - \frac{3x^2\log\left(cx\right)^2}{4} + \frac{3x^2\log\left(cx\right)}{4} - \frac{3x^2}{8}$$

```
[In] integrate(x*ln(c*x)**3,x)
[Out] x**2*log(c*x)**3/2 - 3*x**2*log(c*x)**2/4 + 3*x**2*log(c*x)/4 - 3*x**2/8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int x\log^3(cx)\,dx = \frac{1}{8}\,\left(4\,\log\left(cx\right)^3 - 6\,\log\left(cx\right)^2 + 6\,\log\left(cx\right) - 3\right)x^2$$

```
[In] integrate(x*log(c*x)^3,x, algorithm="maxima")
[Out] 1/8*(4*log(c*x)^3 - 6*log(c*x)^2 + 6*log(c*x) - 3)*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x\log^3(cx)\,dx = \frac{1}{2}\,x^2\log\left(cx\right)^3 - \frac{3}{4}\,x^2\log\left(cx\right)^2 + \frac{3}{4}\,x^2\log\left(cx\right) - \frac{3}{8}\,x^2$$

```
[In] integrate(x*log(c*x)^3,x, algorithm="giac")
[Out] 1/2*x^2*log(c*x)^3 - 3/4*x^2*log(c*x)^2 + 3/4*x^2*log(c*x) - 3/8*x^2
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int x \log^3(cx)\,dx = \frac{x^2\left(4\ln(c\,x)^3-6\ln(c\,x)^2+6\,\ln(c\,x)-3\right)}{8}$$

```
[In] int(x*log(c*x)^3,x)
[Out] (x^2*(6*log(c*x) - 6*log(c*x)^2 + 4*log(c*x)^3 - 3))/8
```

3.18 $\int \log^3(cx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 28

$$\int \log^3(cx)\,dx = -6x + 6x\log(cx) - 3x\log^2(cx) + x\log^3(cx)$$

```
[Out] -6*x+6*x*ln(c*x)-3*x*ln(c*x)^2+x*ln(c*x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2333, 2332}

$$\int \log^3(cx)\,dx = x\log^3(cx) - 3x\log^2(cx) + 6x\log(cx) - 6x$$

```
[In] Int[Log[c*x]^3,x]
```

```
[Out] -6*x + 6*x*Log[c*x] - 3*x*Log[c*x]^2 + x*Log[c*x]^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= x\log^3(cx) - 3\int \log^2(cx)\,dx \\ &= -3x\log^2(cx) + x\log^3(cx) + 6\int \log(cx)\,dx \\ &= -6x + 6x\log(cx) - 3x\log^2(cx) + x\log^3(cx) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log^3(cx)\,dx = -6x + 6x\log(cx) - 3x\log^2(cx) + x\log^3(cx)$$

```
[In] Integrate[Log[c*x]^3,x]
[Out] -6*x + 6*x*Log[c*x] - 3*x*Log[c*x]^2 + x*Log[c*x]^3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
norman	$-6x+6x\ln\left(xc\right)-3x\ln\left(xc\right)^{2}+x\ln\left(xc\right)^{3}$	29
risch	$-6x+6x\ln\left(xc\right)-3x\ln\left(xc\right)^{2}+x\ln\left(xc\right)^{3}$	29
parallelrisch	$-6x+6x\ln\left(xc\right)-3x\ln\left(xc\right)^{2}+x\ln\left(xc\right)^{3}$	29
derivativedivides	$\frac{xc\ln(xc)^{3}-3xc\ln(xc)^{2}+6xc\ln(xc)-6xc}{c}$	37
default	$\frac{xc\ln(xc)^{3}-3xc\ln(xc)^{2}+6xc\ln(xc)-6xc}{c}$	37

```
[In] int(ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] -6*x+6*x*ln(x*c)-3*x*ln(x*c)^2+x*ln(x*c)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log^3(cx)\,dx = x\log(cx)^3 - 3\,x\log(cx)^2 + 6\,x\log(cx) - 6\,x$$

```
[In] integrate(log(c*x)^3,x, algorithm="fricas")
[Out] x*log(c*x)^3 - 3*x*log(c*x)^2 + 6*x*log(c*x) - 6*x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \log^3(cx)\,dx = x\log(cx)^3 - 3x\log(cx)^2 + 6x\log(cx) - 6x$$

```
[In] integrate(ln(c*x)**3,x)
[Out] x*log(c*x)**3 - 3*x*log(c*x)**2 + 6*x*log(c*x) - 6*x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \log^3(cx)\,dx = \left(\log(cx)^3 - 3\,\log(cx)^2 + 6\,\log(cx) - 6\right)x$$

```
[In] integrate(log(c*x)^3,x, algorithm="maxima")
[Out] (log(c*x)^3 - 3*log(c*x)^2 + 6*log(c*x) - 6)*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \log^3(cx)\,dx = x\log(cx)^3 - 3\,x\log(cx)^2 + 6\,x\log(cx) - 6\,x$$

```
[In] integrate(log(c*x)^3,x, algorithm="giac")
[Out] x*log(c*x)^3 - 3*x*log(c*x)^2 + 6*x*log(c*x) - 6*x
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \log^3(cx)\, dx = x\left(\ln(c\,x)^3 - 3\ln(c\,x)^2 + 6\,\ln(c\,x) - 6\right)$$

```
[In] int(log(c*x)^3,x)
[Out] x*(6*log(c*x) - 3*log(c*x)^2 + log(c*x)^3 - 6)
```

3.19 $\int \frac{\log^3(cx)}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\log^3(cx)}{x} \, dx = \frac{1}{4} \log^4(cx)$$

```
[Out] 1/4*ln(c*x)^4
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2339, 30}

$$\int \frac{\log^3(cx)}{x} \, dx = \frac{1}{4} \log^4(cx)$$

```
[In] Int[Log[c*x]^3/x,x]
```

```
[Out] Log[c*x]^4/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int x^3\,dx, x, \log(cx)\right)\\
&= \frac{1}{4}\log^4(cx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(cx)}{x}\,dx = \frac{1}{4}\log^4(cx)$$

```
[In] Integrate[Log[c*x]^3/x,x]
[Out] Log[c*x]^4/4
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\ln(xc)^4}{4}$	9
default	$\frac{\ln(xc)^4}{4}$	9
norman	$\frac{\ln(xc)^4}{4}$	9
risch	$\frac{\ln(xc)^4}{4}$	9
parts	$\ln(xc)^3\ln(x) - \frac{3\ln(xc)^2\ln(x)^2}{2} + \ln(xc)\ln(x)^3 - \frac{\ln(x)^4}{4}$	38

```
[In] int(ln(x*c)^3/x,x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x*c)^4
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log^3(cx)}{x}\,dx = \frac{1}{4}\,\log\left(cx\right)^4$$

```
[In] integrate(log(c*x)^3/x,x, algorithm="fricas")
[Out] 1/4*log(c*x)^4
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\log^3(cx)}{x}\,dx = \frac{\log\left(cx\right)^4}{4}$$

```
[In] integrate(ln(c*x)**3/x,x)
[Out] log(c*x)**4/4
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log^3(cx)}{x}\,dx = \frac{1}{4}\,\log\left(cx\right)^4$$

```
[In] integrate(log(c*x)^3/x,x, algorithm="maxima")
[Out] 1/4*log(c*x)^4
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log^3(cx)}{x}\,dx = \frac{1}{4}\,\log\left(cx\right)^4$$

```
[In] integrate(log(c*x)^3/x,x, algorithm="giac")
[Out] 1/4*log(c*x)^4
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\log^3(cx)}{x}\,dx = \frac{\ln\left(c\,x\right)^4}{4}$$

```
[In] int(log(c*x)^3/x,x)
[Out] log(c*x)^4/4
```

3.20 $\int \frac{\log^3(cx)}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{\log^3(cx)}{x^2} dx = -\frac{6}{x} - \frac{6\log(cx)}{x} - \frac{3\log^2(cx)}{x} - \frac{\log^3(cx)}{x}$$

```
[Out] -6/x-6*ln(c*x)/x-3*ln(c*x)^2/x-ln(c*x)^3/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2342, 2341}

$$\int \frac{\log^3(cx)}{x^2} dx = -\frac{\log^3(cx)}{x} - \frac{3\log^2(cx)}{x} - \frac{6\log(cx)}{x} - \frac{6}{x}$$

```
[In] Int[Log[c*x]^3/x^2,x]
```

```
[Out] -6/x - (6*Log[c*x])/x - (3*Log[c*x]^2)/x - Log[c*x]^3/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
```

```
c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log^3(cx)}{x}+3\int\frac{\log^2(cx)}{x^2}\,dx\\ &= -\frac{3\log^2(cx)}{x}-\frac{\log^3(cx)}{x}+6\int\frac{\log(cx)}{x^2}\,dx\\ &= -\frac{6}{x}-\frac{6\log(cx)}{x}-\frac{3\log^2(cx)}{x}-\frac{\log^3(cx)}{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int\frac{\log^3(cx)}{x^2}\,dx = -\frac{6}{x}-\frac{6\log(cx)}{x}-\frac{3\log^2(cx)}{x}-\frac{\log^3(cx)}{x}$$

```
[In] Integrate[Log[c*x]^3/x^2,x]
[Out] -6/x - (6*Log[c*x])/x - (3*Log[c*x]^2)/x - Log[c*x]^3/x
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

method	result	size
norman	$\frac{-6-3\ln(xc)^2-\ln(xc)^3-6\ln(xc)}{x}$	29
parallelrisch	$\frac{-6-3\ln(xc)^2-\ln(xc)^3-6\ln(xc)}{x}$	29
risch	$-\frac{6}{x}-\frac{6\ln(xc)}{x}-\frac{3\ln(xc)^2}{x}-\frac{\ln(xc)^3}{x}$	38
parts	$-\frac{\ln(xc)^3}{x}+3c\left(-\frac{\ln(xc)^2}{xc}-\frac{2\ln(xc)}{xc}-\frac{2}{xc}\right)$	51
derivativedivides	$c\left(-\frac{\ln(xc)^3}{xc}-\frac{3\ln(xc)^2}{xc}-\frac{6\ln(xc)}{xc}-\frac{6}{xc}\right)$	52
default	$c\left(-\frac{\ln(xc)^3}{xc}-\frac{3\ln(xc)^2}{xc}-\frac{6\ln(xc)}{xc}-\frac{6}{xc}\right)$	52

```
[In] int(ln(x*c)^3/x^2,x,method=_RETURNVERBOSE)
[Out] (-6-3*ln(x*c)^2-ln(x*c)^3-6*ln(x*c))/x
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\log^3(cx)}{x^2}\,dx = -\frac{\log(cx)^3 + 3\,\log(cx)^2 + 6\,\log(cx) + 6}{x}$$

```
[In] integrate(log(c*x)^3/x^2,x, algorithm="fricas")
[Out] -(log(c*x)^3 + 3*log(c*x)^2 + 6*log(c*x) + 6)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\log^3(cx)}{x^2}\,dx = -\frac{\log(cx)^3}{x} - \frac{3\log(cx)^2}{x} - \frac{6\log(cx)}{x} - \frac{6}{x}$$

```
[In] integrate(ln(c*x)**3/x**2,x)
[Out] -log(c*x)**3/x - 3*log(c*x)**2/x - 6*log(c*x)/x - 6/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\log^3(cx)}{x^2}\,dx = -\frac{\log(cx)^3 + 3\,\log(cx)^2 + 6\,\log(cx) + 6}{x}$$

```
[In] integrate(log(c*x)^3/x^2,x, algorithm="maxima")
[Out] -(log(c*x)^3 + 3*log(c*x)^2 + 6*log(c*x) + 6)/x
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(cx)}{x^2}\,dx = -\frac{\log(cx)^3}{x} - \frac{3\,\log(cx)^2}{x} - \frac{6\,\log(cx)}{x} - \frac{6}{x}$$

```
[In] integrate(log(c*x)^3/x^2,x, algorithm="giac")
[Out] -log(c*x)^3/x - 3*log(c*x)^2/x - 6*log(c*x)/x - 6/x
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\log^3(cx)}{x^2} \, dx = -\frac{\ln(c\,x)^3 + 3\ln(c\,x)^2 + 6\,\ln(c\,x) + 6}{x}$$

```
[In] int(log(c*x)^3/x^2,x)
[Out] -(6*log(c*x) + 3*log(c*x)^2 + log(c*x)^3 + 6)/x
```

3.21 $\int \frac{\log^3(cx)}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 45

$$\int \frac{\log^3(cx)}{x^3}\,dx = -\frac{3}{8x^2}-\frac{3\log(cx)}{4x^2}-\frac{3\log^2(cx)}{4x^2}-\frac{\log^3(cx)}{2x^2}$$

```
[Out] -3/8/x^2-3/4*ln(c*x)/x^2-3/4*ln(c*x)^2/x^2-1/2*ln(c*x)^3/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2342, 2341}

$$\int \frac{\log^3(cx)}{x^3}\,dx = -\frac{\log^3(cx)}{2x^2}-\frac{3\log^2(cx)}{4x^2}-\frac{3\log(cx)}{4x^2}-\frac{3}{8x^2}$$

```
[In] Int[Log[c*x]^3/x^3,x]
```

```
[Out] -3/(8*x^2) - (3*Log[c*x])/(4*x^2) - (3*Log[c*x]^2)/(4*x^2) - Log[c*x]^3/(2*
x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\log^3(cx)}{2x^2} + \frac{3}{2}\int \frac{\log^2(cx)}{x^3}\,dx \\ &= -\frac{3\log^2(cx)}{4x^2} - \frac{\log^3(cx)}{2x^2} + \frac{3}{2}\int \frac{\log(cx)}{x^3}\,dx \\ &= -\frac{3}{8x^2} - \frac{3\log(cx)}{4x^2} - \frac{3\log^2(cx)}{4x^2} - \frac{\log^3(cx)}{2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{\log^3(cx)}{x^3}\,dx = -\frac{3}{8x^2} - \frac{3\log(cx)}{4x^2} - \frac{3\log^2(cx)}{4x^2} - \frac{\log^3(cx)}{2x^2}$$

```
[In] Integrate[Log[c*x]^3/x^3,x]
[Out] -3/(8*x^2) - (3*Log[c*x])/(4*x^2) - (3*Log[c*x]^2)/(4*x^2) - Log[c*x]^3/(2*
x^2)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

method	result	size
norman	$\frac{-\frac{3}{8}-\frac{3\ln(xc)^2}{4}-\frac{\ln(xc)^3}{2}-\frac{3\ln(xc)}{4}}{x^2}$	29
parallelrisch	$\frac{-3-4\ln(xc)^3-6\ln(xc)^2-6\ln(xc)}{8x^2}$	30
risch	$-\frac{3}{8x^2}-\frac{3\ln(xc)}{4x^2}-\frac{3\ln(xc)^2}{4x^2}-\frac{\ln(xc)^3}{2x^2}$	38
parts	$-\frac{\ln(xc)^3}{2x^2}+\frac{3c^2\left(-\frac{\ln(xc)^2}{2x^2c^2}-\frac{\ln(xc)}{2x^2c^2}-\frac{1}{4x^2c^2}\right)}{2}$	53
derivativedivides	$c^2\left(-\frac{\ln(xc)^3}{2x^2c^2}-\frac{3\ln(xc)^2}{4x^2c^2}-\frac{3\ln(xc)}{4x^2c^2}-\frac{3}{8x^2c^2}\right)$	54
default	$c^2\left(-\frac{\ln(xc)^3}{2x^2c^2}-\frac{3\ln(xc)^2}{4x^2c^2}-\frac{3\ln(xc)}{4x^2c^2}-\frac{3}{8x^2c^2}\right)$	54

```
[In] int(ln(x*c)^3/x^3,x,method=_RETURNVERBOSE)
[Out] (-3/8-3/4*ln(x*c)^2-1/2*ln(x*c)^3-3/4*ln(x*c))/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{\log^3(cx)}{x^3} \, dx = -\frac{4 \log (cx)^3 + 6 \log (cx)^2 + 6 \log (cx) + 3}{8 x^2}$$

```
[In] integrate(log(c*x)^3/x^3,x, algorithm="fricas")
[Out] -1/8*(4*log(c*x)^3 + 6*log(c*x)^2 + 6*log(c*x) + 3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{\log^3(cx)}{x^3} \, dx = -\frac{\log (cx)^3}{2x^2} - \frac{3 \log (cx)^2}{4x^2} - \frac{3 \log (cx)}{4x^2} - \frac{3}{8x^2}$$

```
[In] integrate(ln(c*x)**3/x**3,x)
[Out] -log(c*x)**3/(2*x**2) - 3*log(c*x)**2/(4*x**2) - 3*log(c*x)/(4*x**2) - 3/(8
*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{\log^3(cx)}{x^3} \, dx = -\frac{4 \log (cx)^3 + 6 \log (cx)^2 + 6 \log (cx) + 3}{8 x^2}$$

```
[In] integrate(log(c*x)^3/x^3,x, algorithm="maxima")
[Out] -1/8*(4*log(c*x)^3 + 6*log(c*x)^2 + 6*log(c*x) + 3)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{\log^3(cx)}{x^3} dx = -\frac{\log(cx)^3}{2x^2} - \frac{3\log(cx)^2}{4x^2} - \frac{3\log(cx)}{4x^2} - \frac{3}{8x^2}$$

```
[In] integrate(log(c*x)^3/x^3,x, algorithm="giac")
[Out] -1/2*log(c*x)^3/x^2 - 3/4*log(c*x)^2/x^2 - 3/4*log(c*x)/x^2 - 3/8/x^2
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int \frac{\log^3(cx)}{x^3} dx = -\frac{\frac{\ln(cx)^3}{2} + \frac{3\ln(cx)^2}{4} + \frac{3\ln(cx)}{4} + \frac{3}{8}}{x^2}$$

```
[In] int(log(c*x)^3/x^3,x)
[Out] -((3*log(c*x))/4 + (3*log(c*x)^2)/4 + log(c*x)^3/2 + 3/8)/x^2
```

3.22 $\int \frac{x^3}{\log(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{x^3}{\log(cx)}\,dx = \frac{\text{ExpIntegralEi}(4\log(cx))}{c^4}$$

```
[Out] Ei(4*ln(c*x))/c^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2346, 2209}

$$\int \frac{x^3}{\log(cx)}\,dx = \frac{\text{ExpIntegralEi}(4\log(cx))}{c^4}$$

```
[In] Int[x^3/Log[c*x],x]
```

```
[Out] ExpIntegralEi[4*Log[c*x]]/c^4
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{e^{4x}}{x}\,dx, x, \log(cx)\right)}{c^4}$$
$$= \frac{\text{Ei}(4\log(cx))}{c^4}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\log(cx)}\,dx = \frac{\text{ExpIntegralEi}(4\log(cx))}{c^4}$$

```
[In] Integrate[x^3/Log[c*x],x]
[Out] ExpIntegralEi[4*Log[c*x]]/c^4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$-\frac{\text{Ei}_1(-4\ln(xc))}{c^4}$	14
default	$-\frac{\text{Ei}_1(-4\ln(xc))}{c^4}$	14
risch	$-\frac{\text{Ei}_1(-4\ln(xc))}{c^4}$	14

```
[In] int(x^3/ln(x*c),x,method=_RETURNVERBOSE)
[Out] -1/c^4*Ei(1,-4*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{\log(cx)}\,dx = \frac{\text{log\_integral}\left(c^4x^4\right)}{c^4}$$

```
[In] integrate(x^3/log(c*x),x, algorithm="fricas")
[Out] log_integral(c^4*x^4)/c^4
```

Sympy [F]

$$\int \frac{x^3}{\log(cx)}\,dx = \int \frac{x^3}{\log{(cx)}}\,dx$$

```
[In] integrate(x**3/ln(c*x),x)
[Out] Integral(x**3/log(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\log(cx)}\,dx = \frac{\mathrm{Ei}(4\log{(cx)})}{c^4}$$

```
[In] integrate(x^3/log(c*x),x, algorithm="maxima")
[Out] Ei(4*log(c*x))/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\log(cx)}\,dx = \frac{\mathrm{Ei}(4\log{(cx)})}{c^4}$$

```
[In] integrate(x^3/log(c*x),x, algorithm="giac")
[Out] Ei(4*log(c*x))/c^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log(cx)}\,dx = \int \frac{x^3}{\ln{(c\,x)}}\,dx$$

```
[In] int(x^3/log(c*x),x)
[Out] int(x^3/log(c*x), x)
```

3.23 $\int \frac{x^2}{\log(cx)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{x^2}{\log(cx)} \, dx = \frac{\text{ExpIntegralEi}(3\log(cx))}{c^3}$$

```
[Out] Ei(3*ln(c*x))/c^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2346, 2209}

$$\int \frac{x^2}{\log(cx)} \, dx = \frac{\text{ExpIntegralEi}(3\log(cx))}{c^3}$$

```
[In] Int[x^2/Log[c*x],x]
```

```
[Out] ExpIntegralEi[3*Log[c*x]]/c^3
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{e^{3x}}{x}\,dx, x, \log(cx)\right)}{c^3}$$
$$= \frac{\text{Ei}(3\log(cx))}{c^3}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\log(cx)}\,dx = \frac{\text{ExpIntegralEi}(3\log(cx))}{c^3}$$

```
[In] Integrate[x^2/Log[c*x],x]
[Out] ExpIntegralEi[3*Log[c*x]]/c^3
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$-\frac{\text{Ei}_1(-3\ln(xc))}{c^3}$	14
default	$-\frac{\text{Ei}_1(-3\ln(xc))}{c^3}$	14
risch	$-\frac{\text{Ei}_1(-3\ln(xc))}{c^3}$	14

```
[In] int(x^2/ln(x*c),x,method=_RETURNVERBOSE)
[Out] -1/c^3*Ei(1,-3*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{\log(cx)}\,dx = \frac{\text{log\_integral}\left(c^3x^3\right)}{c^3}$$

```
[In] integrate(x^2/log(c*x),x, algorithm="fricas")
[Out] log_integral(c^3*x^3)/c^3
```

Sympy [F]

$$\int \frac{x^2}{\log(cx)}\, dx = \int \frac{x^2}{\log{(cx)}}\, dx$$

```
[In] integrate(x**2/ln(c*x),x)
[Out] Integral(x**2/log(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\log(cx)}\, dx = \frac{\mathrm{Ei}(3 \log{(cx)})}{c^3}$$

```
[In] integrate(x^2/log(c*x),x, algorithm="maxima")
[Out] Ei(3*log(c*x))/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\log(cx)}\, dx = \frac{\mathrm{Ei}(3 \log{(cx)})}{c^3}$$

```
[In] integrate(x^2/log(c*x),x, algorithm="giac")
[Out] Ei(3*log(c*x))/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\log(cx)}\, dx = \int \frac{x^2}{\ln{(c\,x)}}\, dx$$

```
[In] int(x^2/log(c*x),x)
[Out] int(x^2/log(c*x), x)
```

3.24 $\int \frac{x}{\log(cx)} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 11

$$\int \frac{x}{\log(cx)} \, dx = \frac{\text{ExpIntegralEi}(2\log(cx))}{c^2}$$

```
[Out] Ei(2*ln(c*x))/c^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2346, 2209}

$$\int \frac{x}{\log(cx)} \, dx = \frac{\text{ExpIntegralEi}(2\log(cx))}{c^2}$$

```
[In] Int[x/Log[c*x],x]
```

```
[Out] ExpIntegralEi[2*Log[c*x]]/c^2
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log(cx)\right)}{c^2}$$
$$= \frac{\text{Ei}(2\log(cx))}{c^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log(cx)}\,dx = \frac{\text{ExpIntegralEi}(2\log(cx))}{c^2}$$

```
[In] Integrate[x/Log[c*x],x]
[Out] ExpIntegralEi[2*Log[c*x]]/c^2
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$-\frac{\text{Ei}_1(-2\ln(xc))}{c^2}$	14
default	$-\frac{\text{Ei}_1(-2\ln(xc))}{c^2}$	14
risch	$-\frac{\text{Ei}_1(-2\ln(xc))}{c^2}$	14

```
[In] int(x/ln(x*c),x,method=_RETURNVERBOSE)
[Out] -1/c^2*Ei(1,-2*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{x}{\log(cx)}\,dx = \frac{\text{log\_integral}\left(c^2x^2\right)}{c^2}$$

```
[In] integrate(x/log(c*x),x, algorithm="fricas")
[Out] log_integral(c^2*x^2)/c^2
```

Sympy [F]

$$\int \frac{x}{\log(cx)} \, dx = \int \frac{x}{\log{(cx)}} \, dx$$

```
[In] integrate(x/ln(c*x),x)
[Out] Integral(x/log(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log(cx)} \, dx = \frac{\operatorname{Ei}(2 \log{(cx)})}{c^2}$$

```
[In] integrate(x/log(c*x),x, algorithm="maxima")
[Out] Ei(2*log(c*x))/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log(cx)} \, dx = \frac{\operatorname{Ei}(2 \log{(cx)})}{c^2}$$

```
[In] integrate(x/log(c*x),x, algorithm="giac")
[Out] Ei(2*log(c*x))/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log(cx)} \, dx = \int \frac{x}{\ln{(c\,x)}} \, dx$$

```
[In] int(x/log(c*x),x)
[Out] int(x/log(c*x), x)
```

3.25 $\int \frac{1}{\log(cx)} dx$

Optimal result

Integrand size = 6, antiderivative size = 8

$$\int \frac{1}{\log(cx)} dx = \frac{\text{LogIntegral}(cx)}{c}$$

```
[Out] Li(c*x)/c
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2335}

$$\int \frac{1}{\log(cx)} dx = \frac{\text{LogIntegral}(cx)}{c}$$

```
[In] Int[Log[c*x]^(-1),x]
```

```
[Out] LogIntegral[c*x]/c
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\text{integral} = \frac{\text{li}(cx)}{c}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(cx)}\,dx = \frac{\text{LogIntegral}(cx)}{c}$$

```
[In] Integrate[Log[c*x]^(-1),x]
[Out] LogIntegral[c*x]/c
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

method	result	size
derivativedivides	$-\frac{\text{Ei}_1(-\ln(xc))}{c}$	14
default	$-\frac{\text{Ei}_1(-\ln(xc))}{c}$	14
risch	$-\frac{\text{Ei}_1(-\ln(xc))}{c}$	14

```
[In] int(1/ln(x*c),x,method=_RETURNVERBOSE)
[Out] -1/c*Ei(1,-ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(cx)}\,dx = \frac{\text{log\_integral}\,(cx)}{c}$$

```
[In] integrate(1/log(c*x),x, algorithm="fricas")
[Out] log_integral(c*x)/c
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{\log(cx)}\,dx = \frac{\mathrm{li}\,(cx)}{c}$$

```
[In] integrate(1/ln(c*x),x)
[Out] li(c*x)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{1}{\log(cx)}\,dx = \frac{\mathrm{Ei}(\log{(cx)})}{c}$$

```
[In] integrate(1/log(c*x),x, algorithm="maxima")
[Out] Ei(log(c*x))/c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{1}{\log(cx)}\,dx = \frac{\mathrm{Ei}(\log{(cx)})}{c}$$

```
[In] integrate(1/log(c*x),x, algorithm="giac")
[Out] Ei(log(c*x))/c
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log(cx)}\,dx = \frac{\mathrm{logint}(c\,x)}{c}$$

```
[In] int(1/log(c*x),x)
[Out] logint(c*x)/c
```

3.26 $\int \frac{1}{x\log(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 5

$$\int \frac{1}{x\log(cx)}\,dx = \log(\log(cx))$$

```
[Out] ln(ln(c*x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2339, 29}

$$\int \frac{1}{x\log(cx)}\,dx = \log(\log(cx))$$

```
[In] Int[1/(x*Log[c*x]),x]
```

```
[Out] Log[Log[c*x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(cx)\right)\\ &= \log(\log(cx)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(cx)}\,dx = \log(\log(cx))$$

```
[In] Integrate[1/(x*Log[c*x]),x]
[Out] Log[Log[c*x]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\ln(\ln(xc))$	6
default	$\ln(\ln(xc))$	6
norman	$\ln(\ln(xc))$	6
risch	$\ln(\ln(xc))$	6
parallelrisch	$\ln(\ln(xc))$	6

```
[In] int(1/x/ln(x*c),x,method=_RETURNVERBOSE)
[Out] ln(ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(cx)}\,dx = \log\left(\log\left(cx\right)\right)$$

```
[In] integrate(1/x/log(c*x),x, algorithm="fricas")
[Out] log(log(c*x))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(cx)}\,dx = \log\left(\log\left(cx\right)\right)$$

```
[In] integrate(1/x/ln(c*x),x)
[Out] log(log(c*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(cx)}\,dx = \log\left(\log\left(cx\right)\right)$$

```
[In] integrate(1/x/log(c*x),x, algorithm="maxima")
[Out] log(log(c*x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(cx)}\,dx = \log\left(\log\left(cx\right)\right)$$

```
[In] integrate(1/x/log(c*x),x, algorithm="giac")
[Out] log(log(c*x))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log(cx)}\,dx = \ln\left(\ln\left(c\,x\right)\right)$$

```
[In] int(1/(x*log(c*x)),x)
[Out] log(log(c*x))
```

3.27 $\int \frac{1}{x^2 \log(cx)} dx$

Optimal result

Integrand size = 10, antiderivative size = 9

$$\int \frac{1}{x^2 \log(cx)} dx = c \operatorname{ExpIntegralEi}(-\log(cx))$$

```
[Out] c*Ei(-ln(c*x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2346, 2209}

$$\int \frac{1}{x^2 \log(cx)} dx = c \operatorname{ExpIntegralEi}(-\log(cx))$$

```
[In] Int[1/(x^2*Log[c*x]),x]
```

```
[Out] c*ExpIntegralEi[-Log[c*x]]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\text{Subst}\left(\int \frac{e^{-x}}{x}\,dx, x, \log(cx)\right)\\ &= c\text{Ei}(-\log(cx))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\log(cx)}\,dx = c\,\text{ExpIntegralEi}(-\log(cx))$$

```
[In] Integrate[1/(x^2*Log[c*x]),x]
[Out] c*ExpIntegralEi[-Log[c*x]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$-c\ \text{Ei}_1\left(\ln\left(xc\right)\right)$	10
default	$-c\ \text{Ei}_1\left(\ln\left(xc\right)\right)$	10
risch	$-c\ \text{Ei}_1\left(\ln\left(xc\right)\right)$	10

```
[In] int(1/x^2/ln(x*c),x,method=_RETURNVERBOSE)
[Out] -c*Ei(1,ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\log(cx)}\,dx = c\log\_integral\left(\frac{1}{cx}\right)$$

```
[In] integrate(1/x^2/log(c*x),x, algorithm="fricas")
[Out] c*log_integral(1/(c*x))
```

Sympy [F]

$$\int \frac{1}{x^2 \log(cx)} \, dx = \int \frac{1}{x^2 \log{(cx)}} \, dx$$

```
[In] integrate(1/x**2/ln(c*x),x)
[Out] Integral(1/(x**2*log(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \log(cx)} \, dx = c\mathrm{Ei}(-\log{(cx)})$$

```
[In] integrate(1/x^2/log(c*x),x, algorithm="maxima")
[Out] c*Ei(-log(c*x))
```

Giac [F]

$$\int \frac{1}{x^2 \log(cx)} \, dx = \int \frac{1}{x^2 \log{(cx)}} \, dx$$

```
[In] integrate(1/x^2/log(c*x),x, algorithm="giac")
[Out] integrate(1/(x^2*log(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \log(cx)} \, dx = \int \frac{1}{x^2 \ln{(c\,x)}} \, dx$$

```
[In] int(1/(x^2*log(c*x)),x)
[Out] int(1/(x^2*log(c*x)), x)
```

3.28 $\int \frac{1}{x^3 \log(cx)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{1}{x^3 \log(cx)} \, dx = c^2 \operatorname{ExpIntegralEi}(-2 \log(cx))$$

```
[Out] c^2*Ei(-2*ln(c*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2346, 2209}

$$\int \frac{1}{x^3 \log(cx)} \, dx = c^2 \operatorname{ExpIntegralEi}(-2 \log(cx))$$

```
[In] Int[1/(x^3*Log[c*x]),x]
[Out] c^2*ExpIntegralEi[-2*Log[c*x]]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c^2 \text{Subst}\left(\int \frac{e^{-2x}}{x}\,dx, x, \log(cx)\right)\\
&= c^2 \text{Ei}(-2\log(cx))
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \log(cx)}\,dx = c^2\,\text{ExpIntegralEi}(-2\log(cx))$$

```
[In] Integrate[1/(x^3*Log[c*x]),x]
[Out] c^2*ExpIntegralEi[-2*Log[c*x]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$-c^2\,\text{Ei}_1\left(2\ln\left(xc\right)\right)$	14
default	$-c^2\,\text{Ei}_1\left(2\ln\left(xc\right)\right)$	14
risch	$-c^2\,\text{Ei}_1\left(2\ln\left(xc\right)\right)$	14

```
[In] int(1/x^3/ln(x*c),x,method=_RETURNVERBOSE)
[Out] -c^2*Ei(1,2*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 \log(cx)}\,dx = c^2\,\text{log\_integral}\left(\frac{1}{c^2x^2}\right)$$

```
[In] integrate(1/x^3/log(c*x),x, algorithm="fricas")
[Out] c^2*log_integral(1/(c^2*x^2))
```

Sympy [F]

$$\int \frac{1}{x^3 \log(cx)}\, dx = \int \frac{1}{x^3 \log{(cx)}}\, dx$$

```
[In] integrate(1/x**3/ln(c*x),x)
[Out] Integral(1/(x**3*log(c*x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \log(cx)}\, dx = c^2 \mathrm{Ei}(-2 \log{(cx)})$$

```
[In] integrate(1/x^3/log(c*x),x, algorithm="maxima")
[Out] c^2*Ei(-2*log(c*x))
```

Giac [F]

$$\int \frac{1}{x^3 \log(cx)}\, dx = \int \frac{1}{x^3 \log{(cx)}}\, dx$$

```
[In] integrate(1/x^3/log(c*x),x, algorithm="giac")
[Out] integrate(1/(x^3*log(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \log(cx)}\, dx = \int \frac{1}{x^3\, \ln{(c\, x)}}\, dx$$

```
[In] int(1/(x^3*log(c*x)),x)
[Out] int(1/(x^3*log(c*x)), x)
```

3.29 $\int \frac{x^3}{\log^2(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{x^3}{\log^2(cx)}\,dx = \frac{4\,\text{ExpIntegralEi}(4\log(cx))}{c^4} - \frac{x^4}{\log(cx)}$$

```
[Out] 4*Ei(4*ln(c*x))/c^4-x^4/ln(c*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2343, 2346, 2209}

$$\int \frac{x^3}{\log^2(cx)}\,dx = \frac{4\,\text{ExpIntegralEi}(4\log(cx))}{c^4} - \frac{x^4}{\log(cx)}$$

```
[In] Int[x^3/Log[c*x]^2,x]
[Out] (4*ExpIntegralEi[4*Log[c*x]])/c^4 - x^4/Log[c*x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{x^4}{\log(cx)} + 4\int \frac{x^3}{\log(cx)}\,dx \\ &= -\frac{x^4}{\log(cx)} + \frac{4\text{Subst}\left(\int \frac{e^{4x}}{x}\,dx, x, \log(cx)\right)}{c^4} \\ &= \frac{4\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{\log(cx)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\log^2(cx)}\,dx = \frac{4\,\text{ExpIntegralEi}(4\log(cx))}{c^4} - \frac{x^4}{\log(cx)}$$

```
[In] Integrate[x^3/Log[c*x]^2,x]
[Out] (4*ExpIntegralEi[4*Log[c*x]])/c^4 - x^4/Log[c*x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{x^4}{\ln(xc)} - \frac{4\ \text{Ei}_1(-4\ln(xc))}{c^4}$	26
derivativedivides	$\frac{-\frac{x^4c^4}{\ln(xc)} - 4\ \text{Ei}_1(-4\ln(xc))}{c^4}$	30
default	$\frac{-\frac{x^4c^4}{\ln(xc)} - 4\ \text{Ei}_1(-4\ln(xc))}{c^4}$	30

```
[In] int(x^3/ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] -x^4/ln(x*c)-4/c^4*Ei(1,-4*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{\log^2(cx)}\,dx = -\frac{c^4x^4 - 4\,\log(cx)\,\text{log\_integral}\left(c^4x^4\right)}{c^4\log(cx)}$$

```
[In] integrate(x^3/log(c*x)^2,x, algorithm="fricas")
[Out] -(c^4*x^4 - 4*log(c*x)*log_integral(c^4*x^4))/(c^4*log(c*x))
```

Sympy [F]

$$\int \frac{x^3}{\log^2(cx)}\,dx = -\frac{x^4}{\log(cx)} + 4\int \frac{x^3}{\log(cx)}\,dx$$

```
[In] integrate(x**3/ln(c*x)**2,x)
[Out] -x**4/log(c*x) + 4*Integral(x**3/log(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{x^3}{\log^2(cx)}\,dx = \frac{4\,\Gamma(-1, -4\,\log(cx))}{c^4}$$

```
[In] integrate(x^3/log(c*x)^2,x, algorithm="maxima")
[Out] 4*gamma(-1, -4*log(c*x))/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\log^2(cx)}\,dx = -\frac{x^4}{\log(cx)} + \frac{4\,\text{Ei}(4\,\log(cx))}{c^4}$$

```
[In] integrate(x^3/log(c*x)^2,x, algorithm="giac")
[Out] -x^4/log(c*x) + 4*Ei(4*log(c*x))/c^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^2(cx)}\,dx = \int \frac{x^3}{\ln\left(c\,x\right)^2}\,dx$$

```
[In] int(x^3/log(c*x)^2,x)
```

```
[Out] int(x^3/log(c*x)^2, x)
```

3.30 $\int \frac{x^2}{\log^2(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{x^2}{\log^2(cx)}\,dx = \frac{3\,\mathrm{ExpIntegralEi}(3\log(cx))}{c^3} - \frac{x^3}{\log(cx)}$$

```
[Out] 3*Ei(3*ln(c*x))/c^3-x^3/ln(c*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2343, 2346, 2209}

$$\int \frac{x^2}{\log^2(cx)}\,dx = \frac{3\,\mathrm{ExpIntegralEi}(3\log(cx))}{c^3} - \frac{x^3}{\log(cx)}$$

```
[In] Int[x^2/Log[c*x]^2,x]
```

```
[Out] (3*ExpIntegralEi[3*Log[c*x]])/c^3 - x^3/Log[c*x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^3}{\log(cx)} + 3\int \frac{x^2}{\log(cx)}\,dx \\ &= -\frac{x^3}{\log(cx)} + \frac{3\text{Subst}\left(\int \frac{e^{3x}}{x}\,dx, x, \log(cx)\right)}{c^3} \\ &= \frac{3\text{Ei}(3\log(cx))}{c^3} - \frac{x^3}{\log(cx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\log^2(cx)}\,dx = \frac{3\,\text{ExpIntegralEi}(3\log(cx))}{c^3} - \frac{x^3}{\log(cx)}$$

```
[In] Integrate[x^2/Log[c*x]^2,x]
[Out] (3*ExpIntegralEi[3*Log[c*x]])/c^3 - x^3/Log[c*x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{x^3}{\ln(xc)} - \frac{3\ \text{Ei}_1(-3\ln(xc))}{c^3}$	26
derivativedivides	$\frac{-\frac{x^3c^3}{\ln(xc)} - 3\ \text{Ei}_1(-3\ln(xc))}{c^3}$	30
default	$\frac{-\frac{x^3c^3}{\ln(xc)} - 3\ \text{Ei}_1(-3\ln(xc))}{c^3}$	30

```
[In] int(x^2/ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] -x^3/ln(x*c)-3/c^3*Ei(1,-3*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{x^2}{\log^2(cx)} \, dx = -\frac{c^3x^3 - 3 \log(cx) \operatorname{log\_integral}(c^3x^3)}{c^3 \log(cx)}$$

```
[In] integrate(x^2/log(c*x)^2,x, algorithm="fricas")
[Out] -(c^3*x^3 - 3*log(c*x)*log_integral(c^3*x^3))/(c^3*log(c*x))
```

Sympy [F]

$$\int \frac{x^2}{\log^2(cx)} \, dx = -\frac{x^3}{\log(cx)} + 3 \int \frac{x^2}{\log(cx)} \, dx$$

```
[In] integrate(x**2/ln(c*x)**2,x)
[Out] -x**3/log(c*x) + 3*Integral(x**2/log(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{x^2}{\log^2(cx)} \, dx = \frac{3 \Gamma(-1, -3 \log(cx))}{c^3}$$

```
[In] integrate(x^2/log(c*x)^2,x, algorithm="maxima")
[Out] 3*gamma(-1, -3*log(c*x))/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\log^2(cx)} \, dx = -\frac{x^3}{\log(cx)} + \frac{3 \operatorname{Ei}(3 \log(cx))}{c^3}$$

```
[In] integrate(x^2/log(c*x)^2,x, algorithm="giac")
[Out] -x^3/log(c*x) + 3*Ei(3*log(c*x))/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\log^2(cx)}\,dx = \int \frac{x^2}{\ln{(c\,x)}^2}\,dx$$

```
[In] int(x^2/log(c*x)^2,x)
[Out] int(x^2/log(c*x)^2, x)
```

3.31 $\int \frac{x}{\log^2(cx)} dx$

Optimal result

Integrand size = 8, antiderivative size = 24

$$\int \frac{x}{\log^2(cx)} dx = \frac{2 \operatorname{ExpIntegralEi}(2 \log(cx))}{c^2} - \frac{x^2}{\log(cx)}$$

```
[Out] 2*Ei(2*ln(c*x))/c^2-x^2/ln(c*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2343, 2346, 2209}

$$\int \frac{x}{\log^2(cx)} dx = \frac{2 \operatorname{ExpIntegralEi}(2 \log(cx))}{c^2} - \frac{x^2}{\log(cx)}$$

```
[In] Int[x/Log[c*x]^2,x]
```

```
[Out] (2*ExpIntegralEi[2*Log[c*x]])/c^2 - x^2/Log[c*x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^2}{\log(cx)} + 2\int \frac{x}{\log(cx)}\,dx \\ &= -\frac{x^2}{\log(cx)} + \frac{2\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log(cx)\right)}{c^2} \\ &= \frac{2\text{Ei}(2\log(cx))}{c^2} - \frac{x^2}{\log(cx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log^2(cx)}\,dx = \frac{2\,\text{ExpIntegralEi}(2\log(cx))}{c^2} - \frac{x^2}{\log(cx)}$$

```
[In] Integrate[x/Log[c*x]^2,x]
[Out] (2*ExpIntegralEi[2*Log[c*x]])/c^2 - x^2/Log[c*x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{x^2}{\ln(xc)} - \frac{2\ \text{Ei}_1(-2\ln(xc))}{c^2}$	26
derivativedivides	$\frac{-\frac{x^2c^2}{\ln(xc)} - 2\ \text{Ei}_1(-2\ln(xc))}{c^2}$	30
default	$\frac{-\frac{x^2c^2}{\ln(xc)} - 2\ \text{Ei}_1(-2\ln(xc))}{c^2}$	30

```
[In] int(x/ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] -x^2/ln(x*c)-2/c^2*Ei(1,-2*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{x}{\log^2(cx)}\,dx = -\frac{c^2x^2 - 2\log(cx)\log\_integral(c^2x^2)}{c^2\log(cx)}$$

```
[In] integrate(x/log(c*x)^2,x, algorithm="fricas")
[Out] -(c^2*x^2 - 2*log(c*x)*log_integral(c^2*x^2))/(c^2*log(c*x))
```

Sympy [F]

$$\int \frac{x}{\log^2(cx)}\,dx = -\frac{x^2}{\log(cx)} + 2\int \frac{x}{\log(cx)}\,dx$$

```
[In] integrate(x/ln(c*x)**2,x)
[Out] -x**2/log(c*x) + 2*Integral(x/log(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{x}{\log^2(cx)}\,dx = \frac{2\,\Gamma(-1, -2\log(cx))}{c^2}$$

```
[In] integrate(x/log(c*x)^2,x, algorithm="maxima")
[Out] 2*gamma(-1, -2*log(c*x))/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log^2(cx)}\,dx = -\frac{x^2}{\log(cx)} + \frac{2\,\mathrm{Ei}(2\log(cx))}{c^2}$$

```
[In] integrate(x/log(c*x)^2,x, algorithm="giac")
[Out] -x^2/log(c*x) + 2*Ei(2*log(c*x))/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log^2(cx)}\,dx = \int \frac{x}{\ln\left(c\,x\right)^2}\,dx$$

```
[In] int(x/log(c*x)^2,x)
[Out] int(x/log(c*x)^2, x)
```

3.32 $\int \frac{1}{\log^2(cx)} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 18

$$\int \frac{1}{\log^2(cx)} \, dx = -\frac{x}{\log(cx)} + \frac{\text{LogIntegral}(cx)}{c}$$

```
[Out] Li(c*x)/c-x/ln(c*x)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2334, 2335}

$$\int \frac{1}{\log^2(cx)} \, dx = \frac{\text{LogIntegral}(cx)}{c} - \frac{x}{\log(cx)}$$

```
[In] Int[Log[c*x]^(-2),x]
```

```
[Out] -(x/Log[c*x]) + LogIntegral[c*x]/c
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x}{\log(cx)} + \int \frac{1}{\log(cx)}\,dx\\ &= -\frac{x}{\log(cx)} + \frac{\text{li}(cx)}{c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^2(cx)}\,dx = -\frac{x}{\log(cx)} + \frac{\text{LogIntegral}(cx)}{c}$$

```
[In] Integrate[Log[c*x]^(-2),x]
[Out] -(x/Log[c*x]) + LogIntegral[c*x]/c
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

method	result	size
risch	$-\frac{x}{\ln(xc)} - \frac{\text{Ei}_1(-\ln(xc))}{c}$	24
derivativedivides	$\frac{-\frac{xc}{\ln(xc)} - \text{Ei}_1(-\ln(xc))}{c}$	26
default	$\frac{-\frac{xc}{\ln(xc)} - \text{Ei}_1(-\ln(xc))}{c}$	26

```
[In] int(1/ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] -x/ln(x*c)-1/c*Ei(1,-ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{\log^2(cx)}\,dx = -\frac{cx - \log(cx)\,\text{log\_integral}(cx)}{c\log(cx)}$$

```
[In] integrate(1/log(c*x)^2,x, algorithm="fricas")
[Out] -(c*x - log(c*x)*log_integral(c*x))/(c*log(c*x))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{\log^2(cx)}\,dx = -\frac{x}{\log(cx)} + \frac{\operatorname{li}(cx)}{c}$$

```
[In] integrate(1/ln(c*x)**2,x)
[Out] -x/log(c*x) + li(c*x)/c
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{\log^2(cx)}\,dx = \frac{\Gamma(-1, -\log(cx))}{c}$$

```
[In] integrate(1/log(c*x)^2,x, algorithm="maxima")
[Out] gamma(-1, -log(c*x))/c
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{\log^2(cx)}\,dx = \frac{\operatorname{Ei}(\log(cx))}{c} - \frac{x}{\log(cx)}$$

```
[In] integrate(1/log(c*x)^2,x, algorithm="giac")
[Out] Ei(log(c*x))/c - x/log(c*x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^2(cx)}\,dx = \frac{\operatorname{logint}(c\,x)}{c} - \frac{x}{\ln(c\,x)}$$

```
[In] int(1/log(c*x)^2,x)
[Out] logint(c*x)/c - x/log(c*x)
```

3.33 $\int \frac{1}{x \log^2(cx)} dx$

Optimal result

Integrand size = 10, antiderivative size = 8

$$\int \frac{1}{x \log^2(cx)} dx = -\frac{1}{\log(cx)}$$

```
[Out] -1/ln(c*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2339, 30}

$$\int \frac{1}{x \log^2(cx)} dx = -\frac{1}{\log(cx)}$$

```
[In] Int[1/(x*Log[c*x]^2),x]
```

```
[Out] -Log[c*x]^(-1)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{x^2}\,dx, x, \log(cx)\right)$$
$$= -\frac{1}{\log(cx)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log^2(cx)}\,dx = -\frac{1}{\log(cx)}$$

```
[In] Integrate[1/(x*Log[c*x]^2),x]
[Out] -Log[c*x]^(-1)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\frac{1}{\ln(xc)}$	9
default	$-\frac{1}{\ln(xc)}$	9
norman	$-\frac{1}{\ln(xc)}$	9
risch	$-\frac{1}{\ln(xc)}$	9
parallelrisch	$-\frac{1}{\ln(xc)}$	9

```
[In] int(1/x/ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] -1/ln(x*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log^2(cx)}\,dx = -\frac{1}{\log(cx)}$$

```
[In] integrate(1/x/log(c*x)^2,x, algorithm="fricas")
[Out] -1/log(c*x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\log^2(cx)}\,dx = -\frac{1}{\log(cx)}$$

```
[In] integrate(1/x/ln(c*x)**2,x)
[Out] -1/log(c*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log^2(cx)}\,dx = -\frac{1}{\log(cx)}$$

```
[In] integrate(1/x/log(c*x)^2,x, algorithm="maxima")
[Out] -1/log(c*x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log^2(cx)}\,dx = -\frac{1}{\log(cx)}$$

```
[In] integrate(1/x/log(c*x)^2,x, algorithm="giac")
[Out] -1/log(c*x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log^2(cx)}\,dx = -\frac{1}{\ln(c\,x)}$$

```
[In] int(1/(x*log(c*x)^2),x)
[Out] -1/log(c*x)
```

3.34 $\int \frac{1}{x^2 \log^2(cx)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 22

$$\int \frac{1}{x^2 \log^2(cx)} \, dx = -c \, \text{ExpIntegralEi}(-\log(cx)) - \frac{1}{x \log(cx)}$$

```
[Out] -c*Ei(-ln(c*x))-1/x/ln(c*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2343, 2346, 2209}

$$\int \frac{1}{x^2 \log^2(cx)} \, dx = -c \, \text{ExpIntegralEi}(-\log(cx)) - \frac{1}{x \log(cx)}$$

```
[In] Int[1/(x^2*Log[c*x]^2),x]
```

```
[Out] -(c*ExpIntegralEi[-Log[c*x]]) - 1/(x*Log[c*x])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
```

```
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{x\log(cx)} - \int \frac{1}{x^2\log(cx)}\,dx \\
&= -\frac{1}{x\log(cx)} - c\text{Subst}\left(\int \frac{e^{-x}}{x}\,dx, x, \log(cx)\right) \\
&= -c\text{Ei}(-\log(cx)) - \frac{1}{x\log(cx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\log^2(cx)}\,dx = -c\,\text{ExpIntegralEi}(-\log(cx)) - \frac{1}{x\log(cx)}$$

```
[In] Integrate[1/(x^2*Log[c*x]^2),x]
[Out] -(c*ExpIntegralEi[-Log[c*x]]) - 1/(x*Log[c*x])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{1}{x\ln(xc)} + c\,\text{Ei}_1\left(\ln(xc)\right)$	21
derivativedivides	$c\left(-\frac{1}{xc\ln(xc)} + \text{Ei}_1\left(\ln(xc)\right)\right)$	24
default	$c\left(-\frac{1}{xc\ln(xc)} + \text{Ei}_1\left(\ln(xc)\right)\right)$	24

```
[In] int(1/x^2/ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] -1/x/ln(x*c)+c*Ei(1,ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^2 \log^2(cx)} \, dx = -\frac{cx \log(cx) \operatorname{log\_integral}\left(\frac{1}{cx}\right) + 1}{x \log(cx)}$$

```
[In] integrate(1/x^2/log(c*x)^2,x, algorithm="fricas")
[Out] -(c*x*log(c*x)*log_integral(1/(c*x)) + 1)/(x*log(c*x))
```

Sympy [F]

$$\int \frac{1}{x^2 \log^2(cx)} \, dx = -\int \frac{1}{x^2 \log(cx)} \, dx - \frac{1}{x \log(cx)}$$

```
[In] integrate(1/x**2/ln(c*x)**2,x)
[Out] -Integral(1/(x**2*log(c*x)), x) - 1/(x*log(c*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^2 \log^2(cx)} \, dx = -c\Gamma(-1, \log(cx))$$

```
[In] integrate(1/x^2/log(c*x)^2,x, algorithm="maxima")
[Out] -c*gamma(-1, log(c*x))
```

Giac [F]

$$\int \frac{1}{x^2 \log^2(cx)} \, dx = \int \frac{1}{x^2 \log(cx)^2} \, dx$$

```
[In] integrate(1/x^2/log(c*x)^2,x, algorithm="giac")
[Out] integrate(1/(x^2*log(c*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \log^2(cx)}\,dx = \int \frac{1}{x^2 \ln(c\,x)^2}\,dx$$

```
[In] int(1/(x^2*log(c*x)^2),x)
[Out] int(1/(x^2*log(c*x)^2), x)
```

3.35 $\int \frac{1}{x^3 \log^2(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{1}{x^3 \log^2(cx)}\,dx = -2c^2\,\text{ExpIntegralEi}(-2\log(cx)) - \frac{1}{x^2\log(cx)}$$

```
[Out] -2*c^2*Ei(-2*ln(c*x))-1/x^2/ln(c*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2343, 2346, 2209}

$$\int \frac{1}{x^3 \log^2(cx)}\,dx = -2c^2\,\text{ExpIntegralEi}(-2\log(cx)) - \frac{1}{x^2\log(cx)}$$

```
[In] Int[1/(x^3*Log[c*x]^2),x]
```

```
[Out] -2*c^2*ExpIntegralEi[-2*Log[c*x]] - 1/(x^2*Log[c*x])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
```

```
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{x^2\log(cx)} - 2\int \frac{1}{x^3\log(cx)}\,dx\\ &= -\frac{1}{x^2\log(cx)} - \left(2c^2\right)\text{Subst}\left(\int \frac{e^{-2x}}{x}\,dx, x, \log(cx)\right)\\ &= -2c^2\text{Ei}(-2\log(cx)) - \frac{1}{x^2\log(cx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\log^2(cx)}\,dx = -2c^2\,\text{ExpIntegralEi}(-2\log(cx)) - \frac{1}{x^2\log(cx)}$$

```
[In] Integrate[1/(x^3*Log[c*x]^2),x]
[Out] -2*c^2*ExpIntegralEi[-2*Log[c*x]] - 1/(x^2*Log[c*x])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

method	result	size
risch	$-\frac{1}{x^2\ln(xc)} + 2c^2\,\text{Ei}_1\left(2\ln\left(xc\right)\right)$	26
derivativedivides	$c^2\left(-\frac{1}{x^2c^2\ln(xc)} + 2\,\text{Ei}_1\left(2\ln\left(xc\right)\right)\right)$	30
default	$c^2\left(-\frac{1}{x^2c^2\ln(xc)} + 2\,\text{Ei}_1\left(2\ln\left(xc\right)\right)\right)$	30

```
[In] int(1/x^3/ln(x*c)^2,x,method=_RETURNVERBOSE)
[Out] -1/x^2/ln(x*c)+2*c^2*Ei(1,2*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^3 \log^2(cx)} \, dx = -\frac{2 c^2 x^2 \log(cx) \operatorname{log\_integral}\left(\frac{1}{c^2 x^2}\right) + 1}{x^2 \log(cx)}$$

```
[In] integrate(1/x^3/log(c*x)^2,x, algorithm="fricas")
[Out] -(2*c^2*x^2*log(c*x)*log_integral(1/(c^2*x^2)) + 1)/(x^2*log(c*x))
```

Sympy [F]

$$\int \frac{1}{x^3 \log^2(cx)} \, dx = -2 \int \frac{1}{x^3 \log(cx)} \, dx - \frac{1}{x^2 \log(cx)}$$

```
[In] integrate(1/x**3/ln(c*x)**2,x)
[Out] -2*Integral(1/(x**3*log(c*x)), x) - 1/(x**2*log(c*x))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^3 \log^2(cx)} \, dx = -2 c^2 \Gamma(-1, 2 \log(cx))$$

```
[In] integrate(1/x^3/log(c*x)^2,x, algorithm="maxima")
[Out] -2*c^2*gamma(-1, 2*log(c*x))
```

Giac [F]

$$\int \frac{1}{x^3 \log^2(cx)} \, dx = \int \frac{1}{x^3 \log(cx)^2} \, dx$$

```
[In] integrate(1/x^3/log(c*x)^2,x, algorithm="giac")
[Out] integrate(1/(x^3*log(c*x)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \log^2(cx)}\,dx = \int \frac{1}{x^3 \ln\left(c\,x\right)^2}\,dx$$

```
[In] int(1/(x^3*log(c*x)^2),x)
[Out] int(1/(x^3*log(c*x)^2), x)
```

3.36 $\int \frac{x^3}{\log^3(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int \frac{x^3}{\log^3(cx)}\,dx = \frac{8\,\text{ExpIntegralEi}(4\log(cx))}{c^4} - \frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)}$$

```
[Out] 8*Ei(4*ln(c*x))/c^4-1/2*x^4/ln(c*x)^2-2*x^4/ln(c*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2343, 2346, 2209}

$$\int \frac{x^3}{\log^3(cx)}\,dx = \frac{8\,\text{ExpIntegralEi}(4\log(cx))}{c^4} - \frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)}$$

```
[In] Int[x^3/Log[c*x]^3,x]
```

```
[Out] (8*ExpIntegralEi[4*Log[c*x]])/c^4 - x^4/(2*Log[c*x]^2) - (2*x^4)/Log[c*x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
```

```
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^4}{2\log^2(cx)} + 2\int \frac{x^3}{\log^2(cx)}\,dx \\
&= -\frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)} + 8\int \frac{x^3}{\log(cx)}\,dx \\
&= -\frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)} + \frac{8\text{Subst}\left(\int \frac{e^{4x}}{x}\,dx, x, \log(cx)\right)}{c^4} \\
&= \frac{8\text{Ei}(4\log(cx))}{c^4} - \frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\log^3(cx)}\,dx = \frac{8\,\text{ExpIntegralEi}(4\log(cx))}{c^4} - \frac{x^4}{2\log^2(cx)} - \frac{2x^4}{\log(cx)}$$

```
[In] Integrate[x^3/Log[c*x]^3,x]
[Out] (8*ExpIntegralEi[4*Log[c*x]])/c^4 - x^4/(2*Log[c*x]^2) - (2*x^4)/Log[c*x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{x^{4}(1+4\ln(xc))}{2\ln(xc)^{2}}-\frac{8\,\mathrm{Ei}_{1}(-4\ln(xc))}{c^{4}}$	34
derivativedivides	$\frac{-\frac{x^{4}c^{4}}{2\ln(xc)^{2}}-\frac{2x^{4}c^{4}}{\ln(xc)}-8\,\mathrm{Ei}_{1}(-4\ln(xc))}{c^{4}}$	44
default	$\frac{-\frac{x^{4}c^{4}}{2\ln(xc)^{2}}-\frac{2x^{4}c^{4}}{\ln(xc)}-8\,\mathrm{Ei}_{1}(-4\ln(xc))}{c^{4}}$	44

```
[In] int(x^3/ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*x^4*(1+4*ln(x*c))/ln(x*c)^2-8/c^4*Ei(1,-4*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{x^3}{\log^3(cx)}\,dx = -\frac{4\,c^4x^4\log(cx)+c^4x^4-16\,\log(cx)^2\,\text{log\_integral}\left(c^4x^4\right)}{2\,c^4\log(cx)^2}$$

```
[In] integrate(x^3/log(c*x)^3,x, algorithm="fricas")
[Out] -1/2*(4*c^4*x^4*log(c*x) + c^4*x^4 - 16*log(c*x)^2*log_integral(c^4*x^4))/(
c^4*log(c*x)^2)
```

Sympy [F]

$$\int \frac{x^3}{\log^3(cx)}\,dx = \frac{-4x^4\log(cx)-x^4}{2\log(cx)^2}+8\int \frac{x^3}{\log(cx)}\,dx$$

```
[In] integrate(x**3/ln(c*x)**3,x)
[Out] (-4*x**4*log(c*x) - x**4)/(2*log(c*x)**2) + 8*Integral(x**3/log(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.35

$$\int \frac{x^3}{\log^3(cx)}\,dx = -\frac{16\,\Gamma(-2,-4\,\log(cx))}{c^4}$$

```
[In] integrate(x^3/log(c*x)^3,x, algorithm="maxima")
[Out] -16*gamma(-2, -4*log(c*x))/c^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{\log^3(cx)}\,dx = -\frac{2\,x^4}{\log{(cx)}} - \frac{x^4}{2\,\log{(cx)}^2} + \frac{8\,\mathrm{Ei}(4\,\log{(cx)})}{c^4}$$

```
[In] integrate(x^3/log(c*x)^3,x, algorithm="giac")
[Out] -2*x^4/log(c*x) - 1/2*x^4/log(c*x)^2 + 8*Ei(4*log(c*x))/c^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^3(cx)}\,dx = \int \frac{x^3}{\ln{(c\,x)}^3}\,dx$$

```
[In] int(x^3/log(c*x)^3,x)
[Out] int(x^3/log(c*x)^3, x)
```

3.37 $\int \frac{x^2}{\log^3(cx)} dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{x^2}{\log^3(cx)} dx = \frac{9\,\text{ExpIntegralEi}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}$$

```
[Out] 9/2*Ei(3*ln(c*x))/c^3-1/2*x^3/ln(c*x)^2-3/2*x^3/ln(c*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2343, 2346, 2209}

$$\int \frac{x^2}{\log^3(cx)} dx = \frac{9\,\text{ExpIntegralEi}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}$$

```
[In] Int[x^2/Log[c*x]^3,x]
```

```
[Out] (9*ExpIntegralEi[3*Log[c*x]])/(2*c^3) - x^3/(2*Log[c*x]^2) - (3*x^3)/(2*Log
[c*x])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
```

```
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3}{2\log^2(cx)} + \frac{3}{2}\int \frac{x^2}{\log^2(cx)}\,dx \\
&= -\frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)} + \frac{9}{2}\int \frac{x^2}{\log(cx)}\,dx \\
&= -\frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)} + \frac{9\text{Subst}\left(\int \frac{e^{3x}}{x}\,dx, x, \log(cx)\right)}{2c^3} \\
&= \frac{9\text{Ei}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\log^3(cx)}\,dx = \frac{9\,\text{ExpIntegralEi}(3\log(cx))}{2c^3} - \frac{x^3}{2\log^2(cx)} - \frac{3x^3}{2\log(cx)}$$

```
[In] Integrate[x^2/Log[c*x]^3,x]
[Out] (9*ExpIntegralEi[3*Log[c*x]])/(2*c^3) - x^3/(2*Log[c*x]^2) - (3*x^3)/(2*Log
[c*x])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{x^3(1+3\ln(xc))}{2\ln(xc)^2}-\frac{9\,\mathrm{Ei}_1(-3\ln(xc))}{2c^3}$	34
derivativedivides	$\frac{-\frac{x^3c^3}{2\ln(xc)^2}-\frac{3x^3c^3}{2\ln(xc)}-\frac{9\,\mathrm{Ei}_1(-3\ln(xc))}{2}}{c^3}$	44
default	$\frac{-\frac{x^3c^3}{2\ln(xc)^2}-\frac{3x^3c^3}{2\ln(xc)}-\frac{9\,\mathrm{Ei}_1(-3\ln(xc))}{2}}{c^3}$	44

```
[In] int(x^2/ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*x^3*(1+3*ln(x*c))/ln(x*c)^2-9/2/c^3*Ei(1,-3*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{\log^3(cx)}\,dx = -\frac{3\,c^3x^3\log(cx)+c^3x^3-9\,\log(cx)^2\,\mathrm{log\_integral}\left(c^3x^3\right)}{2\,c^3\log(cx)^2}$$

```
[In] integrate(x^2/log(c*x)^3,x, algorithm="fricas")
[Out] -1/2*(3*c^3*x^3*log(c*x) + c^3*x^3 - 9*log(c*x)^2*log_integral(c^3*x^3))/(c
^3*log(c*x)^2)
```

Sympy [F]

$$\int \frac{x^2}{\log^3(cx)}\,dx = \frac{-3x^3\log(cx)-x^3}{2\log(cx)^2}+\frac{9\int\frac{x^2}{\log(cx)}\,dx}{2}$$

```
[In] integrate(x**2/ln(c*x)**3,x)
[Out] (-3*x**3*log(c*x) - x**3)/(2*log(c*x)**2) + 9*Integral(x**2/log(c*x), x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.32

$$\int \frac{x^2}{\log^3(cx)}\,dx = -\frac{9\,\Gamma(-2,-3\,\log(cx))}{c^3}$$

```
[In] integrate(x^2/log(c*x)^3,x, algorithm="maxima")
[Out] -9*gamma(-2, -3*log(c*x))/c^3
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\log^3(cx)}\,dx = -\frac{3\,x^3}{2\,\log{(cx)}} - \frac{x^3}{2\,\log{(cx)}^2} + \frac{9\,\mathrm{Ei}(3\,\log{(cx)})}{2\,c^3}$$

```
[In] integrate(x^2/log(c*x)^3,x, algorithm="giac")
[Out] -3/2*x^3/log(c*x) - 1/2*x^3/log(c*x)^2 + 9/2*Ei(3*log(c*x))/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\log^3(cx)}\,dx = \int \frac{x^2}{\ln{(c\,x)}^3}\,dx$$

```
[In] int(x^2/log(c*x)^3,x)
[Out] int(x^2/log(c*x)^3, x)
```

3.38 $\int \frac{x}{\log^3(cx)} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{x}{\log^3(cx)} \, dx = \frac{2 \, \text{ExpIntegralEi}(2\log(cx))}{c^2} - \frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)}$$

```
[Out] 2*Ei(2*ln(c*x))/c^2-1/2*x^2/ln(c*x)^2-x^2/ln(c*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2343, 2346, 2209}

$$\int \frac{x}{\log^3(cx)} \, dx = \frac{2 \, \text{ExpIntegralEi}(2\log(cx))}{c^2} - \frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)}$$

```
[In] Int[x/Log[c*x]^3,x]
```

```
[Out] (2*ExpIntegralEi[2*Log[c*x]])/c^2 - x^2/(2*Log[c*x]^2) - x^2/Log[c*x]
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
```

```
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2}{2\log^2(cx)} + \int \frac{x}{\log^2(cx)}\,dx \\
&= -\frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)} + 2\int \frac{x}{\log(cx)}\,dx \\
&= -\frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)} + \frac{2\text{Subst}\left(\int \frac{e^{2x}}{x}\,dx, x, \log(cx)\right)}{c^2} \\
&= \frac{2\text{Ei}(2\log(cx))}{c^2} - \frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{x}{\log^3(cx)}\,dx = \frac{2\,\text{ExpIntegralEi}(2\log(cx))}{c^2} - \frac{x^2}{2\log^2(cx)} - \frac{x^2}{\log(cx)}$$

```
[In] Integrate[x/Log[c*x]^3,x]
[Out] (2*ExpIntegralEi[2*Log[c*x]])/c^2 - x^2/(2*Log[c*x]^2) - x^2/Log[c*x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{x^2(1+2\ln(xc))}{2\ln(xc)^2} - \frac{2\,\mathrm{Ei}_1(-2\ln(xc))}{c^2}$	34
derivativedivides	$\frac{-\frac{x^2c^2}{2\ln(xc)^2}-\frac{x^2c^2}{\ln(xc)}-2\,\mathrm{Ei}_1(-2\ln(xc))}{c^2}$	44
default	$\frac{-\frac{x^2c^2}{2\ln(xc)^2}-\frac{x^2c^2}{\ln(xc)}-2\,\mathrm{Ei}_1(-2\ln(xc))}{c^2}$	44

```
[In] int(x/ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*x^2*(1+2*ln(x*c))/ln(x*c)^2-2/c^2*Ei(1,-2*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{x}{\log^3(cx)}\,dx = -\frac{2\,c^2x^2\log(cx) + c^2x^2 - 4\,\log(cx)^2\,\mathrm{log\_integral}\left(c^2x^2\right)}{2\,c^2\log(cx)^2}$$

```
[In] integrate(x/log(c*x)^3,x, algorithm="fricas")
[Out] -1/2*(2*c^2*x^2*log(c*x) + c^2*x^2 - 4*log(c*x)^2*log_integral(c^2*x^2))/(c
^2*log(c*x)^2)
```

Sympy [F]

$$\int \frac{x}{\log^3(cx)}\,dx = \frac{-2x^2\log(cx) - x^2}{2\log(cx)^2} + 2\int \frac{x}{\log(cx)}\,dx$$

```
[In] integrate(x/ln(c*x)**3,x)
[Out] (-2*x**2*log(c*x) - x**2)/(2*log(c*x)**2) + 2*Integral(x/log(c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.35

$$\int \frac{x}{\log^3(cx)}\,dx = -\frac{4\,\Gamma(-2, -2\,\log(cx))}{c^2}$$

```
[In] integrate(x/log(c*x)^3,x, algorithm="maxima")
[Out] -4*gamma(-2, -2*log(c*x))/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{x}{\log^3(cx)} \, dx = -\frac{x^2}{\log(cx)} - \frac{x^2}{2 \log(cx)^2} + \frac{2 \operatorname{Ei}(2 \log(cx))}{c^2}$$

```
[In] integrate(x/log(c*x)^3,x, algorithm="giac")
[Out] -x^2/log(c*x) - 1/2*x^2/log(c*x)^2 + 2*Ei(2*log(c*x))/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log^3(cx)} \, dx = \int \frac{x}{\ln(c\,x)^3} \, dx$$

```
[In] int(x/log(c*x)^3,x)
[Out] int(x/log(c*x)^3, x)
```

3.39 $\int \frac{1}{\log^3(cx)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 34

$$\int \frac{1}{\log^3(cx)}\,dx = -\frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)} + \frac{\text{LogIntegral}(cx)}{2c}$$

```
[Out] 1/2*Li(c*x)/c-1/2*x/ln(c*x)^2-1/2*x/ln(c*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2334, 2335}

$$\int \frac{1}{\log^3(cx)}\,dx = \frac{\text{LogIntegral}(cx)}{2c} - \frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)}$$

```
[In] Int[Log[c*x]^(-3),x]
```

```
[Out] -1/2*x/Log[c*x]^2 - x/(2*Log[c*x]) + LogIntegral[c*x]/(2*c)
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x}{2\log^2(cx)} + \frac{1}{2}\int \frac{1}{\log^2(cx)}\,dx\\ &= -\frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)} + \frac{1}{2}\int \frac{1}{\log(cx)}\,dx\\ &= -\frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)} + \frac{\operatorname{li}(cx)}{2c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^3(cx)}\,dx = -\frac{x}{2\log^2(cx)} - \frac{x}{2\log(cx)} + \frac{\text{LogIntegral}(cx)}{2c}$$

```
[In] Integrate[Log[c*x]^(-3),x]
[Out] -1/2*x/Log[c*x]^2 - x/(2*Log[c*x]) + LogIntegral[c*x]/(2*c)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{x(1+\ln(xc))}{2\ln(xc)^2} - \frac{\operatorname{Ei}_1(-\ln(xc))}{2c}$	30
derivativedivides	$\frac{-\frac{xc}{2\ln(xc)^2} - \frac{xc}{2\ln(xc)} - \frac{\operatorname{Ei}_1(-\ln(xc))}{2}}{c}$	36
default	$\frac{-\frac{xc}{2\ln(xc)^2} - \frac{xc}{2\ln(xc)} - \frac{\operatorname{Ei}_1(-\ln(xc))}{2}}{c}$	36

```
[In] int(1/ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*x*(1+ln(x*c))/ln(x*c)^2-1/2/c*Ei(1,-ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\log^3(cx)}\, dx = -\frac{cx\log(cx) - \log(cx)^2 \operatorname{log\_integral}(cx) + cx}{2\,c\log(cx)^2}$$

```
[In] integrate(1/log(c*x)^3,x, algorithm="fricas")
[Out] -1/2*(c*x*log(c*x) - log(c*x)^2*log_integral(c*x) + c*x)/(c*log(c*x)^2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{\log^3(cx)}\, dx = \frac{-x\log(cx) - x}{2\log(cx)^2} + \frac{\operatorname{li}(cx)}{2c}$$

```
[In] integrate(1/ln(c*x)**3,x)
[Out] (-x*log(c*x) - x)/(2*log(c*x)**2) + li(c*x)/(2*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.38

$$\int \frac{1}{\log^3(cx)}\, dx = -\frac{\Gamma(-2, -\log(cx))}{c}$$

```
[In] integrate(1/log(c*x)^3,x, algorithm="maxima")
[Out] -gamma(-2, -log(c*x))/c
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{\log^3(cx)}\, dx = \frac{\operatorname{Ei}(\log(cx))}{2\,c} - \frac{x}{2\,\log(cx)} - \frac{x}{2\,\log(cx)^2}$$

```
[In] integrate(1/log(c*x)^3,x, algorithm="giac")
[Out] 1/2*Ei(log(c*x))/c - 1/2*x/log(c*x) - 1/2*x/log(c*x)^2
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{\log^3(cx)}\,dx = \frac{\operatorname{logint}(c\,x)}{2\,c} - \frac{\frac{x}{2} + \frac{x\,\ln(c\,x)}{2}}{\ln{(c\,x)}^2}$$

```
[In] int(1/log(c*x)^3,x)
[Out] logint(c*x)/(2*c) - (x/2 + (x*log(c*x))/2)/log(c*x)^2
```

3.40 $\int \frac{1}{x\log^3(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x\log^3(cx)}\,dx = -\frac{1}{2\log^2(cx)}$$

```
[Out] -1/2/ln(c*x)^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2339, 30}

$$\int \frac{1}{x\log^3(cx)}\,dx = -\frac{1}{2\log^2(cx)}$$

```
[In] Int[1/(x*Log[c*x]^3),x]
```

```
[Out] -1/2*1/Log[c*x]^2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{x^3}\,dx, x, \log(cx)\right)$$
$$= -\frac{1}{2\log^2(cx)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log^3(cx)}\,dx = -\frac{1}{2\log^2(cx)}$$

```
[In] Integrate[1/(x*Log[c*x]^3),x]
[Out] -1/2*1/Log[c*x]^2
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{1}{2\ln(xc)^2}$	9
default	$-\frac{1}{2\ln(xc)^2}$	9
norman	$-\frac{1}{2\ln(xc)^2}$	9
risch	$-\frac{1}{2\ln(xc)^2}$	9
parallelrisch	$-\frac{1}{2\ln(xc)^2}$	9

```
[In] int(1/x/ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/ln(x*c)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x \log^3(cx)} \, dx = -\frac{1}{2 \log(cx)^2}$$

```
[In] integrate(1/x/log(c*x)^3,x, algorithm="fricas")
[Out] -1/2/log(c*x)^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \log^3(cx)} \, dx = -\frac{1}{2 \log(cx)^2}$$

```
[In] integrate(1/x/ln(c*x)**3,x)
[Out] -1/(2*log(c*x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x \log^3(cx)} \, dx = -\frac{1}{2 \log(cx)^2}$$

```
[In] integrate(1/x/log(c*x)^3,x, algorithm="maxima")
[Out] -1/2/log(c*x)^2
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x \log^3(cx)} \, dx = -\frac{1}{2 \log(cx)^2}$$

```
[In] integrate(1/x/log(c*x)^3,x, algorithm="giac")
[Out] -1/2/log(c*x)^2
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x \log^3(cx)}\,dx = -\frac{1}{2\ln{(c\,x)}^2}$$

```
[In] int(1/(x*log(c*x)^3),x)
[Out] -1/(2*log(c*x)^2)
```

3.41 $\int \frac{1}{x^2 \log^3(cx)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{1}{x^2 \log^3(cx)} \, dx = \frac{1}{2} c \,\text{ExpIntegralEi}(-\log(cx)) - \frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)}$$

```
[Out] 1/2*c*Ei(-ln(c*x))-1/2/x/ln(c*x)^2+1/2/x/ln(c*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2343, 2346, 2209}

$$\int \frac{1}{x^2 \log^3(cx)} \, dx = \frac{1}{2} c \,\text{ExpIntegralEi}(-\log(cx)) - \frac{1}{2x \log^2(cx)} + \frac{1}{2x \log(cx)}$$

```
[In] Int[1/(x^2*Log[c*x]^3),x]
```

```
[Out] (c*ExpIntegralEi[-Log[c*x]])/2 - 1/(2*x*Log[c*x]^2) + 1/(2*x*Log[c*x])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
```

```
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2x\log^2(cx)} - \frac{1}{2}\int \frac{1}{x^2\log^2(cx)}\,dx \\ &= -\frac{1}{2x\log^2(cx)} + \frac{1}{2x\log(cx)} + \frac{1}{2}\int \frac{1}{x^2\log(cx)}\,dx \\ &= -\frac{1}{2x\log^2(cx)} + \frac{1}{2x\log(cx)} + \frac{1}{2}c\text{Subst}\left(\int \frac{e^{-x}}{x}\,dx, x, \log(cx)\right) \\ &= \frac{1}{2}c\text{Ei}(-\log(cx)) - \frac{1}{2x\log^2(cx)} + \frac{1}{2x\log(cx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\log^3(cx)}\,dx = \frac{1}{2}c\,\text{ExpIntegralEi}(-\log(cx)) - \frac{1}{2x\log^2(cx)} + \frac{1}{2x\log(cx)}$$

```
[In] Integrate[1/(x^2*Log[c*x]^3),x]
[Out] (c*ExpIntegralEi[-Log[c*x]])/2 - 1/(2*x*Log[c*x]^2) + 1/(2*x*Log[c*x])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{\ln(xc)-1}{2x\ln(xc)^2} - \frac{c\,\text{Ei}_1(\ln(xc))}{2}$	28
derivativedivides	$c\left(-\frac{1}{2xc\ln(xc)^2} + \frac{1}{2xc\ln(xc)} - \frac{\text{Ei}_1(\ln(xc))}{2}\right)$	40
default	$c\left(-\frac{1}{2xc\ln(xc)^2} + \frac{1}{2xc\ln(xc)} - \frac{\text{Ei}_1(\ln(xc))}{2}\right)$	40

```
[In] int(1/x^2/ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] 1/2*(ln(x*c)-1)/x/ln(x*c)^2-1/2*c*Ei(1,ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2 \log^3(cx)} \, dx = \frac{cx \log{(cx)}^2 \operatorname{log\_integral}\left(\frac{1}{cx}\right) + \log{(cx)} - 1}{2 \, x \log{(cx)}^2}$$

```
[In] integrate(1/x^2/log(c*x)^3,x, algorithm="fricas")
[Out] 1/2*(c*x*log(c*x)^2*log_integral(1/(c*x)) + log(c*x) - 1)/(x*log(c*x)^2)
```

Sympy [F]

$$\int \frac{1}{x^2 \log^3(cx)} \, dx = \frac{\int \frac{1}{x^2 \log{(cx)}} \, dx}{2} + \frac{\log{(cx)} - 1}{2x \log{(cx)}^2}$$

```
[In] integrate(1/x**2/ln(c*x)**3,x)
[Out] Integral(1/(x**2*log(c*x)), x)/2 + (log(c*x) - 1)/(2*x*log(c*x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^2 \log^3(cx)} \, dx = -c\Gamma(-2, \log{(cx)})$$

```
[In] integrate(1/x^2/log(c*x)^3,x, algorithm="maxima")
[Out] -c*gamma(-2, log(c*x))
```

Giac [F]

$$\int \frac{1}{x^2 \log^3(cx)} \, dx = \int \frac{1}{x^2 \log{(cx)}^3} \, dx$$

```
[In] integrate(1/x^2/log(c*x)^3,x, algorithm="giac")
[Out] integrate(1/(x^2*log(c*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \log^3(cx)} \, dx = \int \frac{1}{x^2 \ln{(c\,x)}^3} \, dx$$

```
[In] int(1/(x^2*log(c*x)^3),x)
[Out] int(1/(x^2*log(c*x)^3), x)
```

3.42 $\int \frac{1}{x^3 \log^3(cx)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{x^3 \log^3(cx)} \, dx = 2c^2 \operatorname{ExpIntegralEi}(-2\log(cx)) - \frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)}$$

```
[Out] 2*c^2*Ei(-2*ln(c*x))-1/2/x^2/ln(c*x)^2+1/x^2/ln(c*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2343, 2346, 2209}

$$\int \frac{1}{x^3 \log^3(cx)} \, dx = 2c^2 \operatorname{ExpIntegralEi}(-2\log(cx)) - \frac{1}{2x^2 \log^2(cx)} + \frac{1}{x^2 \log(cx)}$$

```
[In] Int[1/(x^3*Log[c*x]^3),x]
```

```
[Out] 2*c^2*ExpIntegralEi[-2*Log[c*x]] - 1/(2*x^2*Log[c*x]^2) + 1/(x^2*Log[c*x])
```

<u>Rule 2209</u>

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

<u>Rule 2343</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
```

```
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{2x^2\log^2(cx)} - \int \frac{1}{x^3\log^2(cx)}\,dx \\
&= -\frac{1}{2x^2\log^2(cx)} + \frac{1}{x^2\log(cx)} + 2\int \frac{1}{x^3\log(cx)}\,dx \\
&= -\frac{1}{2x^2\log^2(cx)} + \frac{1}{x^2\log(cx)} + \left(2c^2\right)\text{Subst}\left(\int \frac{e^{-2x}}{x}\,dx, x, \log(cx)\right) \\
&= 2c^2\text{Ei}(-2\log(cx)) - \frac{1}{2x^2\log^2(cx)} + \frac{1}{x^2\log(cx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\log^3(cx)}\,dx = 2c^2\,\text{ExpIntegralEi}(-2\log(cx)) - \frac{1}{2x^2\log^2(cx)} + \frac{1}{x^2\log(cx)}$$

```
[In] Integrate[1/(x^3*Log[c*x]^3),x]
[Out] 2*c^2*ExpIntegralEi[-2*Log[c*x]] - 1/(2*x^2*Log[c*x]^2) + 1/(x^2*Log[c*x])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{-1+2\ln(xc)}{2x^2\ln(xc)^2} - 2c^2\,\text{Ei}_1\left(2\ln\left(xc\right)\right)$	34
derivativedivides	$c^2\left(-\frac{1}{2x^2c^2\ln(xc)^2} + \frac{1}{x^2c^2\ln(xc)} - 2\ \text{Ei}_1\left(2\ln\left(xc\right)\right)\right)$	43
default	$c^2\left(-\frac{1}{2x^2c^2\ln(xc)^2} + \frac{1}{x^2c^2\ln(xc)} - 2\ \text{Ei}_1\left(2\ln\left(xc\right)\right)\right)$	43

```
[In] int(1/x^3/ln(x*c)^3,x,method=_RETURNVERBOSE)
[Out] 1/2*(-1+2*ln(x*c))/x^2/ln(x*c)^2-2*c^2*Ei(1,2*ln(x*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3 \log^3(cx)} \, dx = \frac{4 \, c^2 x^2 \log(cx)^2 \operatorname{log\_integral}\left(\frac{1}{c^2 x^2}\right) + 2 \, \log(cx) - 1}{2 \, x^2 \log(cx)^2}$$

```
[In] integrate(1/x^3/log(c*x)^3,x, algorithm="fricas")
[Out] 1/2*(4*c^2*x^2*log(c*x)^2*log_integral(1/(c^2*x^2)) + 2*log(c*x) - 1)/(x^2*
log(c*x)^2)
```

Sympy [F]

$$\int \frac{1}{x^3 \log^3(cx)} \, dx = 2 \int \frac{1}{x^3 \log(cx)} \, dx + \frac{2 \log(cx) - 1}{2x^2 \log(cx)^2}$$

```
[In] integrate(1/x**3/ln(c*x)**3,x)
[Out] 2*Integral(1/(x**3*log(c*x)), x) + (2*log(c*x) - 1)/(2*x**2*log(c*x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^3 \log^3(cx)} \, dx = -4 \, c^2 \Gamma(-2, 2 \, \log(cx))$$

```
[In] integrate(1/x^3/log(c*x)^3,x, algorithm="maxima")
[Out] -4*c^2*gamma(-2, 2*log(c*x))
```

Giac [F]

$$\int \frac{1}{x^3 \log^3(cx)} \, dx = \int \frac{1}{x^3 \log{(cx)}^3} \, dx$$

```
[In] integrate(1/x^3/log(c*x)^3,x, algorithm="giac")
[Out] integrate(1/(x^3*log(c*x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \log^3(cx)} \, dx = \int \frac{1}{x^3 \ln{(c\,x)}^3} \, dx$$

```
[In] int(1/(x^3*log(c*x)^3),x)
[Out] int(1/(x^3*log(c*x)^3), x)
```

3.43 $\int x^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int x^3(a+b\log(cx^n))\,dx=-\frac{1}{16}bnx^4+\frac{1}{4}x^4(a+b\log(cx^n))$$

```
[Out] -1/16*b*n*x^4+1/4*x^4*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2341}

$$\int x^3(a+b\log(cx^n))\,dx=\frac{1}{4}x^4(a+b\log(cx^n))-\frac{1}{16}bnx^4$$

```
[In] Int[x^3*(a + b*Log[c*x^n]),x]
[Out] -1/16*(b*n*x^4) + (x^4*(a + b*Log[c*x^n]))/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=-\frac{1}{16}bnx^4+\frac{1}{4}x^4(a+b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^3(a+b\log(cx^n))\,dx = \frac{ax^4}{4} - \frac{1}{16}bnx^4 + \frac{1}{4}bx^4\log(cx^n)$$

```
[In] Integrate[x^3*(a + b*Log[c*x^n]),x]
[Out] (a*x^4)/4 - (b*n*x^4)/16 + (b*x^4*Log[c*x^n])/4
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{x^4\ln(cx^n)b}{4}-\frac{bn\,x^4}{16}+\frac{a\,x^4}{4}$
risch	$\frac{b\,x^4\ln(x^n)}{4}+\frac{x^4\left(-2ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+2ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+2ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-2ib\pi\ \mathrm{csgn}(ic\right.}{16}$

```
[In] int(x^3*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*ln(c*x^n)*b-1/16*b*n*x^4+1/4*a*x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int x^3(a+b\log(cx^n))\,dx = \frac{1}{4}bnx^4\log(x) + \frac{1}{4}bx^4\log(c) - \frac{1}{16}(bn-4a)x^4$$

```
[In] integrate(x^3*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] 1/4*b*n*x^4*log(x) + 1/4*b*x^4*log(c) - 1/16*(b*n - 4*a)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^3(a+b\log(cx^n))\,dx=\frac{ax^4}{4}-\frac{bnx^4}{16}+\frac{bx^4\log(cx^n)}{4}$$

```
[In] integrate(x**3*(a+b*ln(c*x**n)),x)
[Out] a*x**4/4 - b*n*x**4/16 + b*x**4*log(c*x**n)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x^3(a+b\log(cx^n))\,dx=-\frac{1}{16}\,bnx^4+\frac{1}{4}\,bx^4\log(cx^n)+\frac{1}{4}\,ax^4$$

```
[In] integrate(x^3*(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] -1/16*b*n*x^4 + 1/4*b*x^4*log(c*x^n) + 1/4*a*x^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int x^3(a+b\log(cx^n))\,dx=\frac{1}{4}\,bnx^4\log(x)-\frac{1}{16}\,bnx^4+\frac{1}{4}\,bx^4\log(c)+\frac{1}{4}\,ax^4$$

```
[In] integrate(x^3*(a+b*log(c*x^n)),x, algorithm="giac")
[Out] 1/4*b*n*x^4*log(x) - 1/16*b*n*x^4 + 1/4*b*x^4*log(c) + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^3(a+b\log(cx^n))\,dx=x^4\left(\frac{a}{4}-\frac{b\,n}{16}\right)+\frac{b\,x^4\,\ln(c\,x^n)}{4}$$

```
[In] int(x^3*(a + b*log(c*x^n)),x)
[Out] x^4*(a/4 - (b*n)/16) + (b*x^4*log(c*x^n))/4
```

3.44 $\int x^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int x^2(a+b\log(cx^n))\,dx=-\frac{1}{9}bnx^3+\frac{1}{3}x^3(a+b\log(cx^n))$$

```
[Out] -1/9*b*n*x^3+1/3*x^3*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2341}

$$\int x^2(a+b\log(cx^n))\,dx=\frac{1}{3}x^3(a+b\log(cx^n))-\frac{1}{9}bnx^3$$

```
[In] Int[x^2*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/9*(b*n*x^3) + (x^3*(a + b*Log[c*x^n]))/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=-\frac{1}{9}bnx^3+\frac{1}{3}x^3(a+b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x^2(a+b\log(cx^n))\,dx=\frac{ax^3}{3}-\frac{1}{9}bnx^3+\frac{1}{3}bx^3\log(cx^n)$$

```
[In] Integrate[x^2*(a + b*Log[c*x^n]),x]
[Out] (a*x^3)/3 - (b*n*x^3)/9 + (b*x^3*Log[c*x^n])/3
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{x^3 b\ln(c\,x^n)}{3}-\frac{bn\,x^3}{9}+\frac{x^3 a}{3}$
risch	$\frac{b\,x^3\ln(x^n)}{3}+\frac{x^3\left(-3ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+3ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+3ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-3ib\pi\ \mathrm{csgn}(\right.}{18}$

```
[In] int(x^2*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*b*ln(c*x^n)-1/9*b*n*x^3+1/3*x^3*a
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int x^2(a+b\log(cx^n))\,dx=\frac{1}{3}\,bnx^3\log(x)+\frac{1}{3}\,bx^3\log(c)-\frac{1}{9}\,(bn-3\,a)x^3$$

```
[In] integrate(x^2*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] 1/3*b*n*x^3*log(x) + 1/3*b*x^3*log(c) - 1/9*(b*n - 3*a)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x^2(a+b\log(cx^n))\,dx = \frac{ax^3}{3} - \frac{bnx^3}{9} + \frac{bx^3\log(cx^n)}{3}$$

```
[In] integrate(x**2*(a+b*ln(c*x**n)),x)
[Out] a*x**3/3 - b*n*x**3/9 + b*x**3*log(c*x**n)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x^2(a+b\log(cx^n))\,dx = -\frac{1}{9}\,bnx^3 + \frac{1}{3}\,bx^3\log(cx^n) + \frac{1}{3}\,ax^3$$

```
[In] integrate(x^2*(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] -1/9*b*n*x^3 + 1/3*b*x^3*log(c*x^n) + 1/3*a*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int x^2(a+b\log(cx^n))\,dx = \frac{1}{3}\,bnx^3\log(x) - \frac{1}{9}\,bnx^3 + \frac{1}{3}\,bx^3\log(c) + \frac{1}{3}\,ax^3$$

```
[In] integrate(x^2*(a+b*log(c*x^n)),x, algorithm="giac")
[Out] 1/3*b*n*x^3*log(x) - 1/9*b*n*x^3 + 1/3*b*x^3*log(c) + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x^2(a+b\log(cx^n))\,dx = x^3\left(\frac{a}{3} - \frac{b\,n}{9}\right) + \frac{b\,x^3\,\ln(c\,x^n)}{3}$$

```
[In] int(x^2*(a + b*log(c*x^n)),x)
[Out] x^3*(a/3 - (b*n)/9) + (b*x^3*log(c*x^n))/3
```

3.45 $\int x(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int x(a+b\log(cx^n))\,dx=-\frac{1}{4}bnx^2+\frac{1}{2}x^2(a+b\log(cx^n))$$

```
[Out] -1/4*b*n*x^2+1/2*x^2*(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2341}

$$\int x(a+b\log(cx^n))\,dx=\frac{1}{2}x^2(a+b\log(cx^n))-\frac{1}{4}bnx^2$$

```
[In] Int[x*(a + b*Log[c*x^n]),x]
```

```
[Out] -1/4*(b*n*x^2) + (x^2*(a + b*Log[c*x^n]))/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=-\frac{1}{4}bnx^2+\frac{1}{2}x^2(a+b\log(cx^n))$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int x(a+b\log(cx^n))\,dx=\frac{ax^2}{2}-\frac{1}{4}bnx^2+\frac{1}{2}bx^2\log(cx^n)$$

```
[In] Integrate[x*(a + b*Log[c*x^n]),x]
[Out] (a*x^2)/2 - (b*n*x^2)/4 + (b*x^2*Log[c*x^n])/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{x^{2} b \ln\left(c\,x^{n}\right)}{2}-\frac{b n\,x^{2}}{4}+\frac{x^{2} a}{2}$
norman	$\left(-\frac{b n}{4}+\frac{a}{2}\right) x^{2}+\frac{b\,x^{2} \ln\left(c\,{\mathrm e}^{n \ln\left(x\right)}\right)}{2}$
default	$\frac{x^{2} a}{2}+\frac{b\,x^{2} \ln\left(c\,{\mathrm e}^{n \ln\left(x\right)}\right)}{2}-\frac{b n\,x^{2}}{4}$
parts	$\frac{x^{2} a}{2}+\frac{b\,x^{2} \ln\left(c\,{\mathrm e}^{n \ln\left(x\right)}\right)}{2}-\frac{b n\,x^{2}}{4}$
risch	$\frac{b\,x^{2} \ln\left(x^{n}\right)}{2}+\frac{x^{2} \left(-i b \pi\ \mathrm{csgn}\left(i c\right) \mathrm{csgn}\left(i x^{n}\right) \mathrm{csgn}\left(i c\,x^{n}\right)+i b \pi\ \mathrm{csgn}\left(i c\right) \mathrm{csgn}\left(i c\,x^{n}\right)^{2}+i b \pi\ \mathrm{csgn}\left(i x^{n}\right) \mathrm{csgn}\left(i c\,x^{n}\right)^{2}-i b \pi\, \mathrm{csgn}\left(i c\,x^{n}\right)^{3}}{4}$

```
[In] int(x*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*b*ln(c*x^n)-1/4*b*n*x^2+1/2*x^2*a
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int x(a+b\log(cx^n))\,dx=\frac{1}{2}\,bnx^2\log(x)+\frac{1}{2}\,bx^2\log(c)-\frac{1}{4}\,(bn-2\,a)x^2$$

```
[In] integrate(x*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] 1/2*b*n*x^2*log(x) + 1/2*b*x^2*log(c) - 1/4*(b*n - 2*a)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int x(a+b\log(cx^n))\,dx = \frac{ax^2}{2} - \frac{bnx^2}{4} + \frac{bx^2\log(cx^n)}{2}$$

```
[In] integrate(x*(a+b*ln(c*x**n)),x)
[Out] a*x**2/2 - b*n*x**2/4 + b*x**2*log(c*x**n)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int x(a+b\log(cx^n))\,dx = -\frac{1}{4}\,bnx^2 + \frac{1}{2}\,bx^2\log(cx^n) + \frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] -1/4*b*n*x^2 + 1/2*b*x^2*log(c*x^n) + 1/2*a*x^2
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int x(a+b\log(cx^n))\,dx = \frac{1}{2}\,bnx^2\log(x) - \frac{1}{4}\,bnx^2 + \frac{1}{2}\,bx^2\log(c) + \frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*log(c*x^n)),x, algorithm="giac")
[Out] 1/2*b*n*x^2*log(x) - 1/4*b*n*x^2 + 1/2*b*x^2*log(c) + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int x(a+b\log(cx^n))\,dx = x^2\left(\frac{a}{2} - \frac{b\,n}{4}\right) + \frac{b\,x^2\,\ln(c\,x^n)}{2}$$

```
[In] int(x*(a + b*log(c*x^n)),x)
[Out] x^2*(a/2 - (b*n)/4) + (b*x^2*log(c*x^n))/2
```

3.46 $\int (a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 18

$$\int (a+b\log(cx^n))\,dx = ax-bnx+bx\log(cx^n)$$

```
[Out] a*x-b*n*x+b*x*ln(c*x^n)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2332}

$$\int (a+b\log(cx^n))\,dx = ax+bx\log(cx^n)-bnx$$

```
[In] Int[a + b*Log[c*x^n],x]
```

```
[Out] a*x - b*n*x + b*x*Log[c*x^n]
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax+b\int \log(cx^n)\,dx\\ &= ax-bnx+bx\log(cx^n)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + b\log(cx^n))\, dx = ax - bnx + bx\log(cx^n)$$

```
[In] Integrate[a + b*Log[c*x^n],x]
[Out] a*x - b*n*x + b*x*Log[c*x^n]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
default	$ax - bnx + bx\ln(c\,x^n)$
parts	$ax - bnx + bx\ln(c\,x^n)$
parallelrisch	$b(x\ln(c\,x^n) - nx) + ax$
norman	$(-bn+a)\,x + bx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)$
risch	$ax + bx\ln(x^n) + \frac{bx\left(-i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n) + i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2 + i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2 - i\pi\ \mathrm{csgn}\right.}{2}$

```
[In] int(a+b*ln(c*x^n),x,method=_RETURNVERBOSE)
[Out] a*x-b*n*x+b*x*ln(c*x^n)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int (a + b\log(cx^n))\, dx = bnx\log(x) + bx\log(c) - (bn - a)x$$

```
[In] integrate(a+b*log(c*x^n),x, algorithm="fricas")
[Out] b*n*x*log(x) + b*x*log(c) - (b*n - a)*x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int (a + b \log (cx^n)) \, dx = ax + b(-nx + x \log (cx^n))$$

```
[In] integrate(a+b*ln(c*x**n),x)
[Out] a*x + b*(-n*x + x*log(c*x**n))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + b \log (cx^n)) \, dx = -bnx + bx \log (cx^n) + ax$$

```
[In] integrate(a+b*log(c*x^n),x, algorithm="maxima")
[Out] -b*n*x + b*x*log(c*x^n) + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a + b \log (cx^n)) \, dx = (nx \log (x) - nx + x \log (c))b + ax$$

```
[In] integrate(a+b*log(c*x^n),x, algorithm="giac")
[Out] (n*x*log(x) - n*x + x*log(c))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (a + b \log (cx^n)) \, dx = x\,(a - b\,n) + b\,x\,\ln (c\,x^n)$$

```
[In] int(a + b*log(c*x^n),x)
[Out] x*(a - b*n) + b*x*log(c*x^n)
```

3.47 $\int \frac{a+b\log(cx^n)}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 22

$$\int \frac{a+b\log(cx^n)}{x}\,dx=\frac{(a+b\log(cx^n))^2}{2bn}$$

```
[Out] 1/2*(a+b*ln(c*x^n))^2/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2338}

$$\int \frac{a+b\log(cx^n)}{x}\,dx=\frac{(a+b\log(cx^n))^2}{2bn}$$

```
[In] Int[(a + b*Log[c*x^n])/x,x]
```

```
[Out] (a + b*Log[c*x^n])^2/(2*b*n)
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\text{integral}=\frac{(a+b\log(cx^n))^2}{2bn}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{a+b\log(cx^n)}{x}\,dx = a\log(x) + \frac{b\log^2(cx^n)}{2n}$$

```
[In] Integrate[(a + b*Log[c*x^n])/x,x]
[Out] a*Log[x] + (b*Log[c*x^n]^2)/(2*n)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

method	result
parts	$\ln(x)\,a+\frac{b\ln(c\,x^{n})^{2}}{2n}$
parallelrisch	$\frac{2\ln(x)an+b\ln(c\,x^{n})^{2}}{2n}$
derivativedivides	$\frac{\frac{b\ln(c\,x^{n})^{2}}{2}+\ln(c\,x^{n})a}{n}$
default	$\frac{\frac{b\ln(c\,x^{n})^{2}}{2}+\ln(c\,x^{n})a}{n}$
norman	$\frac{a\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)}{n}+\frac{b\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^{2}}{2n}$
risch	$b\ln(x)\ln(x^{n})-\frac{bn\ln(x)^{2}}{2}-\frac{i\ln(x)\pi b\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})}{2}+\frac{i\ln(x)\pi b\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^{n})^{2}}{2}+$ i ln(

```
[In] int((a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
[Out] ln(x)*a+1/2*b/n*ln(c*x^n)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{a+b\log(cx^n)}{x}\,dx = \frac{1}{2}bn\log(x)^2 + (b\log(c)+a)\log(x)$$

```
[In] integrate((a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] 1/2*b*n*log(x)^2 + (b*log(c) + a)*log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 1.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{a+b\log(cx^n)}{x}\,dx = \begin{cases} a\log(x) & \text{for}\ b=0 \\ -(-a-b\log(c))\log(x) & \text{for}\ n=0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln(c*x**n))/x,x)
[Out] Piecewise((a*log(x), Eq(b, 0)), (-(-a - b*log(c))*log(x), Eq(n, 0)), ((-a -
 b*log(c*x**n))**2/(2*b*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a+b\log(cx^n)}{x}\,dx = \frac{(b\log(cx^n)+a)^2}{2\,bn}$$

```
[In] integrate((a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] 1/2*(b*log(c*x^n) + a)^2/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{a+b\log(cx^n)}{x}\,dx = \frac{1}{2}\,bn\log(x)^2 + b\log(c)\log(x) + a\log(x)$$

```
[In] integrate((a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] 1/2*b*n*log(x)^2 + b*log(c)*log(x) + a*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{a + b\log\left(cx^n\right)}{x}\,dx = a\,\ln\left(x\right) + \frac{b\ln\left(c\,x^n\right)^2}{2\,n}$$

```
[In] int((a + b*log(c*x^n))/x,x)
[Out] a*log(x) + (b*log(c*x^n)^2)/(2*n)
```

3.48 $\int \frac{a+b\log(cx^n)}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{a+b\log(cx^n)}{x^2}\,dx = -\frac{bn}{x} - \frac{a+b\log(cx^n)}{x}$$

```
[Out] -b*n/x+(-a-b*ln(c*x^n))/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2341}

$$\int \frac{a+b\log(cx^n)}{x^2}\,dx = -\frac{a+b\log(cx^n)}{x} - \frac{bn}{x}$$

```
[In] Int[(a + b*Log[c*x^n])/x^2,x]
```

```
[Out] -((b*n)/x) - (a + b*Log[c*x^n])/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{bn}{x} - \frac{a+b\log(cx^n)}{x}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{a+b\log\left(cx^n\right)}{x^2}\,dx=-\frac{a}{x}-\frac{bn}{x}-\frac{b\log\left(cx^n\right)}{x}$$

```
[In] Integrate[(a + b*Log[c*x^n])/x^2,x]
[Out] -(a/x) - (b*n)/x - (b*Log[c*x^n])/x
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result
parallelrisch	$-\frac{b\ln(c\,x^n)+bn+a}{x}$
risch	$-\frac{b\ln(x^n)}{x}-\frac{-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2}{2x}$

```
[In] int((a+b*ln(c*x^n))/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x*(b*ln(c*x^n)+b*n+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(cx^n\right)}{x^2}\,dx=-\frac{bn\log\left(x\right)+bn+b\log\left(c\right)+a}{x}$$

```
[In] integrate((a+b*log(c*x^n))/x^2,x, algorithm="fricas")
[Out] -(b*n*log(x) + b*n + b*log(c) + a)/x
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{a+b\log\left(cx^n\right)}{x^2}\,dx = -\frac{a}{x} - \frac{bn}{x} - \frac{b\log\left(cx^n\right)}{x}$$

```
[In] integrate((a+b*ln(c*x**n))/x**2,x)
[Out] -a/x - b*n/x - b*log(c*x**n)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{a+b\log\left(cx^n\right)}{x^2}\,dx = -\frac{bn}{x} - \frac{b\log\left(cx^n\right)}{x} - \frac{a}{x}$$

```
[In] integrate((a+b*log(c*x^n))/x^2,x, algorithm="maxima")
[Out] -b*n/x - b*log(c*x^n)/x - a/x
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{a+b\log\left(cx^n\right)}{x^2}\,dx = -\frac{bn\log\left(x\right)}{x} - \frac{bn+b\log\left(c\right)+a}{x}$$

```
[In] integrate((a+b*log(c*x^n))/x^2,x, algorithm="giac")
[Out] -b*n*log(x)/x - (b*n + b*log(c) + a)/x
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(cx^n\right)}{x^2}\,dx = -\frac{a+b\,n}{x} - \frac{b\,\ln\left(c\,x^n\right)}{x}$$

```
[In] int((a + b*log(c*x^n))/x^2,x)
[Out] - (a + b*n)/x - (b*log(c*x^n))/x
```

3.49 $\int \frac{a+b\log(cx^n)}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{a+b\log(cx^n)}{x^3}\,dx = -\frac{bn}{4x^2}-\frac{a+b\log(cx^n)}{2x^2}$$

```
[Out] -1/4*b*n/x^2+1/2*(-a-b*ln(c*x^n))/x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2341}

$$\int \frac{a+b\log(cx^n)}{x^3}\,dx = -\frac{a+b\log(cx^n)}{2x^2}-\frac{bn}{4x^2}$$

```
[In] Int[(a + b*Log[c*x^n])/x^3,x]
```

```
[Out] -1/4*(b*n)/x^2 - (a + b*Log[c*x^n])/(2*x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{bn}{4x^2}-\frac{a+b\log(cx^n)}{2x^2}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log(cx^n)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{bn}{4x^2}-\frac{b\log(cx^n)}{2x^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])/x^3,x]
[Out] -1/2*a/x^2 - (b*n)/(4*x^2) - (b*Log[c*x^n])/(2*x^2)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result
parallelrisch	$-\frac{2b\ln(c\,x^n)+bn+2a}{4x^2}$
risch	$-\frac{b\ln(x^n)}{2x^2}-\frac{-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+}{4x^2}$

```
[In] int((a+b*ln(c*x^n))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/4/x^2*(2*b*ln(c*x^n)+b*n+2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{a+b\log(cx^n)}{x^3}\,dx=-\frac{2\,bn\log(x)+bn+2\,b\log(c)+2\,a}{4\,x^2}$$

```
[In] integrate((a+b*log(c*x^n))/x^3,x, algorithm="fricas")
[Out] -1/4*(2*b*n*log(x) + b*n + 2*b*log(c) + 2*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{a + b\log(cx^n)}{x^3}\,dx = -\frac{a}{2x^2} - \frac{bn}{4x^2} - \frac{b\log(cx^n)}{2x^2}$$

```
[In] integrate((a+b*ln(c*x**n))/x**3,x)
[Out] -a/(2*x**2) - b*n/(4*x**2) - b*log(c*x**n)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{a + b\log(cx^n)}{x^3}\,dx = -\frac{bn}{4\,x^2} - \frac{b\log(cx^n)}{2\,x^2} - \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*log(c*x^n))/x^3,x, algorithm="maxima")
[Out] -1/4*b*n/x^2 - 1/2*b*log(c*x^n)/x^2 - 1/2*a/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{a + b\log(cx^n)}{x^3}\,dx = -\frac{bn\log(x)}{2\,x^2} - \frac{bn + 2\,b\log(c) + 2\,a}{4\,x^2}$$

```
[In] integrate((a+b*log(c*x^n))/x^3,x, algorithm="giac")
[Out] -1/2*b*n*log(x)/x^2 - 1/4*(b*n + 2*b*log(c) + 2*a)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{a + b\log(cx^n)}{x^3}\,dx = -\frac{\frac{a}{2} + \frac{b\,n}{4}}{x^2} - \frac{b\,\ln(c\,x^n)}{2\,x^2}$$

```
[In] int((a + b*log(c*x^n))/x^3,x)
[Out] - (a/2 + (b*n)/4)/x^2 - (b*log(c*x^n))/(2*x^2)
```

3.50 $\int x^3(a+b\log(cx^n))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int x^3(a+b\log(cx^n))^2\,dx=\frac{1}{32}b^2n^2x^4-\frac{1}{8}bnx^4(a+b\log(cx^n))+\frac{1}{4}x^4(a+b\log(cx^n))^2$$

```
[Out] 1/32*b^2*n^2*x^4-1/8*b*n*x^4*(a+b*ln(c*x^n))+1/4*x^4*(a+b*ln(c*x^n))^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2342, 2341}

$$\int x^3(a+b\log(cx^n))^2\,dx=\frac{1}{4}x^4(a+b\log(cx^n))^2-\frac{1}{8}bnx^4(a+b\log(cx^n))+\frac{1}{32}b^2n^2x^4$$

```
[In] Int[x^3*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (b^2*n^2*x^4)/32 - (b*n*x^4*(a + b*Log[c*x^n]))/8 + (x^4*(a + b*Log[c*x^n])
^2)/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^4(a+b\log(cx^n))^2 - \frac{1}{2}(bn)\int x^3(a+b\log(cx^n))\,dx \\ &= \frac{1}{32}b^2n^2x^4 - \frac{1}{8}bnx^4(a+b\log(cx^n)) + \frac{1}{4}x^4(a+b\log(cx^n))^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int x^3(a+b\log(cx^n))^2\,dx = \frac{1}{32}x^4\left(b^2n^2 - 4bn(a+b\log(cx^n)) + 8(a+b\log(cx^n))^2\right)$$

```
[In] Integrate[x^3*(a + b*Log[c*x^n])^2,x]
[Out] (x^4*(b^2*n^2 - 4*b*n*(a + b*Log[c*x^n]) + 8*(a + b*Log[c*x^n])^2))/32
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

method	result
parallelrisch	$\frac{x^4\ln(cx^n)^2b^2}{4} - \frac{\ln(cx^n)x^4b^2n}{8} + \frac{b^2n^2x^4}{32} + \frac{\ln(cx^n)x^4ab}{2} - \frac{abn\,x^4}{8} + \frac{a^2x^4}{4}$
risch	$\frac{x^4b^2\ln(x^n)^2}{4} + \frac{b\,x^4\left(-2ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+2ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+2ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-2ib\pi\ \mathrm{csg}\right.}{8}$

```
[In] int(x^3*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*ln(c*x^n)^2*b^2-1/8*ln(c*x^n)*x^4*b^2*n+1/32*b^2*n^2*x^4+1/2*ln(c*x
^n)*x^4*a*b-1/8*a*b*n*x^4+1/4*a^2*x^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(46) = 92$.

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.96

$$\int x^3(a+b\log(cx^n))^2\,dx = \frac{1}{4}\,b^2n^2x^4\log(x)^2 + \frac{1}{4}\,b^2x^4\log(c)^2 - \frac{1}{8}\,(b^2n-4\,ab)x^4\log(c) + \frac{1}{32}\,(b^2n^2-4\,abn+8\,a^2)x^4 + \frac{1}{8}\,(4\,b^2nx^4\log(c)-(b^2n^2-4\,abn)x^4)\log(x)$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*n^2*x^4*log(x)^2 + 1/4*b^2*x^4*log(c)^2 - 1/8*(b^2*n - 4*a*b)*x^4*l
og(c) + 1/32*(b^2*n^2 - 4*a*b*n + 8*a^2)*x^4 + 1/8*(4*b^2*n*x^4*log(c) - (b
^2*n^2 - 4*a*b*n)*x^4)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int x^3(a+b\log(cx^n))^2\,dx = \frac{a^2x^4}{4} - \frac{abnx^4}{8} + \frac{abx^4\log(cx^n)}{2} + \frac{b^2n^2x^4}{32} - \frac{b^2nx^4\log(cx^n)}{8} + \frac{b^2x^4\log(cx^n)^2}{4}$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*n*x**4/8 + a*b*x**4*log(c*x**n)/2 + b**2*n**2*x**4/32 - b
**2*n*x**4*log(c*x**n)/8 + b**2*x**4*log(c*x**n)**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int x^3(a+b\log(cx^n))^2\,dx = \frac{1}{4}\,b^2x^4\log(cx^n)^2 - \frac{1}{8}\,abnx^4 + \frac{1}{2}\,abx^4\log(cx^n) + \frac{1}{4}\,a^2x^4 + \frac{1}{32}\,(n^2x^4-4\,nx^4\log(cx^n))b^2$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*log(c*x^n)^2 - 1/8*a*b*n*x^4 + 1/2*a*b*x^4*log(c*x^n) + 1/4*a^2
*x^4 + 1/32*(n^2*x^4 - 4*n*x^4*log(c*x^n))*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(46) = 92.

Time = 0.34 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.13

$$\int x^3(a+b\log(cx^n))^2\,dx = \frac{1}{4}b^2n^2x^4\log(x)^2 - \frac{1}{8}b^2n^2x^4\log(x) + \frac{1}{2}b^2nx^4\log(c)\log(x) + \frac{1}{32}b^2n^2x^4 - \frac{1}{8}b^2nx^4\log(c) + \frac{1}{4}b^2x^4\log(c)^2 + \frac{1}{2}abnx^4\log(x) - \frac{1}{8}abnx^4 + \frac{1}{2}abx^4\log(c) + \frac{1}{4}a^2x^4$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*n^2*x^4*log(x)^2 - 1/8*b^2*n^2*x^4*log(x) + 1/2*b^2*n*x^4*log(c)*lo
g(x) + 1/32*b^2*n^2*x^4 - 1/8*b^2*n*x^4*log(c) + 1/4*b^2*x^4*log(c)^2 + 1/2
*a*b*n*x^4*log(x) - 1/8*a*b*n*x^4 + 1/2*a*b*x^4*log(c) + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int x^3(a+b\log(cx^n))^2\,dx = x^4\left(\frac{a^2}{4} - \frac{a\,b\,n}{8} + \frac{b^2\,n^2}{32}\right) + \frac{x^4\,\ln(c\,x^n)\,\left(a\,b - \frac{b^2\,n}{4}\right)}{2} + \frac{b^2\,x^4\,\ln(c\,x^n)^2}{4}$$

```
[In] int(x^3*(a + b*log(c*x^n))^2,x)
```

```
[Out] x^4*(a^2/4 + (b^2*n^2)/32 - (a*b*n)/8) + (x^4*log(c*x^n)*(a*b - (b^2*n)/4))
/2 + (b^2*x^4*log(c*x^n)^2)/4
```

3.51 $\int x^2(a+b\log(cx^n))^2\,dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int x^2(a+b\log(cx^n))^2\,dx=\frac{2}{27}b^2n^2x^3-\frac{2}{9}bnx^3(a+b\log(cx^n))+\frac{1}{3}x^3(a+b\log(cx^n))^2$$

```
[Out] 2/27*b^2*n^2*x^3-2/9*b*n*x^3*(a+b*ln(c*x^n))+1/3*x^3*(a+b*ln(c*x^n))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2342, 2341}

$$\int x^2(a+b\log(cx^n))^2\,dx=\frac{1}{3}x^3(a+b\log(cx^n))^2-\frac{2}{9}bnx^3(a+b\log(cx^n))+\frac{2}{27}b^2n^2x^3$$

```
[In] Int[x^2*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*b^2*n^2*x^3)/27 - (2*b*n*x^3*(a + b*Log[c*x^n]))/9 + (x^3*(a + b*Log[c*x
^n])^2)/3
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3(a+b\log(cx^n))^2-\frac{1}{3}(2bn)\int x^2(a+b\log(cx^n))\,dx \\
&= \frac{2}{27}b^2n^2x^3-\frac{2}{9}bnx^3(a+b\log(cx^n))+\frac{1}{3}x^3(a+b\log(cx^n))^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$
\int x^2(a+b\log(cx^n))^2\,dx=\frac{1}{3}\left(\frac{2}{9}bnx^3(-3a+bn-3b\log(cx^n))+x^3(a+b\log(cx^n))^2\right)
$$

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^2,x]
[Out] ((2*b*n*x^3*(-3*a + b*n - 3*b*Log[c*x^n]))/9 + x^3*(a + b*Log[c*x^n])^2)/3
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

method	result
parallelrisch	$\frac{x^3b^2\ln(cx^n)^2}{3}-\frac{2\ln(cx^n)x^3b^2n}{9}+\frac{2b^2n^2x^3}{27}+\frac{2x^3ab\ln(cx^n)}{3}-\frac{2abn\,x^3}{9}+\frac{x^3a^2}{3}$
risch	$\frac{x^3b^2\ln(x^n)^2}{3}+\frac{b\,x^3\left(-3ib\pi\ \text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)+3ib\pi\ \text{csgn}(ic)\,\text{csgn}(ic\,x^n)^2+3ib\pi\ \text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)^2-3ib\pi\ \text{csgn}\right.}{9}$

```
[In] int(x^2*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*b^2*ln(c*x^n)^2-2/9*ln(c*x^n)*x^3*b^2*n+2/27*b^2*n^2*x^3+2/3*x^3*a*
b*ln(c*x^n)-2/9*a*b*n*x^3+1/3*x^3*a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(46) = 92$.

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.98

$$\int x^2(a+b\log(cx^n))^2\,dx = \frac{1}{3}b^2n^2x^3\log(x)^2 + \frac{1}{3}b^2x^3\log(c)^2 - \frac{2}{9}\left(b^2n - 3\,ab\right)x^3\log(c) + \frac{1}{27}\left(2\,b^2n^2 - 6\,abn + 9\,a^2\right)x^3 + \frac{2}{9}\left(3\,b^2nx^3\log(c) - \left(b^2n^2 - 3\,abn\right)x^3\right)\log(x)$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*n^2*x^3*log(x)^2 + 1/3*b^2*x^3*log(c)^2 - 2/9*(b^2*n - 3*a*b)*x^3*l
og(c) + 1/27*(2*b^2*n^2 - 6*a*b*n + 9*a^2)*x^3 + 2/9*(3*b^2*n*x^3*log(c) -
(b^2*n^2 - 3*a*b*n)*x^3)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.63

$$\int x^2(a+b\log(cx^n))^2\,dx = \frac{a^2x^3}{3} - \frac{2abnx^3}{9} + \frac{2abx^3\log(cx^n)}{3} + \frac{2b^2n^2x^3}{27} - \frac{2b^2nx^3\log(cx^n)}{9} + \frac{b^2x^3\log(cx^n)^2}{3}$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*x**3/3 - 2*a*b*n*x**3/9 + 2*a*b*x**3*log(c*x**n)/3 + 2*b**2*n**2*x**3/
27 - 2*b**2*n*x**3*log(c*x**n)/9 + b**2*x**3*log(c*x**n)**2/3
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int x^2(a+b\log(cx^n))^2\,dx = \frac{1}{3}b^2x^3\log(cx^n)^2 - \frac{2}{9}abnx^3 + \frac{2}{3}abx^3\log(cx^n) + \frac{1}{3}a^2x^3 + \frac{2}{27}\left(n^2x^3 - 3\,nx^3\log(cx^n)\right)b^2$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*log(c*x^n)^2 - 2/9*a*b*n*x^3 + 2/3*a*b*x^3*log(c*x^n) + 1/3*a^2
*x^3 + 2/27*(n^2*x^3 - 3*n*x^3*log(c*x^n))*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(46) = 92.

Time = 0.32 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.13

$$\int x^2(a+b\log{(cx^n)})^2\,dx = \frac{1}{3}\,b^2n^2x^3\log{(x)}^2 - \frac{2}{9}\,b^2n^2x^3\log{(x)} + \frac{2}{3}\,b^2nx^3\log{(c)}\log{(x)} + \frac{2}{27}\,b^2n^2x^3 - \frac{2}{9}\,b^2nx^3\log{(c)} + \frac{1}{3}\,b^2x^3\log{(c)}^2 + \frac{2}{3}\,abnx^3\log{(x)} - \frac{2}{9}\,abnx^3 + \frac{2}{3}\,abx^3\log{(c)} + \frac{1}{3}\,a^2x^3$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*n^2*x^3*log(x)^2 - 2/9*b^2*n^2*x^3*log(x) + 2/3*b^2*n*x^3*log(c)*lo
g(x) + 2/27*b^2*n^2*x^3 - 2/9*b^2*n*x^3*log(c) + 1/3*b^2*x^3*log(c)^2 + 2/3
*a*b*n*x^3*log(x) - 2/9*a*b*n*x^3 + 2/3*a*b*x^3*log(c) + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

$$\int x^2(a+b\log{(cx^n)})^2\,dx = x^3\left(\frac{a^2}{3} - \frac{2\,a\,b\,n}{9} + \frac{2\,b^2\,n^2}{27}\right) + \frac{x^3\,\ln{(c\,x^n)}\,\left(2\,a\,b - \frac{2\,b^2\,n}{3}\right)}{3} + \frac{b^2\,x^3\,\ln{(c\,x^n)}^2}{3}$$

```
[In] int(x^2*(a + b*log(c*x^n))^2,x)
```

```
[Out] x^3*(a^2/3 + (2*b^2*n^2)/27 - (2*a*b*n)/9) + (x^3*log(c*x^n)*(2*a*b - (2*b^
2*n)/3))/3 + (b^2*x^3*log(c*x^n)^2)/3
```

3.52 $\int x(a+b\log(cx^n))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int x(a+b\log(cx^n))^2\,dx=\frac{1}{4}b^2n^2x^2-\frac{1}{2}bnx^2(a+b\log(cx^n))+\frac{1}{2}x^2(a+b\log(cx^n))^2$$

```
[Out] 1/4*b^2*n^2*x^2-1/2*b*n*x^2*(a+b*ln(c*x^n))+1/2*x^2*(a+b*ln(c*x^n))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2342, 2341}

$$\int x(a+b\log(cx^n))^2\,dx=\frac{1}{2}x^2(a+b\log(cx^n))^2-\frac{1}{2}bnx^2(a+b\log(cx^n))+\frac{1}{4}b^2n^2x^2$$

```
[In] Int[x*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (b^2*n^2*x^2)/4 - (b*n*x^2*(a + b*Log[c*x^n]))/2 + (x^2*(a + b*Log[c*x^n])^
2)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2(a+b\log(cx^n))^2-(bn)\int x(a+b\log(cx^n))\,dx\\ &= \frac{1}{4}b^2n^2x^2-\frac{1}{2}bnx^2(a+b\log(cx^n))+\frac{1}{2}x^2(a+b\log(cx^n))^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int x(a+b\log(cx^n))^2\,dx=\frac{1}{4}x^2\left(bn(-2a+bn-2b\log(cx^n))+2(a+b\log(cx^n))^2\right)$$

```
[In] Integrate[x*(a + b*Log[c*x^n])^2,x]
[Out] (x^2*(b*n*(-2*a + b*n - 2*b*Log[c*x^n]) + 2*(a + b*Log[c*x^n])^2))/4
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.38

method	result
parallelrisch	$\frac{x^2b^2\ln(c\,x^n)^2}{2}-\frac{\ln(c\,x^n)x^2b^2n}{2}+\frac{b^2n^2x^2}{4}+x^2ab\ln(c\,x^n)-\frac{abn\,x^2}{2}+\frac{x^2a^2}{2}$
risch	$\frac{b^2x^2\ln(x^n)^2}{2}+\frac{b\,x^2\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x\right.}{2}$

```
[In] int(x*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*b^2*ln(c*x^n)^2-1/2*ln(c*x^n)*x^2*b^2*n+1/4*b^2*n^2*x^2+x^2*a*b*ln(
c*x^n)-1/2*a*b*n*x^2+1/2*x^2*a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(46) = 92$.

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.96

$$\int x(a+b\log(cx^n))^2\,dx = \frac{1}{2}\,b^2n^2x^2\log(x)^2 + \frac{1}{2}\,b^2x^2\log(c)^2 - \frac{1}{2}\,(b^2n-2\,ab)x^2\log(c) + \frac{1}{4}\,(b^2n^2-2\,abn+2\,a^2)x^2 + \frac{1}{2}\,(2\,b^2nx^2\log(c) - (b^2n^2-2\,abn)x^2)\log(x)$$

```
[In] integrate(x*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*b^2*n^2*x^2*log(x)^2 + 1/2*b^2*x^2*log(c)^2 - 1/2*(b^2*n - 2*a*b)*x^2*l
og(c) + 1/4*(b^2*n^2 - 2*a*b*n + 2*a^2)*x^2 + 1/2*(2*b^2*n*x^2*log(c) - (b^
2*n^2 - 2*a*b*n)*x^2)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int x(a+b\log(cx^n))^2\,dx = \frac{a^2x^2}{2} - \frac{abnx^2}{2} + abx^2\log(cx^n) + \frac{b^2n^2x^2}{4} - \frac{b^2nx^2\log(cx^n)}{2} + \frac{b^2x^2\log(cx^n)^2}{2}$$

```
[In] integrate(x*(a+b*ln(c*x**n))**2,x)
```

```
[Out] a**2*x**2/2 - a*b*n*x**2/2 + a*b*x**2*log(c*x**n) + b**2*n**2*x**2/4 - b**2
*n*x**2*log(c*x**n)/2 + b**2*x**2*log(c*x**n)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

$$\int x(a+b\log(cx^n))^2\,dx = \frac{1}{2}\,b^2x^2\log(cx^n)^2 - \frac{1}{2}\,abnx^2 + abx^2\log(cx^n) + \frac{1}{2}\,a^2x^2 + \frac{1}{4}\,(n^2x^2 - 2\,nx^2\log(cx^n))b^2$$

```
[In] integrate(x*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*log(c*x^n)^2 - 1/2*a*b*n*x^2 + a*b*x^2*log(c*x^n) + 1/2*a^2*x^2
 + 1/4*(n^2*x^2 - 2*n*x^2*log(c*x^n))*b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(46) = 92$.

Time = 0.34 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.08

$$\int x(a+b\log(cx^n))^2\,dx = \frac{1}{2}b^2n^2x^2\log(x)^2 - \frac{1}{2}b^2n^2x^2\log(x) + b^2nx^2\log(c)\log(x) + \frac{1}{4}b^2n^2x^2 - \frac{1}{2}b^2nx^2\log(c) + \frac{1}{2}b^2x^2\log(c)^2 + abnx^2\log(x) - \frac{1}{2}abnx^2 + abx^2\log(c) + \frac{1}{2}a^2x^2$$

```
[In] integrate(x*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*n^2*x^2*log(x)^2 - 1/2*b^2*n^2*x^2*log(x) + b^2*n*x^2*log(c)*log(x)
 + 1/4*b^2*n^2*x^2 - 1/2*b^2*n*x^2*log(c) + 1/2*b^2*x^2*log(c)^2 + a*b*n*x^
2*log(x) - 1/2*a*b*n*x^2 + a*b*x^2*log(c) + 1/2*a^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int x(a+b\log(cx^n))^2\,dx = x^2\left(\frac{a^2}{2}-\frac{a\,b\,n}{2}+\frac{b^2\,n^2}{4}\right)+x^2\,\ln\left(c\,x^n\right)\,\left(a\,b-\frac{b^2\,n}{2}\right)+\frac{b^2\,x^2\ln\left(c\,x^n\right)^2}{2}$$

```
[In] int(x*(a + b*log(c*x^n))^2,x)
```

```
[Out] x^2*(a^2/2 + (b^2*n^2)/4 - (a*b*n)/2) + x^2*log(c*x^n)*(a*b - (b^2*n)/2) +
(b^2*x^2*log(c*x^n)^2)/2
```

3.53 $\int (a+b\log(cx^n))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 43

$$\int (a+b\log(cx^n))^2\,dx = -2abnx+2b^2n^2x-2b^2nx\log(cx^n)+x(a+b\log(cx^n))^2$$

```
[Out] -2*a*b*n*x+2*b^2*n^2*x-2*b^2*n*x*ln(c*x^n)+x*(a+b*ln(c*x^n))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2333, 2332}

$$\int (a+b\log(cx^n))^2\,dx = x(a+b\log(cx^n))^2-2abnx-2b^2nx\log(cx^n)+2b^2n^2x$$

```
[In] Int[(a + b*Log[c*x^n])^2,x]
```

```
[Out] -2*a*b*n*x + 2*b^2*n^2*x - 2*b^2*n*x*Log[c*x^n] + x*(a + b*Log[c*x^n])^2
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x(a+b\log(cx^n))^2-(2bn)\int (a+b\log(cx^n))\,dx\\ &= -2abnx+x(a+b\log(cx^n))^2-\left(2b^2n\right)\int \log(cx^n)\,dx\\ &= -2abnx+2b^2n^2x-2b^2nx\log(cx^n)+x(a+b\log(cx^n))^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int (a+b\log(cx^n))^2\,dx = x\left((a+b\log(cx^n))^2-2bn(a-bn+b\log(cx^n))\right)$$

```
[In] Integrate[(a + b*Log[c*x^n])^2,x]
[Out] x*((a + b*Log[c*x^n])^2 - 2*b*n*(a - b*n + b*Log[c*x^n]))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

method	result
norman	$\left(2b^2n^2-2abn+a^2\right)x+b^2x\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2+\left(-2b^2n+2ab\right)x\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)$
parallelrisch	$x\,b^2\ln(c\,x^n)^2-2b^2nx\ln(c\,x^n)+2b^2n^2x+2xab\ln(c\,x^n)-2abnx+a^2x$
default	$a^2x+b^2x\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2+2b^2n^2x-2b^2nx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+2xab\ln(c\,x^n)-2abnx$
parts	$a^2x+b^2x\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2+2b^2n^2x-2b^2nx\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+2xab\ln(c\,x^n)-2abnx$
risch	$b^2x\ln(x^n)^2+xb\big(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csg}$

```
[In] int((a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
[Out] (2*b^2*n^2-2*a*b*n+a^2)*x+b^2*x*ln(c*exp(n*ln(x)))^2+(-2*b^2*n+2*a*b)*x*ln(
c*exp(n*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int (a+b\log(cx^n))^2 \, dx = b^2n^2x\log(x)^2 + b^2x\log(c)^2 - 2\left(b^2n - ab\right)x\log(c) + \left(2\,b^2n^2 - 2\,abn + a^2\right)x + 2\left(b^2nx\log(c) - \left(b^2n^2 - abn\right)x\right)\log(x)$$

```
[In] integrate((a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] b^2*n^2*x*log(x)^2 + b^2*x*log(c)^2 - 2*(b^2*n - a*b)*x*log(c) + (2*b^2*n^2
 - 2*a*b*n + a^2)*x + 2*(b^2*n*x*log(c) - (b^2*n^2 - a*b*n)*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int (a+b\log(cx^n))^2 \, dx = a^2x - 2abnx + 2abx\log(cx^n) + 2b^2n^2x - 2b^2nx\log(cx^n) + b^2x\log(cx^n)^2$$

```
[In] integrate((a+b*ln(c*x**n))**2,x)
[Out] a**2*x - 2*a*b*n*x + 2*a*b*x*log(c*x**n) + 2*b**2*n**2*x - 2*b**2*n*x*log(c
*x**n) + b**2*x*log(c*x**n)**2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int (a+b\log(cx^n))^2 \, dx = b^2x\log(cx^n)^2 - 2\,abnx + 2\,abx\log(cx^n) + 2\left(n^2x - nx\log(cx^n)\right)b^2 + a^2x$$

```
[In] integrate((a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] b^2*x*log(c*x^n)^2 - 2*a*b*n*x + 2*a*b*x*log(c*x^n) + 2*(n^2*x - n*x*log(c*
x^n))*b^2 + a^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(43) = 86$.

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.05

$$\int (a + b\log(cx^n))^2\,dx = b^2n^2x\log(x)^2 - 2\,b^2n^2x\log(x) + 2\,b^2nx\log(c)\log(x) + 2\,b^2n^2x - 2\,b^2nx\log(c) + b^2x\log(c)^2 + 2\,abnx\log(x) - 2\,abnx + 2\,abx\log(c) + a^2x$$

```
[In] integrate((a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] b^2*n^2*x*log(x)^2 - 2*b^2*n^2*x*log(x) + 2*b^2*n*x*log(c)*log(x) + 2*b^2*n
^2*x - 2*b^2*n*x*log(c) + b^2*x*log(c)^2 + 2*a*b*n*x*log(x) - 2*a*b*n*x + 2
*a*b*x*log(c) + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int (a + b\log(cx^n))^2\,dx = x\left(a^2 - 2\,a\,b\,n + 2\,b^2\,n^2\right) + b^2\,x\ln\left(c\,x^n\right)^2 + 2\,b\,x\,\ln\left(c\,x^n\right)\,(a - b\,n)$$

```
[In] int((a + b*log(c*x^n))^2,x)
```

```
[Out] x*(a^2 + 2*b^2*n^2 - 2*a*b*n) + b^2*x*log(c*x^n)^2 + 2*b*x*log(c*x^n)*(a -
b*n)
```

3.54 $\int \frac{(a+b\log(cx^n))^2}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{(a+b\log(cx^n))^2}{x}\,dx = \frac{(a+b\log(cx^n))^3}{3bn}$$

```
[Out] 1/3*(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2339, 30}

$$\int \frac{(a+b\log(cx^n))^2}{x}\,dx = \frac{(a+b\log(cx^n))^3}{3bn}$$

```
[In] Int[(a + b*Log[c*x^n])^2/x,x]
```

```
[Out] (a + b*Log[c*x^n])^3/(3*b*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^2\,dx, x, a + b\log(cx^n)\right)}{bn}$$
$$= \frac{(a + b\log(cx^n))^3}{3bn}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\log(cx^n))^2}{x}\,dx = \frac{(a + b\log(cx^n))^3}{3bn}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/x,x]
[Out] (a + b*Log[c*x^n])^3/(3*b*n)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{(a+b\ln(c\,x^n))^3}{3bn}$	21
default	$\frac{(a+b\ln(c\,x^n))^3}{3bn}$	21
parts	$\ln(x)\,a^2 + \frac{b^2\ln(c\,x^n)^3}{3n} + \frac{ab\ln(c\,x^n)^2}{n}$	38
parallelrisch	$\frac{b^2\ln(c\,x^n)^3+3\ln(x)a^2n+3ab\ln(c\,x^n)^2}{3n}$	39
risch	Expression too large to display	774

```
[In] int((a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
[Out] 1/3*(a+b*ln(c*x^n))^3/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(20) = 40$.

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{(a+b\log(cx^n))^2}{x}\,dx = \frac{1}{3}\,b^2n^2\log(x)^3 + \left(b^2n\log(c)+abn\right)\log(x)^2 + \left(b^2\log(c)^2+2\,ab\log(c)+a^2\right)\log(x)$$

```
[In] integrate((a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] 1/3*b^2*n^2*log(x)^3 + (b^2*n*log(c) + a*b*n)*log(x)^2 + (b^2*log(c)^2 + 2*
a*b*log(c) + a^2)*log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(15) = 30$.

Time = 5.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{(a+b\log(cx^n))^2}{x}\,dx = \begin{cases} \frac{a^2\log(cx^n)+ab\log(cx^n)^2+\frac{b^2\log(cx^n)^3}{3}}{n} & \text{for } n \neq 0 \\ \left(a^2+2ab\log(c)+b^2\log(c)^2\right)\log(x) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Piecewise(((a**2*log(c*x**n) + a*b*log(c*x**n)**2 + b**2*log(c*x**n)**3/3)/
n, Ne(n, 0)), ((a**2 + 2*a*b*log(c) + b**2*log(c)**2)*log(x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\log(cx^n))^2}{x}\,dx = \frac{(b\log(cx^n)+a)^3}{3\,bn}$$

```
[In] integrate((a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*(b*log(c*x^n) + a)^3/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(20) = 40$.

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{(a + b\log(cx^n))^2}{x}\,dx = \frac{1}{3}b^2n^2\log(x)^3 + b^2n\log(c)\log(x)^2 + b^2\log(c)^2\log(x) + abn\log(x)^2 + 2\,ab\log(c)\log(x) + a^2\log(x)$$

```
[In] integrate((a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

```
[Out] 1/3*b^2*n^2*log(x)^3 + b^2*n*log(c)*log(x)^2 + b^2*log(c)^2*log(x) + a*b*n*
log(x)^2 + 2*a*b*log(c)*log(x) + a^2*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{(a + b\log(cx^n))^2}{x}\,dx = a^2\,\ln(x) + \frac{b^2\ln\left(c\,x^n\right)^3}{3\,n} + \frac{a\,b\ln\left(c\,x^n\right)^2}{n}$$

```
[In] int((a + b*log(c*x^n))^2/x,x)
```

```
[Out] a^2*log(x) + (b^2*log(c*x^n)^3)/(3*n) + (a*b*log(c*x^n)^2)/n
```

3.55 $\int \frac{(a+b\log(cx^n))^2}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx = -\frac{2b^2n^2}{x}-\frac{2bn(a+b\log(cx^n))}{x}-\frac{(a+b\log(cx^n))^2}{x}$$

```
[Out] -2*b^2*n^2/x-2*b*n*(a+b*ln(c*x^n))/x-(a+b*ln(c*x^n))^2/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2342, 2341}

$$\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx = -\frac{2bn(a+b\log(cx^n))}{x}-\frac{(a+b\log(cx^n))^2}{x}-\frac{2b^2n^2}{x}$$

```
[In] Int[(a + b*Log[c*x^n])^2/x^2,x]
```

```
[Out] (-2*b^2*n^2)/x - (2*b*n*(a + b*Log[c*x^n]))/x - (a + b*Log[c*x^n])^2/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\log(cx^n))^2}{x}+(2bn)\int \frac{a+b\log(cx^n)}{x^2}\,dx \\
&= -\frac{2b^2n^2}{x}-\frac{2bn(a+b\log(cx^n))}{x}-\frac{(a+b\log(cx^n))^2}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx = -\frac{(a+b\log(cx^n))^2+2bn(a+bn+b\log(cx^n))}{x}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/x^2,x]
[Out] -(((a + b*Log[c*x^n])^2 + 2*b*n*(a + b*n + b*Log[c*x^n]))/x)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

method	result
parallelrisch	$-\frac{b^2\ln(cx^n)^2+2\ln(cx^n)b^2n+2b^2n^2+2ab\ln(cx^n)+2abn+a^2}{x}$
risch	$-\frac{b^2\ln(x^n)^2}{x}-\frac{\left(-i\pi b^2\operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)+i\pi b^2\operatorname{csgn}(ic)\operatorname{csgn}(icx^n)^2+i\pi b^2\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)^2-i\pi b^2\operatorname{csgn}(ic\right.}{x}$

```
[In] int((a+b*ln(c*x^n))^2/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x*(b^2*ln(c*x^n)^2+2*ln(c*x^n)*b^2*n+2*b^2*n^2+2*a*b*ln(c*x^n)+2*a*b*n+a
^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.67

$$\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx = -\frac{b^2n^2\log(x)^2 + 2\,b^2n^2 + b^2\log(c)^2 + 2\,abn + a^2 + 2\,(b^2n + ab)\log(c) + 2\,(b^2n^2 + b^2n\log(c) + abn)\log}{x}$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2,x, algorithm="fricas")
[Out] -(b^2*n^2*log(x)^2 + 2*b^2*n^2 + b^2*log(c)^2 + 2*a*b*n + a^2 + 2*(b^2*n +
a*b)*log(c) + 2*(b^2*n^2 + b^2*n*log(c) + a*b*n)*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx = -\frac{a^2}{x} - \frac{2abn}{x} - \frac{2ab\log(cx^n)}{x} - \frac{2b^2n^2}{x} - \frac{2b^2n\log(cx^n)}{x} - \frac{b^2\log(cx^n)^2}{x}$$

```
[In] integrate((a+b*ln(c*x**n))**2/x**2,x)
[Out] -a**2/x - 2*a*b*n/x - 2*a*b*log(c*x**n)/x - 2*b**2*n**2/x - 2*b**2*n*log(c*
x**n)/x - b**2*log(c*x**n)**2/x
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.52

$$\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx = -2\,b^2\left(\frac{n^2}{x} + \frac{n\log(cx^n)}{x}\right) - \frac{b^2\log(cx^n)^2}{x} - \frac{2\,abn}{x} - \frac{2\,ab\log(cx^n)}{x} - \frac{a^2}{x}$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2,x, algorithm="maxima")
[Out] -2*b^2*(n^2/x + n*log(c*x^n)/x) - b^2*log(c*x^n)^2/x - 2*a*b*n/x - 2*a*b*lo
g(c*x^n)/x - a^2/x
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.87

$$\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx = -\frac{b^2n^2\log(x)^2}{x} - \frac{2\left(b^2n^2+b^2n\log(c)+abn\right)\log(x)}{x} - \frac{2\,b^2n^2+2\,b^2n\log(c)+b^2\log(c)^2+2\,abn+2\,ab\log(c)+a^2}{x}$$

```
[In] integrate((a+b*log(c*x^n))^2/x^2,x, algorithm="giac")
```

```
[Out] -b^2*n^2*log(x)^2/x - 2*(b^2*n^2 + b^2*n*log(c) + a*b*n)*log(x)/x - (2*b^2*
n^2 + 2*b^2*n*log(c) + b^2*log(c)^2 + 2*a*b*n + 2*a*b*log(c) + a^2)/x
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx = -\frac{a^2+2\,a\,b\,n+2\,b^2\,n^2}{x} - \frac{b^2\ln(c\,x^n)^2}{x} - \frac{2\,b\,\ln(c\,x^n)\,\left(a+b\,n\right)}{x}$$

```
[In] int((a + b*log(c*x^n))^2/x^2,x)
```

```
[Out] - (a^2 + 2*b^2*n^2 + 2*a*b*n)/x - (b^2*log(c*x^n)^2)/x - (2*b*log(c*x^n)*(a
 + b*n))/x
```

3.56 $\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx = -\frac{b^2n^2}{4x^2}-\frac{bn(a+b\log(cx^n))}{2x^2}-\frac{(a+b\log(cx^n))^2}{2x^2}$$

```
[Out] -1/4*b^2*n^2/x^2-1/2*b*n*(a+b*ln(c*x^n))/x^2-1/2*(a+b*ln(c*x^n))^2/x^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2342, 2341}

$$\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx = -\frac{bn(a+b\log(cx^n))}{2x^2}-\frac{(a+b\log(cx^n))^2}{2x^2}-\frac{b^2n^2}{4x^2}$$

```
[In] Int[(a + b*Log[c*x^n])^2/x^3,x]
```

```
[Out] -1/4*(b^2*n^2)/x^2 - (b*n*(a + b*Log[c*x^n]))/(2*x^2) - (a + b*Log[c*x^n])^
2/(2*x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\log(cx^n))^2}{2x^2}+(bn)\int\frac{a+b\log(cx^n)}{x^3}\,dx\\
&= -\frac{b^2n^2}{4x^2}-\frac{bn(a+b\log(cx^n))}{2x^2}-\frac{(a+b\log(cx^n))^2}{2x^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int\frac{(a+b\log(cx^n))^2}{x^3}\,dx=-\frac{2(a+b\log(cx^n))^2+bn(2a+bn+2b\log(cx^n))}{4x^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/x^3,x]
[Out] -1/4*(2*(a + b*Log[c*x^n])^2 + b*n*(2*a + b*n + 2*b*Log[c*x^n]))/x^2
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

method	result
parallelrisch	$-\frac{2b^2\ln(c\,x^n)^2+2\ln(c\,x^n)b^2n+b^2n^2+4ab\ln(c\,x^n)+2abn+2a^2}{4x^2}$
risch	$-\frac{b^2\ln(x^n)^2}{2x^2}-\frac{\left(-i\pi\,b^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+i\pi\,b^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+i\pi\,b^2\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-i\pi\,b^2\,\mathrm{csgn}(ic\right.}{2x^2}$

```
[In] int((a+b*ln(c*x^n))^2/x^3,x,method=_RETURNVERBOSE)
[Out] -1/4/x^2*(2*b^2*ln(c*x^n)^2+2*ln(c*x^n)*b^2*n+b^2*n^2+4*a*b*ln(c*x^n)+2*a*b
*n+2*a^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.60

$$\int \frac{(a + b\log{(cx^n)})^2}{x^3}\,dx =$$
$$-\frac{2\,b^2n^2\log{(x)}^2 + b^2n^2 + 2\,b^2\log{(c)}^2 + 2\,abn + 2\,a^2 + 2\,(b^2n + 2\,ab)\log{(c)} + 2\,(b^2n^2 + 2\,b^2n\log{(c)} + 2\,a}{4\,x^2}$$

```
[In] integrate((a+b*log(c*x^n))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*n^2*log(x)^2 + b^2*n^2 + 2*b^2*log(c)^2 + 2*a*b*n + 2*a^2 + 2*(
b^2*n + 2*a*b)*log(c) + 2*(b^2*n^2 + 2*b^2*n*log(c) + 2*a*b*n)*log(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int \frac{(a + b\log{(cx^n)})^2}{x^3}\,dx = -\frac{a^2}{2x^2} - \frac{abn}{2x^2} - \frac{ab\log{(cx^n)}}{x^2} - \frac{b^2n^2}{4x^2} - \frac{b^2n\log{(cx^n)}}{2x^2} - \frac{b^2\log{(cx^n)}^2}{2x^2}$$

```
[In] integrate((a+b*ln(c*x**n))**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) - a*b*n/(2*x**2) - a*b*log(c*x**n)/x**2 - b**2*n**2/(4*x**2)
 - b**2*n*log(c*x**n)/(2*x**2) - b**2*log(c*x**n)**2/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int \frac{(a + b\log{(cx^n)})^2}{x^3}\,dx = -\frac{1}{4}\,b^2\left(\frac{n^2}{x^2} + \frac{2\,n\log{(cx^n)}}{x^2}\right) - \frac{b^2\log{(cx^n)}^2}{2\,x^2} - \frac{abn}{2\,x^2} - \frac{ab\log{(cx^n)}}{x^2} - \frac{a^2}{2\,x^2}$$

```
[In] integrate((a+b*log(c*x^n))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(n^2/x^2 + 2*n*log(c*x^n)/x^2) - 1/2*b^2*log(c*x^n)^2/x^2 - 1/2*a*
b*n/x^2 - a*b*log(c*x^n)/x^2 - 1/2*a^2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73

$$\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx = -\frac{b^2n^2\log(x)^2}{2\,x^2} - \frac{(b^2n^2+2\,b^2n\log(c)+2\,abn)\log(x)}{2\,x^2} - \frac{b^2n^2+2\,b^2n\log(c)+2\,b^2\log(c)^2+2\,abn+4\,ab\log(c)+2\,a^2}{4\,x^2}$$

```
[In] integrate((a+b*log(c*x^n))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b^2*n^2*log(x)^2/x^2 - 1/2*(b^2*n^2 + 2*b^2*n*log(c) + 2*a*b*n)*log(x)
/x^2 - 1/4*(b^2*n^2 + 2*b^2*n*log(c) + 2*b^2*log(c)^2 + 2*a*b*n + 4*a*b*log
(c) + 2*a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\log(cx^n))^2}{x^3}\,dx = -\frac{\frac{a^2}{2}+\frac{a\,b\,n}{2}+\frac{b^2\,n^2}{4}}{x^2} - \frac{\ln(c\,x^n)\,\left(\frac{n\,b^2}{2}+a\,b\right)}{x^2} - \frac{b^2\ln(c\,x^n)^2}{2\,x^2}$$

```
[In] int((a + b*log(c*x^n))^2/x^3,x)
```

```
[Out] - (a^2/2 + (b^2*n^2)/4 + (a*b*n)/2)/x^2 - (log(c*x^n)*(a*b + (b^2*n)/2))/x^
2 - (b^2*log(c*x^n)^2)/(2*x^2)
```

3.57 $\int x^3(a+b\log(cx^n))^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int x^3(a+b\log(cx^n))^3\,dx = -\frac{3}{128}b^3n^3x^4+\frac{3}{32}b^2n^2x^4(a+b\log(cx^n)) - \frac{3}{16}bnx^4(a+b\log(cx^n))^2+\frac{1}{4}x^4(a+b\log(cx^n))^3$$

```
[Out] -3/128*b^3*n^3*x^4+3/32*b^2*n^2*x^4*(a+b*ln(c*x^n))-3/16*b*n*x^4*(a+b*ln(c*
x^n))^2+1/4*x^4*(a+b*ln(c*x^n))^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2342, 2341}

$$\int x^3(a+b\log(cx^n))^3\,dx = \frac{3}{32}b^2n^2x^4(a+b\log(cx^n))+\frac{1}{4}x^4(a+b\log(cx^n))^3 - \frac{3}{16}bnx^4(a+b\log(cx^n))^2-\frac{3}{128}b^3n^3x^4$$

```
[In] Int[x^3*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-3*b^3*n^3*x^4)/128 + (3*b^2*n^2*x^4*(a + b*Log[c*x^n]))/32 - (3*b*n*x^4*(
a + b*Log[c*x^n])^2)/16 + (x^4*(a + b*Log[c*x^n])^3)/4
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4(a+b\log(cx^n))^3 - \frac{1}{4}(3bn)\int x^3(a+b\log(cx^n))^2\,dx \\
&= -\frac{3}{16}bnx^4(a+b\log(cx^n))^2 + \frac{1}{4}x^4(a+b\log(cx^n))^3 + \frac{1}{8}\left(3b^2n^2\right)\int x^3(a+b\log(cx^n))\,dx \\
&= -\frac{3}{128}b^3n^3x^4 + \frac{3}{32}b^2n^2x^4(a+b\log(cx^n)) - \frac{3}{16}bnx^4(a+b\log(cx^n))^2 + \frac{1}{4}x^4(a+b\log(cx^n))^3
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$
\int x^3(a+b\log(cx^n))^3\,dx = \frac{1}{4}\left(x^4(a+b\log(cx^n))^3 - \frac{3}{32}bnx^4\left(b^2n^2 - 4bn(a+b\log(cx^n)) + 8(a+b\log(cx^n))^2\right)\right)
$$

```
[In] Integrate[x^3*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (x^4*(a + b*Log[c*x^n])^3 - (3*b*n*x^4*(b^2*n^2 - 4*b*n*(a + b*Log[c*x^n])
+ 8*(a + b*Log[c*x^n])^2))/32)/4
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(69) = 138.

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.83

method	result
parallelrisch	$\frac{x^4\ln(cx^n)^3b^3}{4} - \frac{3\ln(cx^n)^2x^4b^3n}{16} + \frac{3\ln(cx^n)x^4b^3n^2}{32} - \frac{3b^3n^3x^4}{128} + \frac{3\ln(cx^n)^2x^4ab^2}{4} - \frac{3\ln(cx^n)x^4ab^2n}{8} + \frac{3ab^2n^2x}{32}$
risch	Expression too large to display

```
[In] int(x^3*(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*ln(c*x^n)^3*b^3-3/16*ln(c*x^n)^2*x^4*b^3*n+3/32*ln(c*x^n)*x^4*b^3*n
^2-3/128*b^3*n^3*x^4+3/4*ln(c*x^n)^2*x^4*a*b^2-3/8*ln(c*x^n)*x^4*a*b^2*n+3/
32*a*b^2*n^2*x^4+3/4*ln(c*x^n)*x^4*a^2*b-3/16*a^2*b*n*x^4+1/4*a^3*x^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(69) = 138$.

Time = 0.31 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.88

$$\begin{aligned}\int x^3(a+b\log(cx^n))^3\,dx &= \frac{1}{4}\,b^3n^3x^4\log(x)^3+\frac{1}{4}\,b^3x^4\log(c)^3-\frac{3}{16}\,(b^3n-4\,ab^2)x^4\log(c)^2\\ &\quad+\frac{3}{32}\,(b^3n^2-4\,ab^2n+8\,a^2b)x^4\log(c)-\frac{1}{128}\,(3\,b^3n^3-12\,ab^2n^2+24\,a^2bn-32\,a^3)x^4\\ &\quad+\frac{3}{16}\,(4\,b^3n^2x^4\log(c)-(b^3n^3-4\,ab^2n^2)x^4)\log(x)^2\\ &\quad+\frac{3}{32}\,(8\,b^3nx^4\log(c)^2-4\,(b^3n^2-4\,ab^2n)x^4\log(c)+(b^3n^3-4\,ab^2n^2+8\,a^2bn)x^4)\log(x)\end{aligned}$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/4*b^3*n^3*x^4*log(x)^3 + 1/4*b^3*x^4*log(c)^3 - 3/16*(b^3*n - 4*a*b^2)*x^
4*log(c)^2 + 3/32*(b^3*n^2 - 4*a*b^2*n + 8*a^2*b)*x^4*log(c) - 1/128*(3*b^3
*n^3 - 12*a*b^2*n^2 + 24*a^2*b*n - 32*a^3)*x^4 + 3/16*(4*b^3*n^2*x^4*log(c)
 - (b^3*n^3 - 4*a*b^2*n^2)*x^4)*log(x)^2 + 3/32*(8*b^3*n*x^4*log(c)^2 - 4*(
b^3*n^2 - 4*a*b^2*n)*x^4*log(c) + (b^3*n^3 - 4*a*b^2*n^2 + 8*a^2*b*n)*x^4)*
log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 0.45 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.17

$$\begin{aligned}\int x^3(a+b\log(cx^n))^3\,dx &= \frac{a^3x^4}{4}-\frac{3a^2bnx^4}{16}+\frac{3a^2bx^4\log(cx^n)}{4}+\frac{3ab^2n^2x^4}{32}\\ &\quad-\frac{3ab^2nx^4\log(cx^n)}{8}+\frac{3ab^2x^4\log(cx^n)^2}{4}-\frac{3b^3n^3x^4}{128}\\ &\quad+\frac{3b^3n^2x^4\log(cx^n)}{32}-\frac{3b^3nx^4\log(cx^n)^2}{16}+\frac{b^3x^4\log(cx^n)^3}{4}\end{aligned}$$

```
[In] integrate(x**3*(a+b*ln(c*x**n))**3,x)
```

```
[Out] a**3*x**4/4 - 3*a**2*b*n*x**4/16 + 3*a**2*b*x**4*log(c*x**n)/4 + 3*a*b**2*n
**2*x**4/32 - 3*a*b**2*n*x**4*log(c*x**n)/8 + 3*a*b**2*x**4*log(c*x**n)**2/
4 - 3*b**3*n**3*x**4/128 + 3*b**3*n**2*x**4*log(c*x**n)/32 - 3*b**3*n*x**4*
log(c*x**n)**2/16 + b**3*x**4*log(c*x**n)**3/4
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.75

$$\int x^3(a+b\log(cx^n))^3\,dx = \frac{1}{4}\,b^3x^4\log(cx^n)^3 + \frac{3}{4}\,ab^2x^4\log(cx^n)^2 - \frac{3}{16}\,a^2bnx^4 + \frac{3}{4}\,a^2bx^4\log(cx^n) + \frac{1}{4}\,a^3x^4 + \frac{3}{32}\,\left(n^2x^4 - 4\,nx^4\log(cx^n)\right)ab^2 - \frac{3}{128}\,\left(8\,nx^4\log(cx^n)^2 + \left(n^2x^4 - 4\,nx^4\log(cx^n)\right)n\right)b^3$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^3,x, algorithm="maxima")
[Out] 1/4*b^3*x^4*log(c*x^n)^3 + 3/4*a*b^2*x^4*log(c*x^n)^2 - 3/16*a^2*b*n*x^4 +
3/4*a^2*b*x^4*log(c*x^n) + 1/4*a^3*x^4 + 3/32*(n^2*x^4 - 4*n*x^4*log(c*x^n)
)*a*b^2 - 3/128*(8*n*x^4*log(c*x^n)^2 + (n^2*x^4 - 4*n*x^4*log(c*x^n))*n)*b
^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(69) = 138.

Time = 0.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.40

$$\int x^3(a+b\log(cx^n))^3\,dx = \frac{1}{4}\,b^3n^3x^4\log(x)^3 - \frac{3}{16}\,b^3n^3x^4\log(x)^2 + \frac{3}{4}\,b^3n^2x^4\log(c)\log(x)^2 + \frac{3}{32}\,b^3n^3x^4\log(x) - \frac{3}{8}\,b^3n^2x^4\log(c)\log(x) + \frac{3}{4}\,b^3nx^4\log(c)^2\log(x) + \frac{3}{4}\,ab^2n^2x^4\log(x)^2 - \frac{3}{128}\,b^3n^3x^4 + \frac{3}{32}\,b^3n^2x^4\log(c) - \frac{3}{16}\,b^3nx^4\log(c)^2 + \frac{1}{4}\,b^3x^4\log(c)^3 - \frac{3}{8}\,ab^2n^2x^4\log(x) + \frac{3}{2}\,ab^2nx^4\log(c)\log(x) + \frac{3}{32}\,ab^2n^2x^4 - \frac{3}{8}\,ab^2nx^4\log(c) + \frac{3}{4}\,ab^2x^4\log(c)^2 + \frac{3}{4}\,a^2bnx^4\log(x) - \frac{3}{16}\,a^2bnx^4 + \frac{3}{4}\,a^2bx^4\log(c) + \frac{1}{4}\,a^3x^4$$

```
[In] integrate(x^3*(a+b*log(c*x^n))^3,x, algorithm="giac")
[Out] 1/4*b^3*n^3*x^4*log(x)^3 - 3/16*b^3*n^3*x^4*log(x)^2 + 3/4*b^3*n^2*x^4*log(
c)*log(x)^2 + 3/32*b^3*n^3*x^4*log(x) - 3/8*b^3*n^2*x^4*log(c)*log(x) + 3/4
*b^3*n*x^4*log(c)^2*log(x) + 3/4*a*b^2*n^2*x^4*log(x)^2 - 3/128*b^3*n^3*x^4
 + 3/32*b^3*n^2*x^4*log(c) - 3/16*b^3*n*x^4*log(c)^2 + 1/4*b^3*x^4*log(c)^3
 - 3/8*a*b^2*n^2*x^4*log(x) + 3/2*a*b^2*n*x^4*log(c)*log(x) + 3/32*a*b^2*n^
2*x^4 - 3/8*a*b^2*n*x^4*log(c) + 3/4*a*b^2*x^4*log(c)^2 + 3/4*a^2*b*n*x^4*l
og(x) - 3/16*a^2*b*n*x^4 + 3/4*a^2*b*x^4*log(c) + 1/4*a^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.43

$$\int x^3(a+b\log(cx^n))^3\,dx = x^4\left(\frac{a^3}{4}-\frac{3\,a^2\,b\,n}{16}+\frac{3\,a\,b^2\,n^2}{32}-\frac{3\,b^3\,n^3}{128}\right)+\frac{x^4\,\ln\left(c\,x^n\right)\,\left(6\,a^2\,b-3\,a\,b^2\,n+\frac{3\,b^3\,n^2}{4}\right)}{8}+\frac{x^4\ln\left(c\,x^n\right)^2\,\left(3\,a\,b^2-\frac{3\,b^3\,n}{4}\right)}{4}+\frac{b^3\,x^4\ln\left(c\,x^n\right)^3}{4}$$

```
[In] int(x^3*(a + b*log(c*x^n))^3,x)
```

```
[Out] x^4*(a^3/4 - (3*b^3*n^3)/128 + (3*a*b^2*n^2)/32 - (3*a^2*b*n)/16) + (x^4*lo
g(c*x^n)*(6*a^2*b + (3*b^3*n^2)/4 - 3*a*b^2*n))/8 + (x^4*log(c*x^n)^2*(3*a*
b^2 - (3*b^3*n)/4))/4 + (b^3*x^4*log(c*x^n)^3)/4
```

3.58 $\int x^2(a+b\log(cx^n))^3\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int x^2(a+b\log(cx^n))^3\,dx = -\frac{2}{27}b^3n^3x^3+\frac{2}{9}b^2n^2x^3(a+b\log(cx^n)) - \frac{1}{3}bnx^3(a+b\log(cx^n))^2+\frac{1}{3}x^3(a+b\log(cx^n))^3$$

```
[Out] -2/27*b^3*n^3*x^3+2/9*b^2*n^2*x^3*(a+b*ln(c*x^n))-1/3*b*n*x^3*(a+b*ln(c*x^n
))^2+1/3*x^3*(a+b*ln(c*x^n))^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2342, 2341}

$$\int x^2(a+b\log(cx^n))^3\,dx = \frac{2}{9}b^2n^2x^3(a+b\log(cx^n))+\frac{1}{3}x^3(a+b\log(cx^n))^3 - \frac{1}{3}bnx^3(a+b\log(cx^n))^2-\frac{2}{27}b^3n^3x^3$$

```
[In] Int[x^2*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-2*b^3*n^3*x^3)/27 + (2*b^2*n^2*x^3*(a + b*Log[c*x^n]))/9 - (b*n*x^3*(a +
b*Log[c*x^n])^2)/3 + (x^3*(a + b*Log[c*x^n])^3)/3
```

<u>Rule 2341</u>

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^3(a+b\log(cx^n))^3-(bn)\int x^2(a+b\log(cx^n))^2\,dx\\ &=-\frac{1}{3}bnx^3(a+b\log(cx^n))^2+\frac{1}{3}x^3(a+b\log(cx^n))^3+\frac{1}{3}\left(2b^2n^2\right)\int x^2(a+b\log(cx^n))\,dx\\ &=-\frac{2}{27}b^3n^3x^3+\frac{2}{9}b^2n^2x^3(a+b\log(cx^n))-\frac{1}{3}bnx^3(a+b\log(cx^n))^2+\frac{1}{3}x^3(a+b\log(cx^n))^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int x^2(a+b\log(cx^n))^3\,dx=\frac{1}{3}\left(x^3(a+b\log(cx^n))^3-bn\left(\frac{2}{9}bnx^3(-3a+bn-3b\log(cx^n))+x^3(a+b\log(cx^n))^2\right)\right)$$

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^3,x]
[Out] (x^3*(a + b*Log[c*x^n])^3 - b*n*((2*b*n*x^3*(-3*a + b*n - 3*b*Log[c*x^n]))/
9 + x^3*(a + b*Log[c*x^n])^2))/3
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.81

method	result
parallelrisch	$\frac{x^3b^3\ln(cx^n)^3}{3}-\frac{\ln(cx^n)^2x^3b^3n}{3}+\frac{2\ln(cx^n)x^3b^3n^2}{9}-\frac{2b^3n^3x^3}{27}+x^3a\,b^2\ln(cx^n)^2-\frac{2\ln(cx^n)x^3a\,b^2n}{3}+\frac{2a\,b^2}{9}$
risch	Expression too large to display

```
[In] int(x^2*(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*b^3*ln(c*x^n)^3-1/3*ln(c*x^n)^2*x^3*b^3*n+2/9*ln(c*x^n)*x^3*b^3*n^2
-2/27*b^3*n^3*x^3+x^3*a*b^2*ln(c*x^n)^2-2/3*ln(c*x^n)*x^3*a*b^2*n+2/9*a*b^2
*n^2*x^3+x^3*a^2*b*ln(c*x^n)-1/3*a^2*b*n*x^3+1/3*x^3*a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(69) = 138$.

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.91

$$\int x^2(a+b\log(cx^n))^3\,dx = \frac{1}{3}b^3n^3x^3\log(x)^3 + \frac{1}{3}b^3x^3\log(c)^3 - \frac{1}{3}(b^3n - 3ab^2)x^3\log(c)^2 + \frac{1}{9}(2b^3n^2 - 6ab^2n + 9a^2b)x^3\log(c) - \frac{1}{27}(2b^3n^3 - 6ab^2n^2 + 9a^2bn - 9a^3)x^3 + \frac{1}{3}\left(3b^3n^2x^3\log(c) - (b^3n^3 - 3ab^2n^2)x^3\right)\log(x)^2 + \frac{1}{9}\left(9b^3nx^3\log(c)^2 - 6(b^3n^2 - 3ab^2n)x^3\log(c) + (2b^3n^3 - 6ab^2n^2 + 9a^2bn)x^3\right)\log(x)$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*n^3*x^3*log(x)^3 + 1/3*b^3*x^3*log(c)^3 - 1/3*(b^3*n - 3*a*b^2)*x^3
*log(c)^2 + 1/9*(2*b^3*n^2 - 6*a*b^2*n + 9*a^2*b)*x^3*log(c) - 1/27*(2*b^3*
n^3 - 6*a*b^2*n^2 + 9*a^2*b*n - 9*a^3)*x^3 + 1/3*(3*b^3*n^2*x^3*log(c) - (b
^3*n^3 - 3*a*b^2*n^2)*x^3)*log(x)^2 + 1/9*(9*b^3*n*x^3*log(c)^2 - 6*(b^3*n^
2 - 3*a*b^2*n)*x^3*log(c) + (2*b^3*n^3 - 6*a*b^2*n^2 + 9*a^2*b*n)*x^3)*log(
x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(73) = 146$.

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.03

$$\int x^2(a+b\log(cx^n))^3\,dx = \frac{a^3x^3}{3} - \frac{a^2bnx^3}{3} + a^2bx^3\log(cx^n) + \frac{2ab^2n^2x^3}{9} - \frac{2ab^2nx^3\log(cx^n)}{3} + ab^2x^3\log(cx^n)^2 - \frac{2b^3n^3x^3}{27} + \frac{2b^3n^2x^3\log(cx^n)}{9} - \frac{b^3nx^3\log(cx^n)^2}{3} + \frac{b^3x^3\log(cx^n)^3}{3}$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))**3,x)
```

```
[Out] a**3*x**3/3 - a**2*b*n*x**3/3 + a**2*b*x**3*log(c*x**n) + 2*a*b**2*n**2*x**
3/9 - 2*a*b**2*n*x**3*log(c*x**n)/3 + a*b**2*x**3*log(c*x**n)**2 - 2*b**3*n
**3*x**3/27 + 2*b**3*n**2*x**3*log(c*x**n)/9 - b**3*n*x**3*log(c*x**n)**2/3
 + b**3*x**3*log(c*x**n)**3/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.74

$$\int x^2(a+b\log(cx^n))^3\,dx = \frac{1}{3}\,b^3x^3\log(cx^n)^3 + ab^2x^3\log(cx^n)^2 - \frac{1}{3}\,a^2bnx^3 + a^2bx^3\log(cx^n) + \frac{1}{3}\,a^3x^3 + \frac{2}{9}\,\left(n^2x^3 - 3\,nx^3\log(cx^n)\right)ab^2 - \frac{1}{27}\,\left(9\,nx^3\log(cx^n)^2 + 2\,\left(n^2x^3 - 3\,nx^3\log(cx^n)\right)n\right)b^3$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log(c*x^n)^3 + a*b^2*x^3*log(c*x^n)^2 - 1/3*a^2*b*n*x^3 + a^2*b
*x^3*log(c*x^n) + 1/3*a^3*x^3 + 2/9*(n^2*x^3 - 3*n*x^3*log(c*x^n))*a*b^2 -
1/27*(9*n*x^3*log(c*x^n)^2 + 2*(n^2*x^3 - 3*n*x^3*log(c*x^n))*n)*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(69) = 138.

Time = 0.33 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.32

$$\int x^2(a+b\log(cx^n))^3\,dx = \frac{1}{3}\,b^3n^3x^3\log(x)^3 - \frac{1}{3}\,b^3n^3x^3\log(x)^2 + b^3n^2x^3\log(c)\log(x)^2 + \frac{2}{9}\,b^3n^3x^3\log(x) - \frac{2}{3}\,b^3n^2x^3\log(c)\log(x) + b^3nx^3\log(c)^2\log(x) + ab^2n^2x^3\log(x)^2 - \frac{2}{27}\,b^3n^3x^3 + \frac{2}{9}\,b^3n^2x^3\log(c) - \frac{1}{3}\,b^3nx^3\log(c)^2 + \frac{1}{3}\,b^3x^3\log(c)^3 - \frac{2}{3}\,ab^2n^2x^3\log(x) + 2\,ab^2nx^3\log(c)\log(x) + \frac{2}{9}\,ab^2n^2x^3 - \frac{2}{3}\,ab^2nx^3\log(c) + ab^2x^3\log(c)^2 + a^2bnx^3\log(x) - \frac{1}{3}\,a^2bnx^3 + a^2bx^3\log(c) + \frac{1}{3}\,a^3x^3$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 1/3*b^3*n^3*x^3*log(x)^3 - 1/3*b^3*n^3*x^3*log(x)^2 + b^3*n^2*x^3*log(c)*lo
g(x)^2 + 2/9*b^3*n^3*x^3*log(x) - 2/3*b^3*n^2*x^3*log(c)*log(x) + b^3*n*x^3
*log(c)^2*log(x) + a*b^2*n^2*x^3*log(x)^2 - 2/27*b^3*n^3*x^3 + 2/9*b^3*n^2*
x^3*log(c) - 1/3*b^3*n*x^3*log(c)^2 + 1/3*b^3*x^3*log(c)^3 - 2/3*a*b^2*n^2*
x^3*log(x) + 2*a*b^2*n*x^3*log(c)*log(x) + 2/9*a*b^2*n^2*x^3 - 2/3*a*b^2*n*
x^3*log(c) + a*b^2*x^3*log(c)^2 + a^2*b*n*x^3*log(x) - 1/3*a^2*b*n*x^3 + a^
2*b*x^3*log(c) + 1/3*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int x^2(a+b\log(cx^n))^3\,dx = x^3\left(\frac{a^3}{3}-\frac{a^2\,b\,n}{3}+\frac{2\,a\,b^2\,n^2}{9}-\frac{2\,b^3\,n^3}{27}\right)+\frac{x^3\,\ln\left(c\,x^n\right)\,\left(3\,a^2\,b-2\,a\,b^2\,n+\frac{2\,b^3\,n^2}{3}\right)}{3}+x^3\ln\left(c\,x^n\right)^2\left(a\,b^2-\frac{b^3\,n}{3}\right)+\frac{b^3\,x^3\ln\left(c\,x^n\right)^3}{3}$$

```
[In] int(x^2*(a + b*log(c*x^n))^3,x)
```

```
[Out] x^3*(a^3/3 - (2*b^3*n^3)/27 + (2*a*b^2*n^2)/9 - (a^2*b*n)/3) + (x^3*log(c*x
^n)*(3*a^2*b + (2*b^3*n^2)/3 - 2*a*b^2*n))/3 + x^3*log(c*x^n)^2*(a*b^2 - (b
^3*n)/3) + (b^3*x^3*log(c*x^n)^3)/3
```

3.59 $\int x(a+b\log(cx^n))^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int x(a+b\log(cx^n))^3\,dx = -\frac{3}{8}b^3n^3x^2+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n)) -\frac{3}{4}bnx^2(a+b\log(cx^n))^2+\frac{1}{2}x^2(a+b\log(cx^n))^3$$

```
[Out] -3/8*b^3*n^3*x^2+3/4*b^2*n^2*x^2*(a+b*ln(c*x^n))-3/4*b*n*x^2*(a+b*ln(c*x^n)
)^2+1/2*x^2*(a+b*ln(c*x^n))^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2342, 2341}

$$\int x(a+b\log(cx^n))^3\,dx = \frac{3}{4}b^2n^2x^2(a+b\log(cx^n))-\frac{3}{4}bnx^2(a+b\log(cx^n))^2 +\frac{1}{2}x^2(a+b\log(cx^n))^3-\frac{3}{8}b^3n^3x^2$$

```
[In] Int[x*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-3*b^3*n^3*x^2)/8 + (3*b^2*n^2*x^2*(a + b*Log[c*x^n]))/4 - (3*b*n*x^2*(a +
 b*Log[c*x^n])^2)/4 + (x^2*(a + b*Log[c*x^n])^3)/2
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2(a+b\log(cx^n))^3 - \frac{1}{2}(3bn)\int x(a+b\log(cx^n))^2\,dx\\ &= -\frac{3}{4}bnx^2(a+b\log(cx^n))^2+\frac{1}{2}x^2(a+b\log(cx^n))^3+\frac{1}{2}(3b^2n^2)\int x(a+b\log(cx^n))\,dx\\ &= -\frac{3}{8}b^3n^3x^2+\frac{3}{4}b^2n^2x^2(a+b\log(cx^n))-\frac{3}{4}bnx^2(a+b\log(cx^n))^2+\frac{1}{2}x^2(a+b\log(cx^n))^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int x(a+b\log(cx^n))^3\,dx = \frac{1}{8}x^2\big(4(a+b\log(cx^n))^3 - 3bn\big(bn(-2a+bn-2b\log(cx^n))+2(a+b\log(cx^n))^2\big)\big)$$

```
[In] Integrate[x*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (x^2*(4*(a + b*Log[c*x^n])^3 - 3*b*n*(b*n*(-2*a + b*n - 2*b*Log[c*x^n]) + 2
*(a + b*Log[c*x^n])^2)))/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(69) = 138.

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.83

method	result
parallelrisch	$\frac{x^2b^3\ln(cx^n)^3}{2}-\frac{3\ln(cx^n)^2x^2b^3n}{4}+\frac{3\ln(cx^n)x^2b^3n^2}{4}-\frac{3b^3n^3x^2}{8}+\frac{3x^2ab^2\ln(cx^n)^2}{2}-\frac{3\ln(cx^n)x^2ab^2n}{2}+\frac{3ab^2n^2x^2}{4}$
risch	Expression too large to display

```
[In] int(x*(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*b^3*ln(c*x^n)^3-3/4*ln(c*x^n)^2*x^2*b^3*n+3/4*ln(c*x^n)*x^2*b^3*n^2
-3/8*b^3*n^3*x^2+3/2*x^2*a*b^2*ln(c*x^n)^2-3/2*ln(c*x^n)*x^2*a*b^2*n+3/4*a*
b^2*n^2*x^2+3/2*x^2*a^2*b*ln(c*x^n)-3/4*a^2*b*n*x^2+1/2*x^2*a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(69) = 138$.

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.88

$$\begin{aligned}\int x(a+b\log(cx^n))^3\,dx &= \frac{1}{2}\,b^3n^3x^2\log(x)^3+\frac{1}{2}\,b^3x^2\log(c)^3-\frac{3}{4}\,(b^3n-2\,ab^2)x^2\log(c)^2\\&\quad+\frac{3}{4}\,(b^3n^2-2\,ab^2n+2\,a^2b)x^2\log(c)-\frac{1}{8}\,(3\,b^3n^3-6\,ab^2n^2+6\,a^2bn-4\,a^3)x^2\\&\quad+\frac{3}{4}\,\left(2\,b^3n^2x^2\log(c)-\left(b^3n^3-2\,ab^2n^2\right)x^2\right)\log(x)^2\\&\quad+\frac{3}{4}\,\left(2\,b^3nx^2\log(c)^2-2\,\left(b^3n^2-2\,ab^2n\right)x^2\log(c)+\left(b^3n^3-2\,ab^2n^2+2\,a^2bn\right)x^2\right)\log(x)\end{aligned}$$

```
[In] integrate(x*(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/2*b^3*n^3*x^2*log(x)^3 + 1/2*b^3*x^2*log(c)^3 - 3/4*(b^3*n - 2*a*b^2)*x^2
*log(c)^2 + 3/4*(b^3*n^2 - 2*a*b^2*n + 2*a^2*b)*x^2*log(c) - 1/8*(3*b^3*n^3
 - 6*a*b^2*n^2 + 6*a^2*b*n - 4*a^3)*x^2 + 3/4*(2*b^3*n^2*x^2*log(c) - (b^3*
n^3 - 2*a*b^2*n^2)*x^2)*log(x)^2 + 3/4*(2*b^3*n*x^2*log(c)^2 - 2*(b^3*n^2 -
 2*a*b^2*n)*x^2*log(c) + (b^3*n^3 - 2*a*b^2*n^2 + 2*a^2*b*n)*x^2)*log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 0.24 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.17

$$\begin{aligned}\int x(a+b\log(cx^n))^3\,dx &= \frac{a^3x^2}{2}-\frac{3a^2bnx^2}{4}+\frac{3a^2bx^2\log(cx^n)}{2}+\frac{3ab^2n^2x^2}{4}\\&\quad-\frac{3ab^2nx^2\log(cx^n)}{2}+\frac{3ab^2x^2\log(cx^n)^2}{2}-\frac{3b^3n^3x^2}{8}\\&\quad+\frac{3b^3n^2x^2\log(cx^n)}{4}-\frac{3b^3nx^2\log(cx^n)^2}{4}+\frac{b^3x^2\log(cx^n)^3}{2}\end{aligned}$$

```
[In] integrate(x*(a+b*ln(c*x**n))**3,x)
```

```
[Out] a**3*x**2/2 - 3*a**2*b*n*x**2/4 + 3*a**2*b*x**2*log(c*x**n)/2 + 3*a*b**2*n*
*2*x**2/4 - 3*a*b**2*n*x**2*log(c*x**n)/2 + 3*a*b**2*x**2*log(c*x**n)**2/2
- 3*b**3*n**3*x**2/8 + 3*b**3*n**2*x**2*log(c*x**n)/4 - 3*b**3*n*x**2*log(c
*x**n)**2/4 + b**3*x**2*log(c*x**n)**3/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.75

$$\int x(a+b\log(cx^n))^3\,dx = \frac{1}{2}\,b^3x^2\log(cx^n)^3 + \frac{3}{2}\,ab^2x^2\log(cx^n)^2 - \frac{3}{4}\,a^2bnx^2 + \frac{3}{2}\,a^2bx^2\log(cx^n) + \frac{1}{2}\,a^3x^2 + \frac{3}{4}\,(n^2x^2 - 2\,nx^2\log(cx^n))ab^2 - \frac{3}{8}\,(2\,nx^2\log(cx^n)^2 + (n^2x^2 - 2\,nx^2\log(cx^n))n)b^3$$

```
[In] integrate(x*(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*log(c*x^n)^3 + 3/2*a*b^2*x^2*log(c*x^n)^2 - 3/4*a^2*b*n*x^2 + 3
/2*a^2*b*x^2*log(c*x^n) + 1/2*a^3*x^2 + 3/4*(n^2*x^2 - 2*n*x^2*log(c*x^n))*
a*b^2 - 3/8*(2*n*x^2*log(c*x^n)^2 + (n^2*x^2 - 2*n*x^2*log(c*x^n))*n)*b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(69) = 138.

Time = 0.34 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.40

$$\int x(a+b\log(cx^n))^3\,dx = \frac{1}{2}\,b^3n^3x^2\log(x)^3 - \frac{3}{4}\,b^3n^3x^2\log(x)^2 + \frac{3}{2}\,b^3n^2x^2\log(c)\log(x)^2 + \frac{3}{4}\,b^3n^3x^2\log(x) - \frac{3}{2}\,b^3n^2x^2\log(c)\log(x) + \frac{3}{2}\,b^3nx^2\log(c)^2\log(x) + \frac{3}{2}\,ab^2n^2x^2\log(x)^2 - \frac{3}{8}\,b^3n^3x^2 + \frac{3}{4}\,b^3n^2x^2\log(c) - \frac{3}{4}\,b^3nx^2\log(c)^2 + \frac{1}{2}\,b^3x^2\log(c)^3 - \frac{3}{2}\,ab^2n^2x^2\log(x) + 3\,ab^2nx^2\log(c)\log(x) + \frac{3}{4}\,ab^2n^2x^2 - \frac{3}{2}\,ab^2nx^2\log(c) + \frac{3}{2}\,ab^2x^2\log(c)^2 + \frac{3}{2}\,a^2bnx^2\log(x) - \frac{3}{4}\,a^2bnx^2 + \frac{3}{2}\,a^2bx^2\log(c) + \frac{1}{2}\,a^3x^2$$

```
[In] integrate(x*(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*n^3*x^2*log(x)^3 - 3/4*b^3*n^3*x^2*log(x)^2 + 3/2*b^3*n^2*x^2*log(c
)*log(x)^2 + 3/4*b^3*n^3*x^2*log(x) - 3/2*b^3*n^2*x^2*log(c)*log(x) + 3/2*b
^3*n*x^2*log(c)^2*log(x) + 3/2*a*b^2*n^2*x^2*log(x)^2 - 3/8*b^3*n^3*x^2 + 3
/4*b^3*n^2*x^2*log(c) - 3/4*b^3*n*x^2*log(c)^2 + 1/2*b^3*x^2*log(c)^3 - 3/2
*a*b^2*n^2*x^2*log(x) + 3*a*b^2*n*x^2*log(c)*log(x) + 3/4*a*b^2*n^2*x^2 - 3
/2*a*b^2*n*x^2*log(c) + 3/2*a*b^2*x^2*log(c)^2 + 3/2*a^2*b*n*x^2*log(x) - 3
/4*a^2*b*n*x^2 + 3/2*a^2*b*x^2*log(c) + 1/2*a^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.43

$$\int x(a+b\log(cx^n))^3\,dx = x^2\left(\frac{a^3}{2}-\frac{3\,a^2\,b\,n}{4}+\frac{3\,a\,b^2\,n^2}{4}-\frac{3\,b^3\,n^3}{8}\right)+\frac{x^2\,\ln(c\,x^n)\,\left(3\,a^2\,b-3\,a\,b^2\,n+\frac{3\,b^3\,n^2}{2}\right)}{2}+\frac{x^2\ln(c\,x^n)^2\left(3\,a\,b^2-\frac{3\,b^3\,n}{2}\right)}{2}+\frac{b^3\,x^2\ln(c\,x^n)^3}{2}$$

```
[In] int(x*(a + b*log(c*x^n))^3,x)
```

```
[Out] x^2*(a^3/2 - (3*b^3*n^3)/8 + (3*a*b^2*n^2)/4 - (3*a^2*b*n)/4) + (x^2*log(c*
x^n)*(3*a^2*b + (3*b^3*n^2)/2 - 3*a*b^2*n))/2 + (x^2*log(c*x^n)^2*(3*a*b^2
- (3*b^3*n)/2))/2 + (b^3*x^2*log(c*x^n)^3)/2
```

3.60 $\int (a+b\log(cx^n))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int (a+b\log(cx^n))^3\,dx = 6ab^2n^2x - 6b^3n^3x + 6b^3n^2x\log(cx^n) - 3bnx(a+b\log(cx^n))^2 + x(a+b\log(cx^n))^3$$

```
[Out] 6*a*b^2*n^2*x-6*b^3*n^3*x+6*b^3*n^2*x*ln(c*x^n)-3*b*n*x*(a+b*ln(c*x^n))^2+x
*(a+b*ln(c*x^n))^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2333, 2332}

$$\int (a+b\log(cx^n))^3\,dx = 6ab^2n^2x - 3bnx(a+b\log(cx^n))^2 + x(a+b\log(cx^n))^3 + 6b^3n^2x\log(cx^n) - 6b^3n^3x$$

```
[In] Int[(a + b*Log[c*x^n])^3,x]
```

```
[Out] 6*a*b^2*n^2*x - 6*b^3*n^3*x + 6*b^3*n^2*x*Log[c*x^n] - 3*b*n*x*(a + b*Log[c
*x^n])^2 + x*(a + b*Log[c*x^n])^3
```

Rule 2332

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x(a+b\log(cx^n))^3-(3bn)\int(a+b\log(cx^n))^2\,dx\\ &= -3bnx(a+b\log(cx^n))^2+x(a+b\log(cx^n))^3+\left(6b^2n^2\right)\int(a+b\log(cx^n))\,dx\\ &= 6ab^2n^2x-3bnx(a+b\log(cx^n))^2+x(a+b\log(cx^n))^3+\left(6b^3n^2\right)\int\log(cx^n)\,dx\\ &= 6ab^2n^2x-6b^3n^3x+6b^3n^2x\log(cx^n)-3bnx(a+b\log(cx^n))^2+x(a+b\log(cx^n))^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int(a+b\log(cx^n))^3\,dx = x\big((a+b\log(cx^n))^3 - 3bn\big((a+b\log(cx^n))^2-2bn(a-bn+b\log(cx^n))\big)\big)$$

```
[In] Integrate[(a + b*Log[c*x^n])^3,x]
[Out] x*((a + b*Log[c*x^n])^3 - 3*b*n*((a + b*Log[c*x^n])^2 - 2*b*n*(a - b*n + b*
Log[c*x^n])))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.80

method	result
parallelrisch	$x\,b^3\ln(c\,x^n)^3-3x\ln(c\,x^n)^2\,b^3n+6b^3n^2x\ln(c\,x^n)-6b^3n^3x+3xa\,b^2\ln(c\,x^n)^2-6x\ln(c\,x^n$
risch	Expression too large to display

```
[In] int((a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
[Out] x*b^3*ln(c*x^n)^3-3*x*ln(c*x^n)^2*b^3*n+6*b^3*n^2*x*ln(c*x^n)-6*b^3*n^3*x+3
*x*a*b^2*ln(c*x^n)^2-6*x*ln(c*x^n)*a*b^2*n+6*a*b^2*n^2*x+3*x*a^2*b*ln(c*x^n
)-3*a^2*b*n*x+a^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(66) = 132$.

Time = 0.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.00

$$\int (a+b\log(cx^n))^3\,dx = b^3n^3x\log(x)^3 + b^3x\log(c)^3 - 3\left(b^3n - ab^2\right)x\log(c)^2 + 3\left(2\,b^3n^2 - 2\,ab^2n + a^2b\right)x\log(c) + 3\left(b^3n^2x\log(c) - \left(b^3n^3 - ab^2n^2\right)x\right)\log(x)^2 - \left(6\,b^3n^3 - 6\,ab^2n^2 + 3\,a^2bn - a^3\right)x + 3\left(b^3nx\log(c)^2 - 2\left(b^3n^2 - ab^2n\right)x\log(c) + \left(2\,b^3n^3 - 2\,ab^2n^2 + a^2bn\right)x\right)\log(x)$$

```
[In] integrate((a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] b^3*n^3*x*log(x)^3 + b^3*x*log(c)^3 - 3*(b^3*n - a*b^2)*x*log(c)^2 + 3*(2*b
^3*n^2 - 2*a*b^2*n + a^2*b)*x*log(c) + 3*(b^3*n^2*x*log(c) - (b^3*n^3 - a*b
^2*n^2)*x)*log(x)^2 - (6*b^3*n^3 - 6*a*b^2*n^2 + 3*a^2*b*n - a^3)*x + 3*(b^
3*n*x*log(c)^2 - 2*(b^3*n^2 - a*b^2*n)*x*log(c) + (2*b^3*n^3 - 2*a*b^2*n^2
+ a^2*b*n)*x)*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.02

$$\int (a+b\log(cx^n))^3\,dx = a^3x - 3a^2bnx + 3a^2bx\log(cx^n) + 6ab^2n^2x - 6ab^2nx\log(cx^n) + 3ab^2x\log(cx^n)^2 - 6b^3n^3x + 6b^3n^2x\log(cx^n) - 3b^3nx\log(cx^n)^2 + b^3x\log(cx^n)^3$$

```
[In] integrate((a+b*ln(c*x**n))**3,x)
```

```
[Out] a**3*x - 3*a**2*b*n*x + 3*a**2*b*x*log(c*x**n) + 6*a*b**2*n**2*x - 6*a*b**2
*n*x*log(c*x**n) + 3*a*b**2*x*log(c*x**n)**2 - 6*b**3*n**3*x + 6*b**3*n**2*
x*log(c*x**n) - 3*b**3*n*x*log(c*x**n)**2 + b**3*x*log(c*x**n)**3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.71

$$\int (a+b\log(cx^n))^3\,dx = b^3x\log(cx^n)^3 + 3\,ab^2x\log(cx^n)^2 - 3\,a^2bnx + 3\,a^2bx\log(cx^n) + 6\left(n^2x - nx\log(cx^n)\right)ab^2 - 3\left(nx\log(cx^n)^2 + 2\left(n^2x - nx\log(cx^n)\right)n\right)b^3 + a^3x$$

```
[In] integrate((a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(c*x^n)^3 + 3*a*b^2*x*log(c*x^n)^2 - 3*a^2*b*n*x + 3*a^2*b*x*log(c
*x^n) + 6*(n^2*x - n*x*log(c*x^n))*a*b^2 - 3*(n*x*log(c*x^n)^2 + 2*(n^2*x -
 n*x*log(c*x^n))*n)*b^3 + a^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(66) = 132.

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.32

$$\begin{aligned}\int (a+b\log (cx^n))^3\,dx = {} & b^3n^3x\log (x)^3 - 3\,b^3n^3x\log (x)^2 + 3\,b^3n^2x\log (c)\log (x)^2 \\ & + 6\,b^3n^3x\log (x) - 6\,b^3n^2x\log (c)\log (x) + 3\,b^3nx\log (c)^2\log (x) \\ & + 3\,ab^2n^2x\log (x)^2 - 6\,b^3n^3x + 6\,b^3n^2x\log (c) - 3\,b^3nx\log (c)^2 \\ & + b^3x\log (c)^3 - 6\,ab^2n^2x\log (x) + 6\,ab^2nx\log (c)\log (x) \\ & + 6\,ab^2n^2x - 6\,ab^2nx\log (c) + 3\,ab^2x\log (c)^2 \\ & + 3\,a^2bnx\log (x) - 3\,a^2bnx + 3\,a^2bx\log (c) + a^3x\end{aligned}$$

```
[In] integrate((a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] b^3*n^3*x*log(x)^3 - 3*b^3*n^3*x*log(x)^2 + 3*b^3*n^2*x*log(c)*log(x)^2 + 6
*b^3*n^3*x*log(x) - 6*b^3*n^2*x*log(c)*log(x) + 3*b^3*n*x*log(c)^2*log(x) +
 3*a*b^2*n^2*x*log(x)^2 - 6*b^3*n^3*x + 6*b^3*n^2*x*log(c) - 3*b^3*n*x*log(
c)^2 + b^3*x*log(c)^3 - 6*a*b^2*n^2*x*log(x) + 6*a*b^2*n*x*log(c)*log(x) +
6*a*b^2*n^2*x - 6*a*b^2*n*x*log(c) + 3*a*b^2*x*log(c)^2 + 3*a^2*b*n*x*log(x
) - 3*a^2*b*n*x + 3*a^2*b*x*log(c) + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\begin{aligned}\int (a+b\log (cx^n))^3\,dx = {} & x\,\left(a^3-3\,a^2\,b\,n+6\,a\,b^2\,n^2-6\,b^3\,n^3\right) \\ & + x\,\ln\left(c\,x^n\right)\,\left(3\,a^2\,b-6\,a\,b^2\,n+6\,b^3\,n^2\right) \\ & + b^3\,x\,\ln\left(c\,x^n\right)^3 + 3\,b^2\,x\,\ln\left(c\,x^n\right)^2\,(a-b\,n)\end{aligned}$$

```
[In] int((a + b*log(c*x^n))^3,x)
```

```
[Out] x*(a^3 - 6*b^3*n^3 + 6*a*b^2*n^2 - 3*a^2*b*n) + x*log(c*x^n)*(3*a^2*b + 6*b
^3*n^2 - 6*a*b^2*n) + b^3*x*log(c*x^n)^3 + 3*b^2*x*log(c*x^n)^2*(a - b*n)
```

3.61 $\int \frac{(a+b\log(cx^n))^3}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{(a+b\log(cx^n))^3}{x}\,dx=\frac{(a+b\log(cx^n))^4}{4bn}$$

```
[Out] 1/4*(a+b*ln(c*x^n))^4/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2339, 30}

$$\int \frac{(a+b\log(cx^n))^3}{x}\,dx=\frac{(a+b\log(cx^n))^4}{4bn}$$

```
[In] Int[(a + b*Log[c*x^n])^3/x,x]
```

```
[Out] (a + b*Log[c*x^n])^4/(4*b*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^3\,dx, x, a+b\log\left(cx^n\right)\right)}{bn}$$
$$= \frac{\left(a+b\log\left(cx^n\right)\right)^4}{4bn}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^3}{x}\,dx = \frac{\left(a+b\log\left(cx^n\right)\right)^4}{4bn}$$

```
[In] Integrate[(a + b*Log[c*x^n])^3/x,x]
[Out] (a + b*Log[c*x^n])^4/(4*b*n)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{4}}{4bn}$	21
default	$\frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{4}}{4bn}$	21
parallelrisch	$\frac{b^{3}\ln\left(c\,x^{n}\right)^{4}+4a\,b^{2}\ln\left(c\,x^{n}\right)^{3}+4\ln\left(x\right)a^{3}n+6a^{2}b\ln\left(c\,x^{n}\right)^{2}}{4n}$	55
parts	$\ln\left(x\right)a^{3}+\frac{b^{3}\ln\left(c\,x^{n}\right)^{4}}{4n}+\frac{a\,b^{2}\ln\left(c\,x^{n}\right)^{3}}{n}+\frac{3a^{2}b\ln\left(c\,x^{n}\right)^{2}}{2n}$	57
risch	Expression too large to display	2945

```
[In] int((a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
[Out] 1/4*(a+b*ln(c*x^n))^4/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(20) = 40$.

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 4.55

$$\int \frac{(a+b\log(cx^n))^3}{x}\,dx = \frac{1}{4}\,b^3n^3\log(x)^4 + \left(b^3n^2\log(c) + ab^2n^2\right)\log(x)^3 + \frac{3}{2}\left(b^3n\log(c)^2 + 2\,ab^2n\log(c) + a^2bn\right)\log(x)^2 + \left(b^3\log(c)^3 + 3\,ab^2\log(c)^2 + 3\,a^2b\log(c) + a^3\right)\log(x)$$

```
[In] integrate((a+b*log(c*x^n))^3/x,x, algorithm="fricas")
[Out] 1/4*b^3*n^3*log(x)^4 + (b^3*n^2*log(c) + a*b^2*n^2)*log(x)^3 + 3/2*(b^3*n*l
og(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n)*log(x)^2 + (b^3*log(c)^3 + 3*a*b^2*lo
g(c)^2 + 3*a^2*b*log(c) + a^3)*log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(15) = 30$.

Time = 7.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.18

$$\int \frac{(a+b\log(cx^n))^3}{x}\,dx = \begin{cases} \frac{a^3\log(cx^n) + \frac{3a^2b\log(cx^n)^2}{2} + ab^2\log(cx^n)^3 + \frac{b^3\log(cx^n)^4}{4}}{n} & \text{for } n \neq 0 \\ \left(a^3 + 3a^2b\log(c) + 3ab^2\log(c)^2 + b^3\log(c)^3\right)\log(x) & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln(c*x**n))**3/x,x)
[Out] Piecewise(((a**3*log(c*x**n) + 3*a**2*b*log(c*x**n)**2/2 + a*b**2*log(c*x**
n)**3 + b**3*log(c*x**n)**4/4)/n, Ne(n, 0)), ((a**3 + 3*a**2*b*log(c) + 3*a
*b**2*log(c)**2 + b**3*log(c)**3)*log(x), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\log(cx^n))^3}{x}\,dx = \frac{(b\log(cx^n) + a)^4}{4\,bn}$$

```
[In] integrate((a+b*log(c*x^n))^3/x,x, algorithm="maxima")
[Out] 1/4*(b*log(c*x^n) + a)^4/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(20) = 40$.

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 5.18

$$\int \frac{(a+b\log(cx^n))^3}{x}\,dx = \frac{1}{4}\,b^3n^3\log(x)^4 + b^3n^2\log(c)\log(x)^3 + \frac{3}{2}\,b^3n\log(c)^2\log(x)^2 + ab^2n^2\log(x)^3 + b^3\log(c)^3\log(x) + 3\,ab^2n\log(c)\log(x)^2 + 3\,ab^2\log(c)^2\log(x) + \frac{3}{2}\,a^2bn\log(x)^2 + 3\,a^2b\log(c)\log(x) + a^3\log(x)$$

```
[In] integrate((a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/4*b^3*n^3*log(x)^4 + b^3*n^2*log(c)*log(x)^3 + 3/2*b^3*n*log(c)^2*log(x)^
2 + a*b^2*n^2*log(x)^3 + b^3*log(c)^3*log(x) + 3*a*b^2*n*log(c)*log(x)^2 +
3*a*b^2*log(c)^2*log(x) + 3/2*a^2*b*n*log(x)^2 + 3*a^2*b*log(c)*log(x) + a^
3*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{(a+b\log(cx^n))^3}{x}\,dx = a^3\,\ln(x) + \frac{b^3\ln(c\,x^n)^4}{4\,n} + \frac{3\,a^2\,b\ln(c\,x^n)^2}{2\,n} + \frac{a\,b^2\ln(c\,x^n)^3}{n}$$

```
[In] int((a + b*log(c*x^n))^3/x,x)
```

```
[Out] a^3*log(x) + (b^3*log(c*x^n)^4)/(4*n) + (3*a^2*b*log(c*x^n)^2)/(2*n) + (a*b
^2*log(c*x^n)^3)/n
```

3.62 $\int \frac{(a+b\log(cx^n))^3}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 69

$$\int \frac{(a+b\log(cx^n))^3}{x^2}\,dx = -\frac{6b^3n^3}{x} - \frac{6b^2n^2(a+b\log(cx^n))}{x} - \frac{3bn(a+b\log(cx^n))^2}{x} - \frac{(a+b\log(cx^n))^3}{x}$$

```
[Out] -6*b^3*n^3/x-6*b^2*n^2*(a+b*ln(c*x^n))/x-3*b*n*(a+b*ln(c*x^n))^2/x-(a+b*ln(
c*x^n))^3/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2342, 2341}

$$\int \frac{(a+b\log(cx^n))^3}{x^2}\,dx = -\frac{6b^2n^2(a+b\log(cx^n))}{x} - \frac{3bn(a+b\log(cx^n))^2}{x} - \frac{(a+b\log(cx^n))^3}{x} - \frac{6b^3n^3}{x}$$

```
[In] Int[(a + b*Log[c*x^n])^3/x^2,x]
```

```
[Out] (-6*b^3*n^3)/x - (6*b^2*n^2*(a + b*Log[c*x^n]))/x - (3*b*n*(a + b*Log[c*x^n
])^2)/x - (a + b*Log[c*x^n])^3/x
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(a+b\log(cx^n))^3}{x} + (3bn)\int \frac{(a+b\log(cx^n))^2}{x^2}\,dx\\ &= -\frac{3bn(a+b\log(cx^n))^2}{x} - \frac{(a+b\log(cx^n))^3}{x} + (6b^2n^2)\int \frac{a+b\log(cx^n)}{x^2}\,dx\\ &= -\frac{6b^3n^3}{x} - \frac{6b^2n^2(a+b\log(cx^n))}{x} - \frac{3bn(a+b\log(cx^n))^2}{x} - \frac{(a+b\log(cx^n))^3}{x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{(a+b\log(cx^n))^3}{x^2}\,dx = -\frac{(a+b\log(cx^n))^3 + 3bn\big((a+b\log(cx^n))^2 + 2bn(a+bn+b\log(cx^n))\big)}{x}$$

```
[In] Integrate[(a + b*Log[c*x^n])^3/x^2,x]
[Out] -(((a + b*Log[c*x^n])^3 + 3*b*n*((a + b*Log[c*x^n])^2 + 2*b*n*(a + b*n + b*
Log[c*x^n])))/x)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

method	result
parallelrisch	$-\frac{b^3\ln(c\,x^n)^3+3\ln(c\,x^n)^2 b^3 n+6\ln(c\,x^n)b^3n^2+6b^3n^3+3a\,b^2\ln(c\,x^n)^2+6\ln(c\,x^n)a\,b^2n+6a\,b^2n^2+3a^2b\ln(c\,x^n)+3a^2bn+a^3}{x}$
risch	Expression too large to display

```
[In] int((a+b*ln(c*x^n))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x*(b^3*ln(c*x^n)^3+3*ln(c*x^n)^2*b^3*n+6*ln(c*x^n)*b^3*n^2+6*b^3*n^3+3*a
*b^2*ln(c*x^n)^2+6*ln(c*x^n)*a*b^2*n+6*a*b^2*n^2+3*a^2*b*ln(c*x^n)+3*a^2*b*
n+a^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(69) = 138.

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.61

$$\int \frac{(a+b\log(cx^n))^3}{x^2}\,dx = -\frac{b^3n^3\log(x)^3+6\,b^3n^3+b^3\log(c)^3+6\,ab^2n^2+3\,a^2bn+a^3+3\,(b^3n+ab^2)\log(c)^2+3\,(b^3n^3+b^3n^2\log(c}{}$$

```
[In] integrate((a+b*log(c*x^n))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*n^3*log(x)^3 + 6*b^3*n^3 + b^3*log(c)^3 + 6*a*b^2*n^2 + 3*a^2*b*n + a
^3 + 3*(b^3*n + a*b^2)*log(c)^2 + 3*(b^3*n^3 + b^3*n^2*log(c) + a*b^2*n^2)*
log(x)^2 + 3*(2*b^3*n^2 + 2*a*b^2*n + a^2*b)*log(c) + 3*(2*b^3*n^3 + b^3*n*
log(c)^2 + 2*a*b^2*n^2 + a^2*b*n + 2*(b^3*n^2 + a*b^2*n)*log(c))*log(x))/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(63) = 126.

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.94

$$\int \frac{(a+b\log(cx^n))^3}{x^2}\,dx = -\frac{a^3}{x}-\frac{3a^2bn}{x}-\frac{3a^2b\log(cx^n)}{x}-\frac{6ab^2n^2}{x}-\frac{6ab^2n\log(cx^n)}{x}-\frac{3ab^2\log(cx^n)^2}{x}-\frac{6b^3n^3}{x}-\frac{6b^3n^2\log(cx^n)}{x}-\frac{3b^3n\log(cx^n)^2}{x}-\frac{b^3\log(cx^n)^3}{x}$$

```
[In] integrate((a+b*ln(c*x**n))**3/x**2,x)
```

```
[Out] -a**3/x - 3*a**2*b*n/x - 3*a**2*b*log(c*x**n)/x - 6*a*b**2*n**2/x - 6*a*b**
2*n*log(c*x**n)/x - 3*a*b**2*log(c*x**n)**2/x - 6*b**3*n**3/x - 6*b**3*n**2
*log(c*x**n)/x - 3*b**3*n*log(c*x**n)**2/x - b**3*log(c*x**n)**3/x
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.93

$$\int \frac{(a+b\log(cx^n))^3}{x^2}\,dx = -\frac{b^3\log(cx^n)^3}{x} - 3\left(2n\left(\frac{n^2}{x}+\frac{n\log(cx^n)}{x}\right)+\frac{n\log(cx^n)^2}{x}\right)b^3 - 6ab^2\left(\frac{n^2}{x}+\frac{n\log(cx^n)}{x}\right) - \frac{3ab^2\log(cx^n)^2}{x} - \frac{3a^2bn}{x} - \frac{3a^2b\log(cx^n)}{x} - \frac{a^3}{x}$$

```
[In] integrate((a+b*log(c*x^n))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*log(c*x^n)^3/x - 3*(2*n*(n^2/x + n*log(c*x^n)/x) + n*log(c*x^n)^2/x)*b
^3 - 6*a*b^2*(n^2/x + n*log(c*x^n)/x) - 3*a*b^2*log(c*x^n)^2/x - 3*a^2*b*n/
x - 3*a^2*b*log(c*x^n)/x - a^3/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(69) = 138.

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.86

$$\int \frac{(a+b\log(cx^n))^3}{x^2}\,dx = -\frac{b^3n^3\log(x)^3}{x} - \frac{3\left(b^3n^3+b^3n^2\log(c)+ab^2n^2\right)\log(x)^2}{x} - \frac{3\left(2b^3n^3+2b^3n^2\log(c)+b^3n\log(c)^2+2ab^2n^2+2ab^2n\log(c)+a^2bn\right)\log(x)}{x} - \frac{6b^3n^3+6b^3n^2\log(c)+3b^3n\log(c)^2+b^3\log(c)^3+6ab^2n^2+6ab^2n\log(c)+3ab^2\log(c)^2+3a^2bn+}{x}$$

```
[In] integrate((a+b*log(c*x^n))^3/x^2,x, algorithm="giac")
```

```
[Out] -b^3*n^3*log(x)^3/x - 3*(b^3*n^3 + b^3*n^2*log(c) + a*b^2*n^2)*log(x)^2/x -
 3*(2*b^3*n^3 + 2*b^3*n^2*log(c) + b^3*n*log(c)^2 + 2*a*b^2*n^2 + 2*a*b^2*n
*log(c) + a^2*b*n)*log(x)/x - (6*b^3*n^3 + 6*b^3*n^2*log(c) + 3*b^3*n*log(c
)^2 + b^3*log(c)^3 + 6*a*b^2*n^2 + 6*a*b^2*n*log(c) + 3*a*b^2*log(c)^2 + 3*
a^2*b*n + 3*a^2*b*log(c) + a^3)/x
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.51

$$\int \frac{(a+b\log(cx^n))^3}{x^2}\,dx = -\frac{a^3+3\,a^2\,b\,n+6\,a\,b^2\,n^2+6\,b^3\,n^3}{x} - \frac{\ln(c\,x^n)\,\left(3\,a^2\,b+6\,a\,b^2\,n+6\,b^3\,n^2\right)}{x} - \frac{b^3\ln(c\,x^n)^3}{x} - \frac{3\,b^2\ln(c\,x^n)^2\,(a+b\,n)}{x}$$

```
[In] int((a + b*log(c*x^n))^3/x^2,x)
```

```
[Out] - (a^3 + 6*b^3*n^3 + 6*a*b^2*n^2 + 3*a^2*b*n)/x - (log(c*x^n)*(3*a^2*b + 6*
b^3*n^2 + 6*a*b^2*n))/x - (b^3*log(c*x^n)^3)/x - (3*b^2*log(c*x^n)^2*(a + b
*n))/x
```

3.63 $\int \frac{(a+b\log(cx^n))^3}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{(a+b\log(cx^n))^3}{x^3}\,dx = -\frac{3b^3n^3}{8x^2} - \frac{3b^2n^2(a+b\log(cx^n))}{4x^2} - \frac{3bn(a+b\log(cx^n))^2}{4x^2} - \frac{(a+b\log(cx^n))^3}{2x^2}$$

```
[Out] -3/8*b^3*n^3/x^2-3/4*b^2*n^2*(a+b*ln(c*x^n))/x^2-3/4*b*n*(a+b*ln(c*x^n))^2/
x^2-1/2*(a+b*ln(c*x^n))^3/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2342, 2341}

$$\int \frac{(a+b\log(cx^n))^3}{x^3}\,dx = -\frac{3b^2n^2(a+b\log(cx^n))}{4x^2} - \frac{3bn(a+b\log(cx^n))^2}{4x^2} - \frac{(a+b\log(cx^n))^3}{2x^2} - \frac{3b^3n^3}{8x^2}$$

```
[In] Int[(a + b*Log[c*x^n])^3/x^3,x]
```

```
[Out] (-3*b^3*n^3)/(8*x^2) - (3*b^2*n^2*(a + b*Log[c*x^n]))/(4*x^2) - (3*b*n*(a +
 b*Log[c*x^n])^2)/(4*x^2) - (a + b*Log[c*x^n])^3/(2*x^2)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\log(cx^n))^3}{2x^2}+\frac{1}{2}(3bn)\int\frac{(a+b\log(cx^n))^2}{x^3}\,dx\\
&= -\frac{3bn(a+b\log(cx^n))^2}{4x^2}-\frac{(a+b\log(cx^n))^3}{2x^2}+\frac{1}{2}(3b^2n^2)\int\frac{a+b\log(cx^n)}{x^3}\,dx\\
&= -\frac{3b^3n^3}{8x^2}-\frac{3b^2n^2(a+b\log(cx^n))}{4x^2}-\frac{3bn(a+b\log(cx^n))^2}{4x^2}-\frac{(a+b\log(cx^n))^3}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int\frac{(a+b\log(cx^n))^3}{x^3}\,dx = -\frac{4(a+b\log(cx^n))^3+3bn\big(2(a+b\log(cx^n))^2+bn(2a+bn+2b\log(cx^n))\big)}{8x^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])^3/x^3,x]
[Out] -1/8*(4*(a + b*Log[c*x^n])^3 + 3*b*n*(2*(a + b*Log[c*x^n])^2 + b*n*(2*a + b
*n + 2*b*Log[c*x^n])))/x^2
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

method	result
parallelrisch	$-\frac{4b^3\ln(cx^n)^3+6\ln(cx^n)^2b^3n+6\ln(cx^n)b^3n^2+3b^3n^3+12ab^2\ln(cx^n)^2+12\ln(cx^n)ab^2n+6ab^2n^2+12a^2b\ln(cx^n)+6a^2bn+4a}{8x^2}$
risch	Expression too large to display

```
[In] int((a+b*ln(c*x^n))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/x^2*(4*b^3*ln(c*x^n)^3+6*ln(c*x^n)^2*b^3*n+6*ln(c*x^n)*b^3*n^2+3*b^3*n
^3+12*a*b^2*ln(c*x^n)^2+12*ln(c*x^n)*a*b^2*n+6*a*b^2*n^2+12*a^2*b*ln(c*x^n)
+6*a^2*b*n+4*a^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(69) = 138$.

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.45

$$\int \frac{(a + b\log(cx^n))^3}{x^3}\,dx =$$

$$-\frac{4\,b^3n^3\log(x)^3 + 3\,b^3n^3 + 4\,b^3\log(c)^3 + 6\,ab^2n^2 + 6\,a^2bn + 4\,a^3 + 6\,(b^3n + 2\,ab^2)\log(c)^2 + 6\,(b^3n^3 + 2\,b}{}$$

```
[In] integrate((a+b*log(c*x^n))^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*n^3*log(x)^3 + 3*b^3*n^3 + 4*b^3*log(c)^3 + 6*a*b^2*n^2 + 6*a^2
*b*n + 4*a^3 + 6*(b^3*n + 2*a*b^2)*log(c)^2 + 6*(b^3*n^3 + 2*b^3*n^2*log(c)
 + 2*a*b^2*n^2)*log(x)^2 + 6*(b^3*n^2 + 2*a*b^2*n + 2*a^2*b)*log(c) + 6*(b^
3*n^3 + 2*b^3*n*log(c)^2 + 2*a*b^2*n^2 + 2*a^2*b*n + 2*(b^3*n^2 + 2*a*b^2*n
)*log(c))*log(x))/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(76) = 152$.

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.18

$$\begin{aligned}\int \frac{(a + b\log(cx^n))^3}{x^3}\,dx = &-\frac{a^3}{2x^2} - \frac{3a^2bn}{4x^2} - \frac{3a^2b\log(cx^n)}{2x^2} - \frac{3ab^2n^2}{4x^2} \\ &- \frac{3ab^2n\log(cx^n)}{2x^2} - \frac{3ab^2\log(cx^n)^2}{2x^2} - \frac{3b^3n^3}{8x^2} \\ &- \frac{3b^3n^2\log(cx^n)}{4x^2} - \frac{3b^3n\log(cx^n)^2}{4x^2} - \frac{b^3\log(cx^n)^3}{2x^2}\end{aligned}$$

```
[In] integrate((a+b*ln(c*x**n))**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) - 3*a**2*b*n/(4*x**2) - 3*a**2*b*log(c*x**n)/(2*x**2) - 3*a*
b**2*n**2/(4*x**2) - 3*a*b**2*n*log(c*x**n)/(2*x**2) - 3*a*b**2*log(c*x**n)
**2/(2*x**2) - 3*b**3*n**3/(8*x**2) - 3*b**3*n**2*log(c*x**n)/(4*x**2) - 3*
b**3*n*log(c*x**n)**2/(4*x**2) - b**3*log(c*x**n)**3/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\log(cx^n))^3}{x^3}\,dx = -\frac{3}{8}\left(n\left(\frac{n^2}{x^2}+\frac{2\,n\log(cx^n)}{x^2}\right)+\frac{2\,n\log(cx^n)^2}{x^2}\right)b^3 - \frac{3}{4}\,ab^2\left(\frac{n^2}{x^2}+\frac{2\,n\log(cx^n)}{x^2}\right)-\frac{b^3\log(cx^n)^3}{2\,x^2} - \frac{3\,ab^2\log(cx^n)^2}{2\,x^2}-\frac{3\,a^2bn}{4\,x^2}-\frac{3\,a^2b\log(cx^n)}{2\,x^2}-\frac{a^3}{2\,x^2}$$

```
[In] integrate((a+b*log(c*x^n))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/8*(n*(n^2/x^2 + 2*n*log(c*x^n)/x^2) + 2*n*log(c*x^n)^2/x^2)*b^3 - 3/4*a*
b^2*(n^2/x^2 + 2*n*log(c*x^n)/x^2) - 1/2*b^3*log(c*x^n)^3/x^2 - 3/2*a*b^2*l
og(c*x^n)^2/x^2 - 3/4*a^2*b*n/x^2 - 3/2*a^2*b*log(c*x^n)/x^2 - 1/2*a^3/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(69) = 138.

Time = 0.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.64

$$\int \frac{(a+b\log(cx^n))^3}{x^3}\,dx = -\frac{b^3n^3\log(x)^3}{2\,x^2}-\frac{3\,(b^3n^3+2\,b^3n^2\log(c)+2\,ab^2n^2)\log(x)^2}{4\,x^2} - \frac{3\left(b^3n^3+2\,b^3n^2\log(c)+2\,b^3n\log(c)^2+2\,ab^2n^2+4\,ab^2n\log(c)+2\,a^2bn\right)\log(x)}{4\,x^2} - \frac{3\,b^3n^3+6\,b^3n^2\log(c)+6\,b^3n\log(c)^2+4\,b^3\log(c)^3+6\,ab^2n^2+12\,ab^2n\log(c)+12\,ab^2\log(c)^2+6\,a^2bn}{8\,x^2}$$

```
[In] integrate((a+b*log(c*x^n))^3/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b^3*n^3*log(x)^3/x^2 - 3/4*(b^3*n^3 + 2*b^3*n^2*log(c) + 2*a*b^2*n^2)*
log(x)^2/x^2 - 3/4*(b^3*n^3 + 2*b^3*n^2*log(c) + 2*b^3*n*log(c)^2 + 2*a*b^2
*n^2 + 4*a*b^2*n*log(c) + 2*a^2*b*n)*log(x)/x^2 - 1/8*(3*b^3*n^3 + 6*b^3*n^
2*log(c) + 6*b^3*n*log(c)^2 + 4*b^3*log(c)^3 + 6*a*b^2*n^2 + 12*a*b^2*n*log
(c) + 12*a*b^2*log(c)^2 + 6*a^2*b*n + 12*a^2*b*log(c) + 4*a^3)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.44

$$\int \frac{(a+b\log(cx^n))^3}{x^3}\,dx = -\frac{\frac{a^3}{2}+\frac{3\,a^2\,b\,n}{4}+\frac{3\,a\,b^2\,n^2}{4}+\frac{3\,b^3\,n^3}{8}}{x^2} - \frac{\ln(c\,x^n)\,\left(3\,a^2\,b+3\,a\,b^2\,n+\frac{3\,b^3\,n^2}{2}\right)}{2\,x^2} - \frac{\ln(c\,x^n)^2\,\left(\frac{3\,n\,b^3}{2}+3\,a\,b^2\right)}{2\,x^2} - \frac{b^3\ln(c\,x^n)^3}{2\,x^2}$$

```
[In] int((a + b*log(c*x^n))^3/x^3,x)
```

```
[Out] - (a^3/2 + (3*b^3*n^3)/8 + (3*a*b^2*n^2)/4 + (3*a^2*b*n)/4)/x^2 - (log(c*x^
n)*(3*a^2*b + (3*b^3*n^2)/2 + 3*a*b^2*n))/(2*x^2) - (log(c*x^n)^2*(3*a*b^2
+ (3*b^3*n)/2))/(2*x^2) - (b^3*log(c*x^n)^3)/(2*x^2)
```

3.64 $\int \frac{(a+b\log(cx^n))^3}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{(a+b\log(cx^n))^3}{x^4}\,dx = -\frac{2b^3n^3}{27x^3} - \frac{2b^2n^2(a+b\log(cx^n))}{9x^3} - \frac{bn(a+b\log(cx^n))^2}{3x^3} - \frac{(a+b\log(cx^n))^3}{3x^3}$$

```
[Out] -2/27*b^3*n^3/x^3-2/9*b^2*n^2*(a+b*ln(c*x^n))/x^3-1/3*b*n*(a+b*ln(c*x^n))^2
/x^3-1/3*(a+b*ln(c*x^n))^3/x^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2342, 2341}

$$\int \frac{(a+b\log(cx^n))^3}{x^4}\,dx = -\frac{2b^2n^2(a+b\log(cx^n))}{9x^3} - \frac{bn(a+b\log(cx^n))^2}{3x^3} - \frac{(a+b\log(cx^n))^3}{3x^3} - \frac{2b^3n^3}{27x^3}$$

```
[In] Int[(a + b*Log[c*x^n])^3/x^4,x]
```

```
[Out] (-2*b^3*n^3)/(27*x^3) - (2*b^2*n^2*(a + b*Log[c*x^n]))/(9*x^3) - (b*n*(a +
b*Log[c*x^n])^2)/(3*x^3) - (a + b*Log[c*x^n])^3/(3*x^3)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\log(cx^n))^3}{3x^3} + (bn)\int \frac{(a+b\log(cx^n))^2}{x^4}\,dx \\
&= -\frac{bn(a+b\log(cx^n))^2}{3x^3} - \frac{(a+b\log(cx^n))^3}{3x^3} + \frac{1}{3}\left(2b^2n^2\right)\int \frac{a+b\log(cx^n)}{x^4}\,dx \\
&= -\frac{2b^3n^3}{27x^3} - \frac{2b^2n^2(a+b\log(cx^n))}{9x^3} - \frac{bn(a+b\log(cx^n))^2}{3x^3} - \frac{(a+b\log(cx^n))^3}{3x^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$
\int \frac{(a+b\log(cx^n))^3}{x^4}\,dx = -\frac{9(a+b\log(cx^n))^3 + bn\left(9(a+b\log(cx^n))^2 + 2bn(3a+bn+3b\log(cx^n))\right)}{27x^3}
$$

```
[In] Integrate[(a + b*Log[c*x^n])^3/x^4,x]
[Out] -1/27*(9*(a + b*Log[c*x^n])^3 + b*n*(9*(a + b*Log[c*x^n])^2 + 2*b*n*(3*a +
b*n + 3*b*Log[c*x^n])))/x^3
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

method	result
parallelrisch	$-\frac{9b^3\ln(cx^n)^3+9\ln(cx^n)^2b^3n+6\ln(cx^n)b^3n^2+2b^3n^3+27ab^2\ln(cx^n)^2+18\ln(cx^n)ab^2n+6ab^2n^2+27a^2b\ln(cx^n)+9a^2bn+}{27x^3}$
risch	Expression too large to display

```
[In] int((a+b*ln(c*x^n))^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/27/x^3*(9*b^3*ln(c*x^n)^3+9*ln(c*x^n)^2*b^3*n+6*ln(c*x^n)*b^3*n^2+2*b^3*
n^3+27*a*b^2*ln(c*x^n)^2+18*ln(c*x^n)*a*b^2*n+6*a*b^2*n^2+27*a^2*b*ln(c*x^n
)+9*a^2*b*n+9*a^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(69) = 138$.

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.48

$$\int \frac{(a + b\log{(cx^n)})^3}{x^4}\,dx =$$
$$-\frac{9\,b^3n^3\log{(x)}^3 + 2\,b^3n^3 + 9\,b^3\log{(c)}^3 + 6\,ab^2n^2 + 9\,a^2bn + 9\,a^3 + 9\,(b^3n + 3\,ab^2)\log{(c)}^2 + 9\,(b^3n^3 + 3\,b^3\ldots}{}$$

```
[In] integrate((a+b*log(c*x^n))^3/x^4,x, algorithm="fricas")
```

```
[Out] -1/27*(9*b^3*n^3*log(x)^3 + 2*b^3*n^3 + 9*b^3*log(c)^3 + 6*a*b^2*n^2 + 9*a^
2*b*n + 9*a^3 + 9*(b^3*n + 3*a*b^2)*log(c)^2 + 9*(b^3*n^3 + 3*b^3*n^2*log(c
) + 3*a*b^2*n^2)*log(x)^2 + 3*(2*b^3*n^2 + 6*a*b^2*n + 9*a^2*b)*log(c) + 3*
(2*b^3*n^3 + 9*b^3*n*log(c)^2 + 6*a*b^2*n^2 + 9*a^2*b*n + 6*(b^3*n^2 + 3*a*
b^2*n)*log(c))*log(x))/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(75) = 150$.

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.05

$$\int \frac{(a + b\log{(cx^n)})^3}{x^4}\,dx = -\frac{a^3}{3x^3} - \frac{a^2bn}{3x^3} - \frac{a^2b\log{(cx^n)}}{x^3} - \frac{2ab^2n^2}{9x^3} - \frac{2ab^2n\log{(cx^n)}}{3x^3} - \frac{ab^2\log{(cx^n)}^2}{x^3} - \frac{2b^3n^3}{27x^3} - \frac{2b^3n^2\log{(cx^n)}}{9x^3} - \frac{b^3n\log{(cx^n)}^2}{3x^3} - \frac{b^3\log{(cx^n)}^3}{3x^3}$$

```
[In] integrate((a+b*ln(c*x**n))**3/x**4,x)
```

```
[Out] -a**3/(3*x**3) - a**2*b*n/(3*x**3) - a**2*b*log(c*x**n)/x**3 - 2*a*b**2*n**
2/(9*x**3) - 2*a*b**2*n*log(c*x**n)/(3*x**3) - a*b**2*log(c*x**n)**2/x**3 -
 2*b**3*n**3/(27*x**3) - 2*b**3*n**2*log(c*x**n)/(9*x**3) - b**3*n*log(c*x*
*n)**2/(3*x**3) - b**3*log(c*x**n)**3/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.77

$$\int \frac{(a+b\log(cx^n))^3}{x^4}\,dx = -\frac{1}{27}\left(2\,n\left(\frac{n^2}{x^3}+\frac{3\,n\log(cx^n)}{x^3}\right)+\frac{9\,n\log(cx^n)^2}{x^3}\right)b^3 - \frac{2}{9}\,ab^2\left(\frac{n^2}{x^3}+\frac{3\,n\log(cx^n)}{x^3}\right)-\frac{b^3\log(cx^n)^3}{3\,x^3} - \frac{ab^2\log(cx^n)^2}{x^3}-\frac{a^2bn}{3\,x^3}-\frac{a^2b\log(cx^n)}{x^3}-\frac{a^3}{3\,x^3}$$

```
[In] integrate((a+b*log(c*x^n))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/27*(2*n*(n^2/x^3 + 3*n*log(c*x^n)/x^3) + 9*n*log(c*x^n)^2/x^3)*b^3 - 2/9
*a*b^2*(n^2/x^3 + 3*n*log(c*x^n)/x^3) - 1/3*b^3*log(c*x^n)^3/x^3 - a*b^2*lo
g(c*x^n)^2/x^3 - 1/3*a^2*b*n/x^3 - a^2*b*log(c*x^n)/x^3 - 1/3*a^3/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(69) = 138.

Time = 0.34 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.65

$$\int \frac{(a+b\log(cx^n))^3}{x^4}\,dx = -\frac{b^3n^3\log(x)^3}{3\,x^3}-\frac{(b^3n^3+3\,b^3n^2\log(c)+3\,ab^2n^2)\log(x)^2}{3\,x^3} - \frac{(2\,b^3n^3+6\,b^3n^2\log(c)+9\,b^3n\log(c)^2+6\,ab^2n^2+18\,ab^2n\log(c)+9\,a^2bn)\log(x)}{9\,x^3} - \frac{2\,b^3n^3+6\,b^3n^2\log(c)+9\,b^3n\log(c)^2+9\,b^3\log(c)^3+6\,ab^2n^2+18\,ab^2n\log(c)+27\,ab^2\log(c)^2+9\,a^2b}{27\,x^3}$$

```
[In] integrate((a+b*log(c*x^n))^3/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b^3*n^3*log(x)^3/x^3 - 1/3*(b^3*n^3 + 3*b^3*n^2*log(c) + 3*a*b^2*n^2)*
log(x)^2/x^3 - 1/9*(2*b^3*n^3 + 6*b^3*n^2*log(c) + 9*b^3*n*log(c)^2 + 6*a*b
^2*n^2 + 18*a*b^2*n*log(c) + 9*a^2*b*n)*log(x)/x^3 - 1/27*(2*b^3*n^3 + 6*b^
3*n^2*log(c) + 9*b^3*n*log(c)^2 + 9*b^3*log(c)^3 + 6*a*b^2*n^2 + 18*a*b^2*n
*log(c) + 27*a*b^2*log(c)^2 + 9*a^2*b*n + 27*a^2*b*log(c) + 9*a^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.43

$$\int \frac{(a+b\log(cx^n))^3}{x^4}\,dx = -\frac{\frac{a^3}{3}+\frac{a^2\,b\,n}{3}+\frac{2\,a\,b^2\,n^2}{9}+\frac{2\,b^3\,n^3}{27}}{x^3} - \frac{\ln\left(c\,x^n\right)\,\left(3\,a^2\,b+2\,a\,b^2\,n+\frac{2\,b^3\,n^2}{3}\right)}{3\,x^3} - \frac{\ln\left(c\,x^n\right)^2\,\left(\frac{n\,b^3}{3}+a\,b^2\right)}{x^3} - \frac{b^3\,\ln\left(c\,x^n\right)^3}{3\,x^3}$$

```
[In] int((a + b*log(c*x^n))^3/x^4,x)
```

```
[Out] - (a^3/3 + (2*b^3*n^3)/27 + (2*a*b^2*n^2)/9 + (a^2*b*n)/3)/x^3 - (log(c*x^n
)*(3*a^2*b + (2*b^3*n^2)/3 + 2*a*b^2*n))/(3*x^3) - (log(c*x^n)^2*(a*b^2 + (
b^3*n)/3))/x^3 - (b^3*log(c*x^n)^3)/(3*x^3)
```

3.65 $\int \frac{x^3}{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{x^3}{a+b\log(cx^n)}\,dx=\frac{e^{-\frac{4a}{bn}}x^4(cx^n)^{-4/n}\,\text{ExpIntegralEi}\left(\frac{4(a+b\log(cx^n))}{bn}\right)}{bn}$$

```
[Out] x^4*Ei(4*(a+b*ln(c*x^n))/b/n)/b/exp(4*a/b/n)/n/((c*x^n)^(4/n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2347, 2209}

$$\int \frac{x^3}{a+b\log(cx^n)}\,dx=\frac{x^4e^{-\frac{4a}{bn}}(cx^n)^{-4/n}\,\text{ExpIntegralEi}\left(\frac{4(a+b\log(cx^n))}{bn}\right)}{bn}$$

```
[In] Int[x^3/(a + b*Log[c*x^n]),x]
```

```
[Out] (x^4*ExpIntegralEi[(4*(a + b*Log[c*x^n]))/(b*n)])/(b*E^((4*a)/(b*n))*n*(c*x
^n)^(4/n))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
```

```
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^4 (cx^n)^{-4/n}\right) \text{Subst}\left(\int \frac{e^{\frac{4x}{n}}}{a+bx} \, dx, x, \log (cx^n)\right)}{n} \\
&= \frac{e^{-\frac{4a}{bn}} x^4 (cx^n)^{-4/n} \, \text{Ei}\left(\frac{4(a+b\log(cx^n))}{bn}\right)}{bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a+b\log(cx^n)} \, dx = \frac{e^{-\frac{4a}{bn}} x^4 (cx^n)^{-4/n} \, \text{ExpIntegralEi}\left(\frac{4(a+b\log(cx^n))}{bn}\right)}{bn}$$

```
[In] Integrate[x^3/(a + b*Log[c*x^n]),x]
[Out] (x^4*ExpIntegralEi[(4*(a + b*Log[c*x^n]))/(b*n)])/(b*E^((4*a)/(b*n))*n*(c*x
^n)^(4/n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.75

method	result
risch	$-\frac{x^4 c^{-\frac{4}{n}} (x^n)^{-\frac{4}{n}} \mathrm{e}^{-\frac{2\left(-ib\pi \,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi \,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi \,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2a\right)}{bn}} \mathrm{Ei}_1\left(-\right.}{}$

```
[In] int(x^3/(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -1/b/n*x^4*c^(-4/n)*(x^n)^(-4/n)*exp(-2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn
(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)
^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-4*ln(x)-2*(-I*b*Pi*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/
b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{a+b\log(cx^n)}\,dx = \frac{e^{\left(-\frac{4\,(b\log(c)+a)}{bn}\right)}\,\mathrm{log\_integral}\left(x^4e^{\left(\frac{4\,(b\log(c)+a)}{bn}\right)}\right)}{bn}$$

```
[In] integrate(x^3/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] e^(-4*(b*log(c) + a)/(b*n))*log_integral(x^4*e^(4*(b*log(c) + a)/(b*n)))/(b
*n)
```

Sympy [F]

$$\int \frac{x^3}{a+b\log(cx^n)}\,dx = \int \frac{x^3}{a+b\log(cx^n)}\,dx$$

```
[In] integrate(x**3/(a+b*ln(c*x**n)),x)
[Out] Integral(x**3/(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^3}{a+b\log(cx^n)}\,dx = \int \frac{x^3}{b\log(cx^n)+a}\,dx$$

```
[In] integrate(x^3/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(x^3/(b*log(c*x^n) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{a+b\log(cx^n)}\,dx = \frac{\mathrm{Ei}\left(\frac{4\log(c)}{n}+\frac{4a}{bn}+4\log(x)\right)e^{\left(-\frac{4a}{bn}\right)}}{bc^{\frac{4}{n}}n}$$

```
[In] integrate(x^3/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] Ei(4*log(c)/n + 4*a/(b*n) + 4*log(x))*e^(-4*a/(b*n))/(b*c^(4/n)*n)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\log\left(cx^n\right)}\,dx=\int \frac{x^3}{a+b\,\ln\left(c\,x^n\right)}\,dx$$

```
[In] int(x^3/(a + b*log(c*x^n)),x)
[Out] int(x^3/(a + b*log(c*x^n)), x)
```

3.66 $\int \frac{x^2}{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{x^2}{a+b\log\left(cx^n\right)}\,dx=\frac{e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{bn}\right)}{bn}$$

```
[Out] x^3*Ei(3*(a+b*ln(c*x^n))/b/n)/b/exp(3*a/b/n)/n/((c*x^n)^(3/n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2347, 2209}

$$\int \frac{x^2}{a+b\log\left(cx^n\right)}\,dx=\frac{x^3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{bn}\right)}{bn}$$

```
[In] Int[x^2/(a + b*Log[c*x^n]),x]
```

```
[Out] (x^3*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(b*n)])/(b*E^((3*a)/(b*n))*n*(c*x
^n)^(3/n))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
```

```
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x^3(cx^n)^{-3/n}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{a+bx}\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\operatorname{Ei}\left(\frac{3(a+b\log(cx^n))}{bn}\right)}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\log\left(cx^n\right)}\,dx = \frac{e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{bn}\right)}{bn}$$

```
[In] Integrate[x^2/(a + b*Log[c*x^n]),x]
[Out] (x^3*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(b*n)])/(b*E^((3*a)/(b*n))*n*(c*x
^n)^(3/n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.75

method	result
risch	$-\frac{x^3 c^{-\frac{3}{n}} (x^n)^{-\frac{3}{n}} e^{-\frac{3\left(-ib\pi \operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)+ib\pi \operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2+ib\pi \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2-ib\pi\operatorname{csgn}(ic\,x^n)^3+2a\right)}{2bn}} \operatorname{Ei}_1\left(-\right.}{}$

```
[In] int(x^2/(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -1/b/n*x^3*c^(-3/n)*(x^n)^(-3/n)*exp(-3/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*cs
gn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^
n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-3*ln(x)-3/2*(-I*b*Pi*csgn(I*c)*
csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^
n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2
*a)/b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{a+b\log(cx^n)}\,dx = \frac{e^{\left(-\frac{3\,(b\log(c)+a)}{bn}\right)}\,\mathrm{log\_integral}\left(x^3e^{\left(\frac{3\,(b\log(c)+a)}{bn}\right)}\right)}{bn}$$

```
[In] integrate(x^2/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] e^(-3*(b*log(c) + a)/(b*n))*log_integral(x^3*e^(3*(b*log(c) + a)/(b*n)))/(b
*n)
```

Sympy [F]

$$\int \frac{x^2}{a+b\log(cx^n)}\,dx = \int \frac{x^2}{a+b\log(cx^n)}\,dx$$

```
[In] integrate(x**2/(a+b*ln(c*x**n)),x)
[Out] Integral(x**2/(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \frac{x^2}{a+b\log(cx^n)}\,dx = \int \frac{x^2}{b\log(cx^n)+a}\,dx$$

```
[In] integrate(x^2/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(x^2/(b*log(c*x^n) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a+b\log(cx^n)}\,dx = \frac{\mathrm{Ei}\left(\frac{3\log(c)}{n}+\frac{3\,a}{bn}+3\,\log(x)\right)e^{\left(-\frac{3\,a}{bn}\right)}}{bc^{\frac{3}{n}}n}$$

```
[In] integrate(x^2/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))/(b*c^(3/n)*n)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\log(cx^n)}\,dx = \int \frac{x^2}{a+b\,\ln(c\,x^n)}\,dx$$

```
[In] int(x^2/(a + b*log(c*x^n)),x)
[Out] int(x^2/(a + b*log(c*x^n)), x)
```

3.67 $\int \frac{x}{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{x}{a+b\log\left(cx^n\right)}\,dx=\frac{e^{-\frac{2a}{bn}}x^2\left(cx^n\right)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(cx^n))}{bn}\right)}{bn}$$

```
[Out] x^2*Ei(2*(a+b*ln(c*x^n))/b/n)/b/exp(2*a/b/n)/n/((c*x^n)^(2/n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2347, 2209}

$$\int \frac{x}{a+b\log\left(cx^n\right)}\,dx=\frac{x^2e^{-\frac{2a}{bn}}\left(cx^n\right)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(cx^n))}{bn}\right)}{bn}$$

```
[In] Int[x/(a + b*Log[c*x^n]),x]
```

```
[Out] (x^2*ExpIntegralEi[(2*(a + b*Log[c*x^n]))/(b*n)])/(b*E^((2*a)/(b*n))*n*(c*x
^n)^(2/n))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
```

```
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{a+bx}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\,\text{Ei}\left(\frac{2(a+b\log(cx^n))}{bn}\right)}{bn}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\log(cx^n)}\,dx = \frac{e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(cx^n))}{bn}\right)}{bn}$$

```
[In] Integrate[x/(a + b*Log[c*x^n]),x]
[Out] (x^2*ExpIntegralEi[(2*(a + b*Log[c*x^n]))/(b*n)])/(b*E^((2*a)/(b*n))*n*(c*x
^n)^(2/n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.75

method	result
risch	$-\frac{x^2(x^n)^{-\frac{2}{n}}c^{-\frac{2}{n}}\,\mathrm{e}^{-\frac{-ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2a}{bn}}\,\mathrm{Ei}_1\left(-2\,\mathrm{l}\right.}{}$

```
[In] int(x/(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -1/b/n*x^2*(x^n)^(-2/n)*c^(-2/n)*exp(-(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I
*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2
-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-2*ln(x)-(-I*b*Pi*csgn(I*c)*csgn(I*x
^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(
I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x}{a+b\log\left(cx^{n}\right)}\,dx=\frac{e^{\left(-\frac{2\,(b\log(c)+a)}{bn}\right)}\,\mathrm{log\_integral}\left(x^{2}e^{\left(\frac{2\,(b\log(c)+a)}{bn}\right)}\right)}{bn}$$

```
[In] integrate(x/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] e^(-2*(b*log(c) + a)/(b*n))*log_integral(x^2*e^(2*(b*log(c) + a)/(b*n)))/(b
*n)
```

Sympy [F]

$$\int \frac{x}{a+b\log\left(cx^{n}\right)}\,dx=\int \frac{x}{a+b\log\left(cx^{n}\right)}\,dx$$

```
[In] integrate(x/(a+b*ln(c*x**n)),x)
[Out] Integral(x/(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \frac{x}{a+b\log\left(cx^{n}\right)}\,dx=\int \frac{x}{b\log\left(cx^{n}\right)+a}\,dx$$

```
[In] integrate(x/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(x/(b*log(c*x^n) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+b\log\left(cx^{n}\right)}\,dx=\frac{\mathrm{Ei}\left(\frac{2\log(c)}{n}+\frac{2\,a}{bn}+2\,\log\left(x\right)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{bc^{\frac{2}{n}}n}$$

```
[In] integrate(x/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] Ei(2*log(c)/n + 2*a/(b*n) + 2*log(x))*e^(-2*a/(b*n))/(b*c^(2/n)*n)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a + b\log(cx^n)}\,dx = \int \frac{x}{a + b\,\ln(c\,x^n)}\,dx$$

```
[In] int(x/(a + b*log(c*x^n)),x)
[Out] int(x/(a + b*log(c*x^n)), x)
```

3.68 $\int \frac{1}{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \frac{1}{a+b\log(cx^n)}\,dx = \frac{e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{bn}\right)}{bn}$$

```
[Out] x*Ei((a+b*ln(c*x^n))/b/n)/b/exp(a/b/n)/n/((c*x^n)^(1/n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2337, 2209}

$$\int \frac{1}{a+b\log(cx^n)}\,dx = \frac{xe^{-\frac{a}{bn}}(cx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{bn}\right)}{bn}$$

```
[In] Int[(a + b*Log[c*x^n])^(-1),x]
```

```
[Out] (x*ExpIntegralEi[(a + b*Log[c*x^n])/(b*n)])/(b*E^(a/(b*n))*n*(c*x^n)^n^(-1)
)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
```

```
{a, b, c, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx, x, \log\left(cx^n\right)\right)}{n}$$
$$= \frac{e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\text{Ei}\left(\frac{a+b\log(cx^n)}{bn}\right)}{bn}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\log\left(cx^n\right)}\,dx = \frac{e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{bn}\right)}{bn}$$

```
[In] Integrate[(a + b*Log[c*x^n])^(-1),x]
[Out] (x*ExpIntegralEi[(a + b*Log[c*x^n])/(b*n)])/(b*E^(a/(b*n))*n*(c*x^n)^n^(-1)
)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 240, normalized size of antiderivative = 5.00

method	result
risch	$-\frac{x\,c^{-\frac{1}{n}}\left(x^n\right)^{-\frac{1}{n}}\,\mathrm{e}^{-\frac{-ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2a}{2bn}}\,\mathrm{Ei}_1\left(-\ln(\right.}{}$

```
[In] int(1/(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -1/b/n*x*c^(-1/n)*(x^n)^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn
(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)
^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-ln(x)-1/2*(-I*b*Pi*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/
b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{a+b\log\left(cx^n\right)}\,dx=\frac{e^{\left(-\frac{b\log(c)+a}{bn}\right)}\,\mathrm{log\_integral}\left(xe^{\left(\frac{b\log(c)+a}{bn}\right)}\right)}{bn}$$

```
[In] integrate(1/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] e^(-(b*log(c) + a)/(b*n))*log_integral(x*e^((b*log(c) + a)/(b*n)))/(b*n)
```

Sympy [F]

$$\int \frac{1}{a+b\log\left(cx^n\right)}\,dx=\int \frac{1}{a+b\log\left(cx^n\right)}\,dx$$

```
[In] integrate(1/(a+b*ln(c*x**n)),x)
[Out] Integral(1/(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{a+b\log\left(cx^n\right)}\,dx=\int \frac{1}{b\log\left(cx^n\right)+a}\,dx$$

```
[In] integrate(1/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(1/(b*log(c*x^n) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+b\log\left(cx^n\right)}\,dx=\frac{\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log\left(x\right)\right)e^{\left(-\frac{a}{bn}\right)}}{bc^{\left(\frac{1}{n}\right)}n}$$

```
[In] integrate(1/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] Ei(log(c)/n + a/(b*n) + log(x))*e^(-a/(b*n))/(b*c^(1/n)*n)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\log(cx^n)}\,dx=\int \frac{1}{a+b\,\ln(c\,x^n)}\,dx$$

```
[In] int(1/(a + b*log(c*x^n)),x)
[Out] int(1/(a + b*log(c*x^n)), x)
```

3.69 $\int \frac{1}{x(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[Out] ln(a+b*ln(c*x^n))/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2339, 29}

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[In] Int[1/(x*(a + b*Log[c*x^n])),x]
```

```
[Out] Log[a + b*Log[c*x^n]]/(b*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log\left(cx^n\right)\right)}{bn}\\ &= \frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{\log\left(a+b\log\left(cx^n\right)\right)}{bn}$$

```
[In] Integrate[1/(x*(a + b*Log[c*x^n])),x]
[Out] Log[a + b*Log[c*x^n]]/(b*n)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\ln(a+b\ln(c\,x^n))}{bn}$
default	$\frac{\ln(a+b\ln(c\,x^n))}{bn}$
parallelrisch	$\frac{\ln(a+b\ln(c\,x^n))}{bn}$
norman	$\frac{\ln\left(a+b\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)\right)}{bn}$
risch	$\frac{\ln\left(\ln(x^n)-\frac{ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\,\mathrm{csgn}(ic\,x^n)^3-2b\ln(c)-2a}{2b}\right.}{bn}$

```
[In] int(1/x/(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*ln(c*x^n))/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\frac{\log\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)}{bn}$$

```
[In] integrate(1/x/(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] log(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\begin{cases}\frac{\log\left(x\right)}{a} & \text{for } b=0\wedge\left(b=0\vee n=0\right)\\ \frac{\log\left(x\right)}{a+b\log\left(c\right)} & \text{for } n=0\\ \frac{\log\left(\frac{a}{b}+\log\left(cx^n\right)\right)}{bn} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/x/(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)/(a + b*log(
c)), Eq(n, 0)), (log(a/b + log(c*x**n))/(b*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)}\,dx=\frac{\log\left(b\log\left(cx^n\right)+a\right)}{bn}$$

```
[In] integrate(1/x/(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] log(b*log(c*x^n) + a)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(18) = 36$.

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\,(a + b\log\left(cx^n\right))}\,dx$$
$$= \frac{\log\left(\frac{1}{4}\left(\pi bn(\mathrm{sgn}(x) - 1) + \pi b(\mathrm{sgn}(c) - 1)\right)^2 + \left(bn\log\left(|x|\right) + b\log\left(|c|\right) + a\right)^2\right)}{2\,bn}$$

```
[In] integrate(1/x/(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*log(1/4*(pi*b*n*(sgn(x) - 1) + pi*b*(sgn(c) - 1))^2 + (b*n*log(abs(x))
+ b*log(abs(c)) + a)^2)/(b*n)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a + b\log\left(cx^n\right))}\,dx = \frac{\ln\left(a + b\,\ln\left(c\,x^n\right)\right)}{b\,n}$$

```
[In] int(1/(x*(a + b*log(c*x^n))),x)
```

```
[Out] log(a + b*log(c*x^n))/(b*n)
```

3.70 $\int \frac{1}{x^2(a+b\log(cx^n))} dx$

Optimal result

Integrand size = 16, antiderivative size = 48

$$\int \frac{1}{x^2 (a + b\log(cx^n))} dx = \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}} \operatorname{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{bn}\right)}{bnx}$$

```
[Out] exp(a/b/n)*(c*x^n)^(1/n)*Ei((-a-b*ln(c*x^n))/b/n)/b/n/x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2347, 2209}

$$\int \frac{1}{x^2 (a + b\log(cx^n))} dx = \frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}} \operatorname{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{bn}\right)}{bnx}$$

```
[In] Int[1/(x^2*(a + b*Log[c*x^n])),x]
```

```
[Out] (E^(a/(b*n))*(c*x^n)^n^(-1)*ExpIntegralEi[-((a + b*Log[c*x^n])/(b*n))])/(b*
n*x)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
```

```
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(cx^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{a+bx} \, dx, x, \log(cx^n)\right)}{nx} \\ &= \frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{a+b\log(cx^n)}{bn}\right)}{bnx}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a+b\log(cx^n))} \, dx = \frac{e^{\frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{bn}\right)}{bnx}$$

```
[In] Integrate[1/(x^2*(a + b*Log[c*x^n])),x]
[Out] (E^(a/(b*n))*(c*x^n)^n^(-1)*ExpIntegralEi[-((a + b*Log[c*x^n])/(b*n))])/(b*
n*x)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.92

method	result
risch	$-\frac{c^{\frac{1}{n}} (x^n)^{\frac{1}{n}} \mathrm{e}^{\frac{-ib\pi \operatorname{csgn}(ic) \operatorname{csgn}(ix^n) \operatorname{csgn}(ic\,x^n)+ib\pi \operatorname{csgn}(ic) \operatorname{csgn}(ic\,x^n)^2+ib\pi \operatorname{csgn}(ix^n) \operatorname{csgn}(ic\,x^n)^2-ib\pi \operatorname{csgn}(ic\,x^n)^3+2a}{2bn}} \operatorname{Ei}_1\left(\ln(x)+\frac{-ib\pi}{}\right.}{bnx}$

```
[In] int(1/x^2/(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -1/b/n/x*c^(1/n)*(x^n)^(1/n)*exp(1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-
I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,ln(x)+1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x
^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(
I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 (a + b \log (cx^n))} \, dx = \frac{e^{\left(\frac{b \log(c)+a}{bn}\right)} \operatorname{log\_integral}\left(\frac{e^{\left(-\frac{b \log(c)+a}{bn}\right)}}{x}\right)}{bn}$$

```
[In] integrate(1/x^2/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] e^((b*log(c) + a)/(b*n))*log_integral(e^(-(b*log(c) + a)/(b*n))/x)/(b*n)
```

Sympy [F]

$$\int \frac{1}{x^2 (a + b \log (cx^n))} \, dx = \int \frac{1}{x^2 (a + b \log (cx^n))} \, dx$$

```
[In] integrate(1/x**2/(a+b*ln(c*x**n)),x)
[Out] Integral(1/(x**2*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + b \log (cx^n))} \, dx = \int \frac{1}{(b \log (cx^n) + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(1/((b*log(c*x^n) + a)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + b \log (cx^n))} \, dx = \int \frac{1}{(b \log (cx^n) + a)x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(1/((b*log(c*x^n) + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+b\log(cx^n))}\,dx = \int \frac{1}{x^2\ (a+b\ \ln(c\,x^n))}\,dx$$

```
[In] int(1/(x^2*(a + b*log(c*x^n))),x)
[Out] int(1/(x^2*(a + b*log(c*x^n))), x)
```

3.71 $\int \frac{1}{x^3(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{1}{x^3\,(a+b\log\left(cx^n\right))}\,dx=\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2(a+b\log(cx^n))}{bn}\right)}{bnx^2}$$

```
[Out] exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-2*(a+b*ln(c*x^n))/b/n)/b/n/x^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2347, 2209}

$$\int \frac{1}{x^3\,(a+b\log\left(cx^n\right))}\,dx=\frac{e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2(a+b\log(cx^n))}{bn}\right)}{bnx^2}$$

```
[In] Int[1/(x^3*(a + b*Log[c*x^n])),x]
```

```
[Out] (E^((2*a)/(b*n))*(c*x^n)^(2/n)*ExpIntegralEi[(-2*(a + b*Log[c*x^n]))/(b*n)]
)/(b*n*x^2)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{a+bx}\,dx, x, \log(cx^n)\right)}{nx^2} \\
&= \frac{e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\text{Ei}\left(-\frac{2(a+b\log(cx^n))}{bn}\right)}{bnx^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$
\int \frac{1}{x^3(a+b\log(cx^n))}\,dx = \frac{e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{2(a+b\log(cx^n))}{bn}\right)}{bnx^2}
$$

```
[In] Integrate[1/(x^3*(a + b*Log[c*x^n])),x]
[Out] (E^((2*a)/(b*n))*(c*x^n)^(2/n)*ExpIntegralEi[(-2*(a + b*Log[c*x^n]))/(b*n)]
)/(b*n*x^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.71

method	result
risch	$-\frac{c^{\frac{2}{n}}(x^n)^{\frac{2}{n}}\,\mathrm{e}^{\frac{-ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)+ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2+ib\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2-ib\pi\,\mathrm{csgn}(icx^n)^3+2a}{bn}}\,\mathrm{Ei}_1\left(2\ln(x)+\frac{-ib}{}\right.}{bn\,x^2}$

```
[In] int(1/x^3/(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -1/b/n/x^2*c^(2/n)*(x^n)^(2/n)*exp((-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*
x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*
b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,2*ln(x)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*
csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*
x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^3 (a + b \log (cx^n))} \, dx = \frac{e^{\left(\frac{2 (b \log(c)+a)}{bn}\right)} \operatorname{log\_integral}\left(\frac{e^{\left(-\frac{2 (b \log(c)+a)}{bn}\right)}}{x^2}\right)}{bn}$$

```
[In] integrate(1/x^3/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] e^(2*(b*log(c) + a)/(b*n))*log_integral(e^(-2*(b*log(c) + a)/(b*n))/x^2)/(b
*n)
```

Sympy [F]

$$\int \frac{1}{x^3 (a + b \log (cx^n))} \, dx = \int \frac{1}{x^3 (a + b \log (cx^n))} \, dx$$

```
[In] integrate(1/x**3/(a+b*ln(c*x**n)),x)
[Out] Integral(1/(x**3*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + b \log (cx^n))} \, dx = \int \frac{1}{(b \log (cx^n) + a)x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(1/((b*log(c*x^n) + a)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + b \log (cx^n))} \, dx = \int \frac{1}{(b \log (cx^n) + a)x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(1/((b*log(c*x^n) + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a+b\log(cx^n))}\,dx = \int \frac{1}{x^3\ (a+b\ \ln(c\,x^n))}\,dx$$

```
[In] int(1/(x^3*(a + b*log(c*x^n))),x)
[Out] int(1/(x^3*(a + b*log(c*x^n))), x)
```

3.72 $\int \frac{1}{x^4(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int \frac{1}{x^4\left(a+b\log\left(cx^n\right)\right)}\,dx=\frac{e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\operatorname{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{bn}\right)}{bnx^3}$$

```
[Out] exp(3*a/b/n)*(c*x^n)^(3/n)*Ei(-3*(a+b*ln(c*x^n))/b/n)/b/n/x^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2347, 2209}

$$\int \frac{1}{x^4\left(a+b\log\left(cx^n\right)\right)}\,dx=\frac{e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\operatorname{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{bn}\right)}{bnx^3}$$

```
[In] Int[1/(x^4*(a + b*Log[c*x^n])),x]
```

```
[Out] (E^((3*a)/(b*n))*(c*x^n)^(3/n)*ExpIntegralEi[(-3*(a + b*Log[c*x^n]))/(b*n)]
)/(b*n*x^3)
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(cx^n)^{3/n}\,\text{Subst}\left(\int \frac{e^{-\frac{3x}{n}}}{a+bx}\,dx, x, \log(cx^n)\right)}{nx^3} \\ &= \frac{e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\text{Ei}\left(-\frac{3(a+b\log(cx^n))}{bn}\right)}{bnx^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(a+b\log(cx^n))}\,dx = \frac{e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{bn}\right)}{bnx^3}$$

```
[In] Integrate[1/(x^4*(a + b*Log[c*x^n])),x]
[Out] (E^((3*a)/(b*n))*(c*x^n)^(3/n)*ExpIntegralEi[(-3*(a + b*Log[c*x^n]))/(b*n)]
)/(b*n*x^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.75

method	result
risch	$-\frac{c^{\frac{3}{n}}(x^n)^{\frac{3}{n}}\,e^{\frac{-\frac{3ib\pi\,\text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(icx^n)}{2}+\frac{3ib\pi\,\text{csgn}(ic)\,\text{csgn}(icx^n)^2}{2}+\frac{3ib\pi\,\text{csgn}(ix^n)\,\text{csgn}(icx^n)^2}{2}-\frac{3ib\pi\,\text{csgn}(icx^n)^3}{2}+3a}{bn}}\,\text{Ei}_1\left(3\ln(\right.}{}$

```
[In] int(1/x^4/(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -1/b/n/x^3*c^(3/n)*(x^n)^(3/n)*exp(3/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^
2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,3*ln(x)+3/2*(-I*b*Pi*csgn(I*c)*csgn
(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*c
sgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/
b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4 (a + b \log(cx^n))} dx = \frac{e^{\left(\frac{3 (b \log(c)+a)}{bn}\right)} \text{log\_integral}\left(\frac{e^{\left(-\frac{3 (b \log(c)+a)}{bn}\right)}}{x^3}\right)}{bn}$$

```
[In] integrate(1/x^4/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] e^(3*(b*log(c) + a)/(b*n))*log_integral(e^(-3*(b*log(c) + a)/(b*n))/x^3)/(b
*n)
```

Sympy [F]

$$\int \frac{1}{x^4 (a + b \log(cx^n))} dx = \int \frac{1}{x^4 (a + b \log(cx^n))} dx$$

```
[In] integrate(1/x**4/(a+b*ln(c*x**n)),x)
[Out] Integral(1/(x**4*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + b \log(cx^n))} dx = \int \frac{1}{(b \log(cx^n) + a)x^4} dx$$

```
[In] integrate(1/x^4/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(1/((b*log(c*x^n) + a)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a + b \log(cx^n))} dx = \int \frac{1}{(b \log(cx^n) + a)x^4} dx$$

```
[In] integrate(1/x^4/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(1/((b*log(c*x^n) + a)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + b \log (cx^n))} \, dx = \int \frac{1}{x^4 \ (a + b \ \ln (c \, x^n))} \, dx$$

```
[In] int(1/(x^4*(a + b*log(c*x^n))),x)
[Out] int(1/(x^4*(a + b*log(c*x^n))), x)
```

3.73 $\int \frac{x^3}{(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{x^3}{(a+b\log(cx^n))^2}\,dx = \frac{4e^{-\frac{4a}{bn}}x^4(cx^n)^{-4/n}\,\text{ExpIntegralEi}\left(\frac{4(a+b\log(cx^n))}{bn}\right)}{b^2n^2} - \frac{x^4}{bn(a+b\log(cx^n))}$$

```
[Out] 4*x^4*Ei(4*(a+b*ln(c*x^n))/b/n)/b^2/exp(4*a/b/n)/n^2/((c*x^n)^(4/n))-x^4/b/
n/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2343, 2347, 2209}

$$\int \frac{x^3}{(a+b\log(cx^n))^2}\,dx = \frac{4x^4e^{-\frac{4a}{bn}}(cx^n)^{-4/n}\,\text{ExpIntegralEi}\left(\frac{4(a+b\log(cx^n))}{bn}\right)}{b^2n^2} - \frac{x^4}{bn(a+b\log(cx^n))}$$

```
[In] Int[x^3/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (4*x^4*ExpIntegralEi[(4*(a + b*Log[c*x^n]))/(b*n)])/(b^2*E^((4*a)/(b*n))*n^
2*(c*x^n)^(4/n)) - x^4/(b*n*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^4}{bn\left(a+b\log\left(cx^n\right)\right)}+\frac{4\int\frac{x^3}{a+b\log(cx^n)}\,dx}{bn}\\
&= -\frac{x^4}{bn\left(a+b\log\left(cx^n\right)\right)}+\frac{\left(4x^4\left(cx^n\right)^{-4/n}\right)\text{Subst}\left(\int\frac{e^{\frac{4x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{bn^2}\\
&= \frac{4e^{-\frac{4a}{bn}}x^4\left(cx^n\right)^{-4/n}\text{Ei}\left(\frac{4(a+b\log(cx^n))}{bn}\right)}{b^2n^2}-\frac{x^4}{bn\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int\frac{x^3}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\frac{x^4\left(4e^{-\frac{4a}{bn}}\left(cx^n\right)^{-4/n}\text{ExpIntegralEi}\left(\frac{4(a+b\log(cx^n))}{bn}\right)-\frac{bn}{a+b\log(cx^n)}\right)}{b^2n^2}$$

```
[In] Integrate[x^3/(a + b*Log[c*x^n])^2,x]
[Out] (x^4*((4*ExpIntegralEi[(4*(a + b*Log[c*x^n]))/(b*n)])/(E^((4*a)/(b*n))*(c*x
^n)^(4/n)) - (b*n)/(a + b*Log[c*x^n])))/(b^2*n^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.66

method	result
risch	$-\frac{2x^4}{\left(-ib\pi \operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)+ib\pi \operatorname{csgn}(ic)\operatorname{csgn}(icx^n)^2+ib\pi \operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)^2-ib\pi \operatorname{csgn}(icx^n)^3+2b\ln(c)+2\ln(x^n)\right.}$

[In] int(x^3/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)

[Out] -2*x^4/(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*ln(x^n)*b+2*a)/b/n-4/b^2/n^2*x^4*c^(-4/n)*(x^n)^(-4/n)*exp(-2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-4*ln(x)-2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/b/n)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(a+b\log(cx^n))^2}\,dx = -\frac{\left(bnx^4e^{\left(\frac{4\,(b\log(c)+a)}{bn}\right)}-4\,(bn\log(x)+b\log(c)+a)\,\mathrm{log\_integral}\left(x^4e^{\left(\frac{4\,(b\log(c)+a)}{bn}\right)}\right)\right)e^{\left(-\frac{4\,(b\log(c)+a)}{bn}\right)}}{b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2}$$

[In] integrate(x^3/(a+b*log(c*x^n))^2,x, algorithm="fricas")

[Out] -(b*n*x^4*e^(4*(b*log(c) + a)/(b*n)) - 4*(b*n*log(x) + b*log(c) + a)*log_integral(x^4*e^(4*(b*log(c) + a)/(b*n))))*e^(-4*(b*log(c) + a)/(b*n))/(b^3*n^3*log(x) + b^3*n^2*log(c) + a*b^2*n^2)

Sympy [F]

$$\int \frac{x^3}{(a+b\log(cx^n))^2}\,dx = \int \frac{x^3}{(a+b\log(cx^n))^2}\,dx$$

```
[In] integrate(x**3/(a+b*ln(c*x**n))**2,x)
[Out] Integral(x**3/(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \frac{x^3}{(a+b\log(cx^n))^2}\,dx = \int \frac{x^3}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate(x^3/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -x^4/(b^2*n*log(c) + b^2*n*log(x^n) + a*b*n) + 4*integrate(x^3/(b^2*n*log(c
) + b^2*n*log(x^n) + a*b*n), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(77) = 154$.

Time = 0.37 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.43

$$\begin{aligned}\int \frac{x^3}{(a+b\log(cx^n))^2}\,dx = &-\frac{bnx^4}{b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2} \\ &+\frac{4\,bn\mathrm{Ei}\left(\frac{4\,\log(c)}{n}+\frac{4\,a}{bn}+4\,\log(x)\right)e^{\left(-\frac{4\,a}{bn}\right)}\log(x)}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\frac{4}{n}}} \\ &+\frac{4\,b\mathrm{Ei}\left(\frac{4\,\log(c)}{n}+\frac{4\,a}{bn}+4\,\log(x)\right)e^{\left(-\frac{4\,a}{bn}\right)}\log(c)}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\frac{4}{n}}} \\ &+\frac{4\,a\mathrm{Ei}\left(\frac{4\,\log(c)}{n}+\frac{4\,a}{bn}+4\,\log(x)\right)e^{\left(-\frac{4\,a}{bn}\right)}}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\frac{4}{n}}}\end{aligned}$$

```
[In] integrate(x^3/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] -b*n*x^4/(b^3*n^3*log(x) + b^3*n^2*log(c) + a*b^2*n^2) + 4*b*n*Ei(4*log(c)/
n + 4*a/(b*n) + 4*log(x))*e^(-4*a/(b*n))*log(x)/((b^3*n^3*log(x) + b^3*n^2*
log(c) + a*b^2*n^2)*c^(4/n)) + 4*b*Ei(4*log(c)/n + 4*a/(b*n) + 4*log(x))*e^
(-4*a/(b*n))*log(c)/((b^3*n^3*log(x) + b^3*n^2*log(c) + a*b^2*n^2)*c^(4/n))
 + 4*a*Ei(4*log(c)/n + 4*a/(b*n) + 4*log(x))*e^(-4*a/(b*n))/((b^3*n^3*log(x
) + b^3*n^2*log(c) + a*b^2*n^2)*c^(4/n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{x^3}{\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

```
[In] int(x^3/(a + b*log(c*x^n))^2,x)
[Out] int(x^3/(a + b*log(c*x^n))^2, x)
```

3.74 $\int \frac{x^2}{(a+b\log(cx^n))^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{x^2}{(a+b\log(cx^n))^2} \, dx = \frac{3e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{bn}\right)}{b^2n^2} - \frac{x^3}{bn(a+b\log(cx^n))}$$

```
[Out] 3*x^3*Ei(3*(a+b*ln(c*x^n))/b/n)/b^2/exp(3*a/b/n)/n^2/((c*x^n)^(3/n))-x^3/b/
n/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2343, 2347, 2209}

$$\int \frac{x^2}{(a+b\log(cx^n))^2} \, dx = \frac{3x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{bn}\right)}{b^2n^2} - \frac{x^3}{bn(a+b\log(cx^n))}$$

```
[In] Int[x^2/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (3*x^3*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(b*n)])/(b^2*E^((3*a)/(b*n))*n^
2*(c*x^n)^(3/n)) - x^3/(b*n*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3}{bn\left(a+b\log\left(cx^n\right)\right)}+\frac{3\int\frac{x^2}{a+b\log(cx^n)}\,dx}{bn}\\
&= -\frac{x^3}{bn\left(a+b\log\left(cx^n\right)\right)}+\frac{\left(3x^3\left(cx^n\right)^{-3/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{bn^2}\\
&= \frac{3e^{-\frac{3a}{bn}}x^3\left(cx^n\right)^{-3/n}\operatorname{Ei}\left(\frac{3(a+b\log(cx^n))}{bn}\right)}{b^2n^2}-\frac{x^3}{bn\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int\frac{x^2}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\frac{x^3\left(3e^{-\frac{3a}{bn}}\left(cx^n\right)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{bn}\right)-\frac{bn}{a+b\log(cx^n)}\right)}{b^2n^2}$$

```
[In] Integrate[x^2/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (x^3*((3*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(b*n)])/(E^((3*a)/(b*n))*(c*x
^n)^(3/n)) - (b*n)/(a + b*Log[c*x^n])))/(b^2*n^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.66

method	result
risch	$-\frac{2x^3}{\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2b\ln(c)+2\ln(x^n)b-\right.}$

[In] int(x^2/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)

[Out] -2*x^3/(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*ln(x^n)*b+2*a)/b/n-3/b^2/n^2*x^3*c^(-3/n)*(x^n)^(-3/n)*exp(-3/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-3*ln(x)-3/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/b/n)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.33

$$\int \frac{x^2}{(a+b\log(cx^n))^2}\,dx = -\frac{\left(bnx^3e^{\left(\frac{3\,(b\log(c)+a)}{bn}\right)} - 3\,(bn\log(x)+b\log(c)+a)\,\mathrm{log\_integral}\left(x^3e^{\left(\frac{3\,(b\log(c)+a)}{bn}\right)}\right)\right)e^{\left(-\frac{3\,(b\log(c)+a)}{bn}\right)}}{b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2}$$

[In] integrate(x^2/(a+b*log(c*x^n))^2,x, algorithm="fricas")

[Out] -(b*n*x^3*e^(3*(b*log(c) + a)/(b*n)) - 3*(b*n*log(x) + b*log(c) + a)*log_integral(x^3*e^(3*(b*log(c) + a)/(b*n))))*e^(-3*(b*log(c) + a)/(b*n))/(b^3*n^3*log(x) + b^3*n^2*log(c) + a*b^2*n^2)

Sympy [F]

$$\int \frac{x^2}{(a+b\log(cx^n))^2}\,dx = \int \frac{x^2}{(a+b\log(cx^n))^2}\,dx$$

```
[In] integrate(x**2/(a+b*ln(c*x**n))**2,x)
[Out] Integral(x**2/(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\log(cx^n))^2}\,dx = \int \frac{x^2}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -x^3/(b^2*n*log(c) + b^2*n*log(x^n) + a*b*n) + 3*integrate(x^2/(b^2*n*log(c
) + b^2*n*log(x^n) + a*b*n), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(77) = 154$.

Time = 0.37 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.43

$$\begin{aligned}\int \frac{x^2}{(a+b\log(cx^n))^2}\,dx =& -\frac{bnx^3}{b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2} \\ &+ \frac{3\,bn\mathrm{Ei}\left(\frac{3\,\log(c)}{n}+\frac{3\,a}{bn}+3\,\log(x)\right)e^{\left(-\frac{3\,a}{bn}\right)}\log(x)}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\frac{3}{n}}} \\ &+ \frac{3\,b\mathrm{Ei}\left(\frac{3\,\log(c)}{n}+\frac{3\,a}{bn}+3\,\log(x)\right)e^{\left(-\frac{3\,a}{bn}\right)}\log(c)}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\frac{3}{n}}} \\ &+ \frac{3\,a\mathrm{Ei}\left(\frac{3\,\log(c)}{n}+\frac{3\,a}{bn}+3\,\log(x)\right)e^{\left(-\frac{3\,a}{bn}\right)}}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\frac{3}{n}}}\end{aligned}$$

```
[In] integrate(x^2/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] -b*n*x^3/(b^3*n^3*log(x) + b^3*n^2*log(c) + a*b^2*n^2) + 3*b*n*Ei(3*log(c)/
n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))*log(x)/((b^3*n^3*log(x) + b^3*n^2*
log(c) + a*b^2*n^2)*c^(3/n)) + 3*b*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^
(-3*a/(b*n))*log(c)/((b^3*n^3*log(x) + b^3*n^2*log(c) + a*b^2*n^2)*c^(3/n))
 + 3*a*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))/((b^3*n^3*log(x
) + b^3*n^2*log(c) + a*b^2*n^2)*c^(3/n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\log(cx^n))^2}\,dx = \int \frac{x^2}{(a+b\,\ln(c\,x^n))^2}\,dx$$

```
[In] int(x^2/(a + b*log(c*x^n))^2,x)
[Out] int(x^2/(a + b*log(c*x^n))^2, x)
```

3.75 $\int \frac{x}{(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 76

$$\int \frac{x}{(a+b\log(cx^n))^2}\,dx = \frac{2e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(cx^n))}{bn}\right)}{b^2n^2} - \frac{x^2}{bn\,(a+b\log(cx^n))}$$

```
[Out] 2*x^2*Ei(2*(a+b*ln(c*x^n))/b/n)/b^2/exp(2*a/b/n)/n^2/((c*x^n)^(2/n))-x^2/b/
n/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2343, 2347, 2209}

$$\int \frac{x}{(a+b\log(cx^n))^2}\,dx = \frac{2x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(cx^n))}{bn}\right)}{b^2n^2} - \frac{x^2}{bn\,(a+b\log(cx^n))}$$

```
[In] Int[x/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*x^2*ExpIntegralEi[(2*(a + b*Log[c*x^n]))/(b*n)])/(b^2*E^((2*a)/(b*n))*n^
2*(c*x^n)^(2/n)) - x^2/(b*n*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2}{bn\,(a+b\log(cx^n))}+\frac{2\int \frac{x}{a+b\log(cx^n)}\,dx}{bn}\\
&= -\frac{x^2}{bn\,(a+b\log(cx^n))}+\frac{\left(2x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{a+bx}\,dx,x,\log(cx^n)\right)}{bn^2}\\
&= \frac{2e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\,\text{Ei}\left(\frac{2(a+b\log(cx^n))}{bn}\right)}{b^2n^2}-\frac{x^2}{bn\,(a+b\log(cx^n))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$
\int \frac{x}{(a+b\log(cx^n))^2}\,dx=\frac{x^2\left(2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(cx^n))}{bn}\right)-\frac{bn}{a+b\log(cx^n)}\right)}{b^2n^2}
$$

```
[In] Integrate[x/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (x^2*((2*ExpIntegralEi[(2*(a + b*Log[c*x^n]))/(b*n)])/(E^((2*a)/(b*n))*(c*x
^n)^(2/n)) - (b*n)/(a + b*Log[c*x^n])))/(b^2*n^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 350, normalized size of antiderivative = 4.61

method	result
risch	$-\frac{2ix^2}{bn\left(b\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)-b\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2-b\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2+b\pi\,\mathrm{csgn}(ic\,x^n)^3+2ib\ln(c)+2ib\ln(x^n)\right.}$

```
[In] int(x/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*x^2/b/n/(b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-b*Pi*csgn(I*c)*csgn(
I*c*x^n)^2-b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+b*Pi*csgn(I*c*x^n)^3+2*I*b*ln(c
)+2*I*b*ln(x^n)+2*I*a)-2/b^2/n^2*x^2*c^(-2/n)*(x^n)^(-2/n)*exp(-(-I*b*Pi*cs
gn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*c
sgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-2*ln(x)+I
*(b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-b
*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2+b*Pi*csgn(I*c*x^n)^3+2*I*b*ln(c)+2*I*b*(ln(
x^n)-n*ln(x))+2*I*a)/b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.33

$$\int \frac{x}{(a+b\log(cx^n))^2}\,dx = -\frac{\left(bnx^2e^{\left(\frac{2\,(b\log(c)+a)}{bn}\right)}-2\,(bn\log(x)+b\log(c)+a)\,\mathrm{log\_integral}\left(x^2e^{\left(\frac{2\,(b\log(c)+a)}{bn}\right)}\right)\right)e^{\left(-\frac{2\,(b\log(c)+a)}{bn}\right)}}{b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2}$$

```
[In] integrate(x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] -(b*n*x^2*e^(2*(b*log(c) + a)/(b*n)) - 2*(b*n*log(x) + b*log(c) + a)*log_in
tegral(x^2*e^(2*(b*log(c) + a)/(b*n))))*e^(-2*(b*log(c) + a)/(b*n))/(b^3*n^
3*log(x) + b^3*n^2*log(c) + a*b^2*n^2)
```

Sympy [F]

$$\int \frac{x}{(a+b\log(cx^n))^2}\,dx = \int \frac{x}{(a+b\log(cx^n))^2}\,dx$$

```
[In] integrate(x/(a+b*ln(c*x**n))**2,x)
[Out] Integral(x/(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \frac{x}{(a+b\log(cx^n))^2}\,dx = \int \frac{x}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -x^2/(b^2*n*log(c) + b^2*n*log(x^n) + a*b*n) + 2*integrate(x/(b^2*n*log(c)
+ b^2*n*log(x^n) + a*b*n), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(77) = 154.

Time = 0.36 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.43

$$\begin{aligned}\int \frac{x}{(a+b\log(cx^n))^2}\,dx = &-\frac{bnx^2}{b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2} \\ &+\frac{2\,bn\text{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log(x)\right)e^{\left(-\frac{2\,a}{bn}\right)}\log(x)}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\frac{2}{n}}} \\ &+\frac{2\,b\text{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log(x)\right)e^{\left(-\frac{2\,a}{bn}\right)}\log(c)}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\frac{2}{n}}} \\ &+\frac{2\,a\text{Ei}\left(\frac{2\,\log(c)}{n}+\frac{2\,a}{bn}+2\,\log(x)\right)e^{\left(-\frac{2\,a}{bn}\right)}}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\frac{2}{n}}}\end{aligned}$$

```
[In] integrate(x/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] -b*n*x^2/(b^3*n^3*log(x) + b^3*n^2*log(c) + a*b^2*n^2) + 2*b*n*Ei(2*log(c)/
n + 2*a/(b*n) + 2*log(x))*e^(-2*a/(b*n))*log(x)/((b^3*n^3*log(x) + b^3*n^2*
log(c) + a*b^2*n^2)*c^(2/n)) + 2*b*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(x))*e^
(-2*a/(b*n))*log(c)/((b^3*n^3*log(x) + b^3*n^2*log(c) + a*b^2*n^2)*c^(2/n))
 + 2*a*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(x))*e^(-2*a/(b*n))/((b^3*n^3*log(x
) + b^3*n^2*log(c) + a*b^2*n^2)*c^(2/n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{x}{\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

```
[In] int(x/(a + b*log(c*x^n))^2,x)
```

```
[Out] int(x/(a + b*log(c*x^n))^2, x)
```

3.76 $\int \frac{1}{(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int \frac{1}{(a+b\log(cx^n))^2}\,dx=\frac{e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{bn}\right)}{b^2n^2}-\frac{x}{bn(a+b\log(cx^n))}$$

```
[Out] x*Ei((a+b*ln(c*x^n))/b/n)/b^2/exp(a/b/n)/n^2/((c*x^n)^(1/n))-x/b/n/(a+b*ln(
c*x^n))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2334, 2337, 2209}

$$\int \frac{1}{(a+b\log(cx^n))^2}\,dx=\frac{xe^{-\frac{a}{bn}}(cx^n)^{-1/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{bn}\right)}{b^2n^2}-\frac{x}{bn(a+b\log(cx^n))}$$

```
[In] Int[(a + b*Log[c*x^n])^(-2),x]
```

```
[Out] (x*ExpIntegralEi[(a + b*Log[c*x^n])/(b*n)])/(b^2*E^(a/(b*n))*n^2*(c*x^n)^n^
(-1)) - x/(b*n*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x}{bn\left(a+b\log\left(cx^n\right)\right)}+\frac{\int\frac{1}{a+b\log(cx^n)}\,dx}{bn}\\
&= -\frac{x}{bn\left(a+b\log\left(cx^n\right)\right)}+\frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{bn^2}\\
&= \frac{e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\text{Ei}\left(\frac{a+b\log(cx^n)}{bn}\right)}{b^2n^2}-\frac{x}{bn\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$
\int\frac{1}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\frac{x\left(e^{-\frac{a}{bn}}(cx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{bn}\right)-\frac{bn}{a+b\log(cx^n)}\right)}{b^2n^2}
$$

```
[In] Integrate[(a + b*Log[c*x^n])^(-2),x]
```

```
[Out] (x*(ExpIntegralEi[(a + b*Log[c*x^n])/(b*n)]/(E^(a/(b*n))*(c*x^n)^n^(-1)) -
(b*n)/(a + b*Log[c*x^n])))/(b^2*n^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 350, normalized size of antiderivative = 5.00

method	result
risch	$-\frac{2x}{\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2b\ln(c)+2\ln(x^n)b-\right.}$

```
[In] int(1/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x
^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2
*ln(x^n)*b+2*a)/b/n*x-1/b^2/n^2*x*(x^n)^(-1/n)*c^(-1/n)*exp(-1/2*(-I*b*Pi*c
sgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*
csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-ln(x)-1/
2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^
n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*
b*(ln(x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+b\log(cx^n))^2}\,dx = -\frac{\left(bnxe^{\left(\frac{b\log(c)+a}{bn}\right)}-(bn\log(x)+b\log(c)+a)\,\mathrm{log\_integral}\left(xe^{\left(\frac{b\log(c)+a}{bn}\right)}\right)\right)e^{\left(-\frac{b\log(c)+a}{bn}\right)}}{b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2}$$

```
[In] integrate(1/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] -(b*n*x*e^((b*log(c) + a)/(b*n)) - (b*n*log(x) + b*log(c) + a)*log_integral
(x*e^((b*log(c) + a)/(b*n))))*e^(-(b*log(c) + a)/(b*n))/(b^3*n^3*log(x) + b
^3*n^2*log(c) + a*b^2*n^2)
```

Sympy [F]

$$\int \frac{1}{(a+b\log(cx^n))^2}\,dx = \int \frac{1}{(a+b\log(cx^n))^2}\,dx$$

```
[In] integrate(1/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral((a + b*log(c*x**n))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\log(cx^n))^2}\,dx = \int \frac{1}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -x/(b^2*n*log(c) + b^2*n*log(x^n) + a*b*n) + integrate(1/(b^2*n*log(c) + b^
2*n*log(x^n) + a*b*n), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(69) = 138$.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.40

$$\begin{aligned}\int \frac{1}{(a+b\log(cx^n))^2}\,dx &= \frac{bn\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(x)\right)e^{\left(-\frac{a}{bn}\right)}\log(x)}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\left(\frac{1}{n}\right)}} \\ &\quad - \frac{bnx}{b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2} \\ &\quad + \frac{b\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(x)\right)e^{\left(-\frac{a}{bn}\right)}\log(c)}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\left(\frac{1}{n}\right)}} \\ &\quad + \frac{a\mathrm{Ei}\left(\frac{\log(c)}{n}+\frac{a}{bn}+\log(x)\right)e^{\left(-\frac{a}{bn}\right)}}{(b^3n^3\log(x)+b^3n^2\log(c)+ab^2n^2)c^{\left(\frac{1}{n}\right)}}\end{aligned}$$

```
[In] integrate(1/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] b*n*Ei(log(c)/n + a/(b*n) + log(x))*e^(-a/(b*n))*log(x)/((b^3*n^3*log(x) +
b^3*n^2*log(c) + a*b^2*n^2)*c^(1/n)) - b*n*x/(b^3*n^3*log(x) + b^3*n^2*log(
c) + a*b^2*n^2) + b*Ei(log(c)/n + a/(b*n) + log(x))*e^(-a/(b*n))*log(c)/((b
^3*n^3*log(x) + b^3*n^2*log(c) + a*b^2*n^2)*c^(1/n)) + a*Ei(log(c)/n + a/(b
*n) + log(x))*e^(-a/(b*n))/((b^3*n^3*log(x) + b^3*n^2*log(c) + a*b^2*n^2)*c
^(1/n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(cx^n))^2}\,dx = \int \frac{1}{(a+b\,\ln(c\,x^n))^2}\,dx$$

```
[In] int(1/(a + b*log(c*x^n))^2,x)
[Out] int(1/(a + b*log(c*x^n))^2, x)
```

3.77 $\int \frac{1}{x(a+b\log(cx^n))^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)^2} dx = -\frac{1}{bn\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] -1/b/n/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2339, 30}

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)^2} dx = -\frac{1}{bn\left(a+b\log\left(cx^n\right)\right)}$$

```
[In] Int[1/(x*(a + b*Log[c*x^n])^2),x]
[Out] -(1/(b*n*(a + b*Log[c*x^n])))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^2}\,dx, x, a + b\log\left(cx^n\right)\right)}{bn}$$
$$= -\frac{1}{bn\left(a + b\log\left(cx^n\right)\right)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a + b\log\left(cx^n\right)\right)^2}\,dx = -\frac{1}{bn\left(a + b\log\left(cx^n\right)\right)}$$

```
[In] Integrate[1/(x*(a + b*Log[c*x^n])^2),x]
[Out] -(1/(b*n*(a + b*Log[c*x^n])))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{1}{bn(a+b\ln(c\,x^n))}$
default	$-\frac{1}{bn(a+b\ln(c\,x^n))}$
parallelrisch	$-\frac{1}{bn(a+b\ln(c\,x^n))}$
risch	$-\frac{2}{nb\left(-ib\pi\ \text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)+ib\pi\ \text{csgn}(ic)\,\text{csgn}(ic\,x^n)^2+ib\pi\ \text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)^2-ib\pi\,\text{csgn}(ic\,x^n)^3+2b\,\text{l}\right.}$

```
[In] int(1/x/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
[Out] -1/b/n/(a+b*ln(c*x^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\left(a+b\log\left(cx^{n}\right)\right)^{2}}\,dx=-\frac{1}{b^{2}n^{2}\log\left(x\right)+b^{2}n\log\left(c\right)+abn}$$

```
[In] integrate(1/x/(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] -1/(b^2*n^2*log(x) + b^2*n*log(c) + a*b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(15) = 30.

Time = 0.87 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{1}{x\left(a+b\log\left(cx^{n}\right)\right)^{2}}\,dx=\begin{cases}\frac{\log\left(x\right)}{a^{2}} & \text{for } b=0\wedge\left(b=0\vee n=0\right)\\ \frac{\log\left(x\right)}{\left(a+b\log\left(c\right)\right)^{2}} & \text{for } n=0\\ -\frac{1}{abn+b^{2}n\log\left(cx^{n}\right)} & \text{otherwise}\end{cases}$$

```
[In] integrate(1/x/(a+b*ln(c*x**n))**2,x)
[Out] Piecewise((log(x)/a**2, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)/(a + b*l
og(c))**2, Eq(n, 0)), (-1/(a*b*n + b**2*n*log(c*x**n)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\log\left(cx^{n}\right)\right)^{2}}\,dx=-\frac{1}{\left(b\log\left(cx^{n}\right)+a\right)bn}$$

```
[In] integrate(1/x/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -1/((b*log(c*x^n) + a)*b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(a+b\log\left(cx^{n}\right)\right)^{2}}\,dx = -\frac{1}{(bn\log\left(x\right)+b\log\left(c\right)+a)bn}$$

```
[In] integrate(1/x/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] -1/((b*n*log(x) + b*log(c) + a)*b*n)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\log\left(cx^{n}\right)\right)^{2}}\,dx = -\frac{1}{n\,\ln\left(c\,x^{n}\right)\,b^{2}+a\,n\,b}$$

```
[In] int(1/(x*(a + b*log(c*x^n))^2),x)
[Out] -1/(b^2*n*log(c*x^n) + a*b*n)
```

3.78 $\int \frac{1}{x^2(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 73

$$\int \frac{1}{x^2 (a+b\log(cx^n))^2}\,dx = -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\,\text{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{bn}\right)}{b^2n^2x} - \frac{1}{bnx\,(a+b\log(cx^n))}$$

```
[Out] -exp(a/b/n)*(c*x^n)^(1/n)*Ei((-a-b*ln(c*x^n))/b/n)/b^2/n^2/x-1/b/n/x/(a+b*l
n(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2343, 2347, 2209}

$$\int \frac{1}{x^2 (a+b\log(cx^n))^2}\,dx = -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\,\text{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{bn}\right)}{b^2n^2x} - \frac{1}{bnx\,(a+b\log(cx^n))}$$

```
[In] Int[1/(x^2*(a + b*Log[c*x^n])^2),x]
```

```
[Out] -((E^(a/(b*n))*(c*x^n)^n^(-1)*ExpIntegralEi[-((a + b*Log[c*x^n])/(b*n))])/(
b^2*n^2*x)) - 1/(b*n*x*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{bnx\left(a+b\log\left(cx^n\right)\right)} - \frac{\int \frac{1}{x^2\left(a+b\log\left(cx^n\right)\right)}\,dx}{bn} \\
&= -\frac{1}{bnx\left(a+b\log\left(cx^n\right)\right)} - \frac{\left(cx^n\right)^{\frac{1}{n}}\operatorname{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{a+bx}\,dx, x, \log\left(cx^n\right)\right)}{bn^2x} \\
&= -\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)}{b^2n^2x} - \frac{1}{bnx\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$
\int \frac{1}{x^2\left(a+b\log\left(cx^n\right)\right)^2}\,dx = -\frac{bn+e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{a+b\log\left(cx^n\right)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)}{b^2n^2x\left(a+b\log\left(cx^n\right)\right)}
$$

```
[In] Integrate[1/(x^2*(a + b*Log[c*x^n])^2),x]
[Out] -((b*n + E^(a/(b*n))*(c*x^n)^n^(-1)*ExpIntegralEi[-((a + b*Log[c*x^n])/(b*n
))]*(a + b*Log[c*x^n]))/(b^2*n^2*x*(a + b*Log[c*x^n])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 347, normalized size of antiderivative = 4.75

method	result
risch	$-\frac{2}{x\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2b\ln(c)+2\ln(x^n\right.}}$

```
[In] int(1/x^2/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/x/(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c
*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)
+2*ln(x^n)*b+2*a)/b/n+1/b^2/n^2/x*(x^n)^(1/n)*c^(1/n)*exp(1/2*(-I*b*Pi*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csg
n(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,ln(x)+1/2*(-
I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2
+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(l
n(x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$
$$= -\frac{(bnx\log(x)+bx\log(c)+ax)e^{\left(\frac{b\log(c)+a}{bn}\right)}\ \mathrm{log\_integral}\left(\frac{e^{\left(-\frac{b\log(c)+a}{bn}\right)}}{x}\right)+bn}{b^3n^3x\log(x)+b^3n^2x\log(c)+ab^2n^2x}$$

```
[In] integrate(1/x^2/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] -((b*n*x*log(x) + b*x*log(c) + a*x)*e^((b*log(c) + a)/(b*n))*log_integral(e
^(-(b*log(c) + a)/(b*n))/x) + b*n)/(b^3*n^3*x*log(x) + b^3*n^2*x*log(c) + a
*b^2*n^2*x)
```

Sympy [F]

$$\int \frac{1}{x^2 \left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{x^2 \left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*ln(c*x**n))**2,x)
[Out] Integral(1/(x**2*(a + b*log(c*x**n))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{\left(b\log\left(cx^n\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -1/(b^2*n*x*log(x^n) + (b^2*n*log(c) + a*b*n)*x) - integrate(1/(b^2*n*x^2*l
og(x^n) + (b^2*n*log(c) + a*b*n)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{\left(b\log\left(cx^n\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(1/((b*log(c*x^n) + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{x^2\,\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

```
[In] int(1/(x^2*(a + b*log(c*x^n))^2),x)
[Out] int(1/(x^2*(a + b*log(c*x^n))^2), x)
```

3.79 $\int \frac{1}{x^3(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{1}{x^3 (a+b\log(cx^n))^2}\,dx = -\frac{2e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{2(a+b\log(cx^n))}{bn}\right)}{b^2n^2x^2} - \frac{1}{bnx^2(a+b\log(cx^n))}$$

```
[Out] -2*exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-2*(a+b*ln(c*x^n))/b/n)/b^2/n^2/x^2-1/b/n/
x^2/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2343, 2347, 2209}

$$\int \frac{1}{x^3 (a+b\log(cx^n))^2}\,dx = -\frac{2e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\text{ExpIntegralEi}\left(-\frac{2(a+b\log(cx^n))}{bn}\right)}{b^2n^2x^2} - \frac{1}{bnx^2(a+b\log(cx^n))}$$

```
[In] Int[1/(x^3*(a + b*Log[c*x^n])^2),x]
```

```
[Out] (-2*E^((2*a)/(b*n))*(c*x^n)^(2/n)*ExpIntegralEi[(-2*(a + b*Log[c*x^n]))/(b*
n)])/(b^2*n^2*x^2) - 1/(b*n*x^2*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{bnx^2\left(a+b\log\left(cx^n\right)\right)}-\frac{2\int\frac{1}{x^3\left(a+b\log\left(cx^n\right)\right)}\,dx}{bn}\\
&= -\frac{1}{bnx^2\left(a+b\log\left(cx^n\right)\right)}-\frac{\left(2\left(cx^n\right)^{2/n}\right)\operatorname{Subst}\left(\int\frac{e^{-\frac{2x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{bn^2x^2}\\
&= -\frac{2e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\operatorname{Ei}\left(-\frac{2(a+b\log(cx^n))}{bn}\right)}{b^2n^2x^2}-\frac{1}{bnx^2\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int\frac{1}{x^3\left(a+b\log\left(cx^n\right)\right)^2}\,dx = -\frac{bn+2e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\operatorname{ExpIntegralEi}\left(-\frac{2(a+b\log(cx^n))}{bn}\right)\left(a+b\log\left(cx^n\right)\right)}{b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}$$

```
[In] Integrate[1/(x^3*(a + b*Log[c*x^n])^2),x]
[Out] -((b*n + 2*E^((2*a)/(b*n))*(c*x^n)^(2/n)*ExpIntegralEi[(-2*(a + b*Log[c*x^n
]))/(b*n)]*(a + b*Log[c*x^n]))/(b^2*n^2*x^2*(a + b*Log[c*x^n])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 352, normalized size of antiderivative = 4.63

method	result
risch	$-\frac{2}{x^2\left(-ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)+ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2+ib\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2-ib\pi\,\mathrm{csgn}(icx^n)^3+2b\ln(c)+2\ln(x\right.}$

```
[In] int(1/x^3/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/x^2/(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I
*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(
c)+2*ln(x^n)*b+2*a)/b/n+2/b^2/n^2/x^2*(x^n)^(2/n)*c^(2/n)*exp((-I*b*Pi*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csg
n(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,2*ln(x)+(-I*
b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I
*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(
x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^3\left(a+b\log\left(cx^n\right)\right)^2}\,dx = -\frac{2\left(bnx^2\log\left(x\right)+bx^2\log\left(c\right)+ax^2\right)e^{\left(\frac{2\left(b\log(c)+a\right)}{bn}\right)}\,\mathrm{log\_integral}\left(\frac{e^{\left(-\frac{2\left(b\log(c)+a\right)}{bn}\right)}}{x^2}\right)+bn}{b^3n^3x^2\log\left(x\right)+b^3n^2x^2\log\left(c\right)+ab^2n^2x^2}$$

```
[In] integrate(1/x^3/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] -(2*(b*n*x^2*log(x) + b*x^2*log(c) + a*x^2)*e^(2*(b*log(c) + a)/(b*n))*log_
integral(e^(-2*(b*log(c) + a)/(b*n))/x^2) + b*n)/(b^3*n^3*x^2*log(x) + b^3*
n^2*x^2*log(c) + a*b^2*n^2*x^2)
```

Sympy [F]

$$\int \frac{1}{x^3 (a + b \log(cx^n))^2} dx = \int \frac{1}{x^3 (a + b \log(cx^n))^2} dx$$

```
[In] integrate(1/x**3/(a+b*ln(c*x**n))**2,x)
[Out] Integral(1/(x**3*(a + b*log(c*x**n))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + b \log(cx^n))^2} dx = \int \frac{1}{(b \log(cx^n) + a)^2 x^3} dx$$

```
[In] integrate(1/x^3/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -1/(b^2*n*x^2*log(x^n) + (b^2*n*log(c) + a*b*n)*x^2) - 2*integrate(1/(b^2*n
*x^3*log(x^n) + (b^2*n*log(c) + a*b*n)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + b \log(cx^n))^2} dx = \int \frac{1}{(b \log(cx^n) + a)^2 x^3} dx$$

```
[In] integrate(1/x^3/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(1/((b*log(c*x^n) + a)^2*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + b \log(cx^n))^2} dx = \int \frac{1}{x^3 (a + b \ln(c x^n))^2} dx$$

```
[In] int(1/(x^3*(a + b*log(c*x^n))^2),x)
[Out] int(1/(x^3*(a + b*log(c*x^n))^2), x)
```

3.80 $\int \frac{1}{x^4(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{1}{x^4(a+b\log(cx^n))^2}\,dx = -\frac{3e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{bn}\right)}{b^2n^2x^3} - \frac{1}{bnx^3(a+b\log(cx^n))}$$

```
[Out] -3*exp(3*a/b/n)*(c*x^n)^(3/n)*Ei(-3*(a+b*ln(c*x^n))/b/n)/b^2/n^2/x^3-1/b/n/
x^3/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2343, 2347, 2209}

$$\int \frac{1}{x^4(a+b\log(cx^n))^2}\,dx = -\frac{3e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{bn}\right)}{b^2n^2x^3} - \frac{1}{bnx^3(a+b\log(cx^n))}$$

```
[In] Int[1/(x^4*(a + b*Log[c*x^n])^2),x]
```

```
[Out] (-3*E^((3*a)/(b*n))*(c*x^n)^(3/n)*ExpIntegralEi[(-3*(a + b*Log[c*x^n]))/(b*
n)])/(b^2*n^2*x^3) - 1/(b*n*x^3*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{bnx^3 (a+b\log(cx^n))} - \frac{3\int \frac{1}{x^4(a+b\log(cx^n))}\,dx}{bn} \\
&= -\frac{1}{bnx^3 (a+b\log(cx^n))} - \frac{\left(3(cx^n)^{3/n}\right)\text{Subst}\left(\int \frac{e^{-\frac{3x}{n}}}{a+bx}\,dx, x, \log(cx^n)\right)}{bn^2x^3} \\
&= -\frac{3e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\text{Ei}\left(-\frac{3(a+b\log(cx^n))}{bn}\right)}{b^2n^2x^3} - \frac{1}{bnx^3(a+b\log(cx^n))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4 (a+b\log(cx^n))^2}\,dx = -\frac{bn+3e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))}{b^2n^2x^3(a+b\log(cx^n))}$$

```
[In] Integrate[1/(x^4*(a + b*Log[c*x^n])^2),x]
```

```
[Out] -((b*n + 3*E^((3*a)/(b*n))*(c*x^n)^(3/n)*ExpIntegralEi[(-3*(a + b*Log[c*x^n
]))/(b*n)]*(a + b*Log[c*x^n]))/(b^2*n^2*x^3*(a + b*Log[c*x^n])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.66 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.66

method	result
risch	$-\frac{2}{x^{3}\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^{n})^{2}+ib\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}-ib\pi\,\mathrm{csgn}(ic\,x^{n})^{3}+2b\ln(c)+2\ln(x\right.}}$

```
[In] int(1/x^4/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/x^3/(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I
*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(
c)+2*ln(x^n)*b+2*a)/b/n+3/b^2/n^2/x^3*(x^n)^(3/n)*c^(3/n)*exp(3/2*(-I*b*Pi*
csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi
*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,3*ln(x)+
3/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*
x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+
2*b*(ln(x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^4\left(a+b\log\left(cx^n\right)\right)^2}\,dx = -\frac{3\left(bnx^3\log\left(x\right)+bx^3\log\left(c\right)+ax^3\right)e^{\left(\frac{3\,(b\log(c)+a)}{bn}\right)}\log\_integral\left(\frac{e^{\left(-\frac{3\,(b\log(c)+a)}{bn}\right)}}{x^3}\right)+bn}{b^3n^3x^3\log\left(x\right)+b^3n^2x^3\log\left(c\right)+ab^2n^2x^3}$$

```
[In] integrate(1/x^4/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] -(3*(b*n*x^3*log(x) + b*x^3*log(c) + a*x^3)*e^(3*(b*log(c) + a)/(b*n))*log_
integral(e^(-3*(b*log(c) + a)/(b*n))/x^3) + b*n)/(b^3*n^3*x^3*log(x) + b^3*
n^2*x^3*log(c) + a*b^2*n^2*x^3)
```

Sympy [F]

$$\int \frac{1}{x^4 (a + b \log (cx^n))^2}\,dx = \int \frac{1}{x^4 (a + b \log (cx^n))^2}\,dx$$

```
[In] integrate(1/x**4/(a+b*ln(c*x**n))**2,x)
[Out] Integral(1/(x**4*(a + b*log(c*x**n))**2), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + b \log (cx^n))^2}\,dx = \int \frac{1}{(b \log (cx^n) + a)^2 x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -1/(b^2*n*x^3*log(x^n) + (b^2*n*log(c) + a*b*n)*x^3) - 3*integrate(1/(b^2*n
*x^4*log(x^n) + (b^2*n*log(c) + a*b*n)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a + b \log (cx^n))^2}\,dx = \int \frac{1}{(b \log (cx^n) + a)^2 x^4}\,dx$$

```
[In] integrate(1/x^4/(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(1/((b*log(c*x^n) + a)^2*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + b \log (cx^n))^2}\,dx = \int \frac{1}{x^4 (a + b \ln (c\,x^n))^2}\,dx$$

```
[In] int(1/(x^4*(a + b*log(c*x^n))^2),x)
[Out] int(1/(x^4*(a + b*log(c*x^n))^2), x)
```

3.81 $\int \frac{x^3}{(a+b\log(cx^n))^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{x^3}{(a+b\log(cx^n))^3}\,dx = \frac{8e^{-\frac{4a}{bn}}x^4(cx^n)^{-4/n}\,\text{ExpIntegralEi}\left(\frac{4(a+b\log(cx^n))}{bn}\right)}{b^3n^3} - \frac{x^4}{2bn(a+b\log(cx^n))^2} - \frac{2x^4}{b^2n^2(a+b\log(cx^n))}$$

```
[Out] 8*x^4*Ei(4*(a+b*ln(c*x^n))/b/n)/b^3/exp(4*a/b/n)/n^3/((c*x^n)^(4/n))-1/2*x^
4/b/n/(a+b*ln(c*x^n))^2-2*x^4/b^2/n^2/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2343, 2347, 2209}

$$\int \frac{x^3}{(a+b\log(cx^n))^3}\,dx = \frac{8x^4e^{-\frac{4a}{bn}}(cx^n)^{-4/n}\,\text{ExpIntegralEi}\left(\frac{4(a+b\log(cx^n))}{bn}\right)}{b^3n^3} - \frac{2x^4}{b^2n^2(a+b\log(cx^n))} - \frac{x^4}{2bn(a+b\log(cx^n))^2}$$

```
[In] Int[x^3/(a + b*Log[c*x^n])^3,x]
```

```
[Out] (8*x^4*ExpIntegralEi[(4*(a + b*Log[c*x^n]))/(b*n)])/(b^3*E^((4*a)/(b*n))*n^
3*(c*x^n)^(4/n)) - x^4/(2*b*n*(a + b*Log[c*x^n])^2) - (2*x^4)/(b^2*n^2*(a +
 b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^4}{2bn\left(a+b\log\left(cx^n\right)\right)^2}+\frac{2\int\frac{x^3}{\left(a+b\log(cx^n)\right)^2}\,dx}{bn}\\
&= -\frac{x^4}{2bn\left(a+b\log\left(cx^n\right)\right)^2}-\frac{2x^4}{b^2n^2\left(a+b\log\left(cx^n\right)\right)}+\frac{8\int\frac{x^3}{a+b\log(cx^n)}\,dx}{b^2n^2}\\
&= -\frac{x^4}{2bn\left(a+b\log\left(cx^n\right)\right)^2}-\frac{2x^4}{b^2n^2\left(a+b\log\left(cx^n\right)\right)}\\
&\quad+\frac{\left(8x^4\left(cx^n\right)^{-4/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{4x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{b^3n^3}\\
&= \frac{8e^{-\frac{4a}{bn}}x^4\left(cx^n\right)^{-4/n}\operatorname{Ei}\left(\frac{4(a+b\log(cx^n))}{bn}\right)}{b^3n^3}-\frac{x^4}{2bn\left(a+b\log\left(cx^n\right)\right)^2}-\frac{2x^4}{b^2n^2\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int\frac{x^3}{\left(a+b\log\left(cx^n\right)\right)^3}\,dx = \frac{x^4\left(16e^{-\frac{4a}{bn}}\left(cx^n\right)^{-4/n}\text{ExpIntegralEi}\left(\frac{4(a+b\log(cx^n))}{bn}\right)-\frac{bn(4a+bn+4b\log(cx^n))}{(a+b\log(cx^n))^2}\right)}{2b^3n^3}$$

```
[In] Integrate[x^3/(a + b*Log[c*x^n])^3,x]
```

```
[Out] (x^4*((16*ExpIntegralEi[(4*(a + b*Log[c*x^n]))/(b*n)])/(E^((4*a)/(b*n))*(c*
x^n)^(4/n)) - (b*n*(4*a + b*n + 4*b*Log[c*x^n]))/(a + b*Log[c*x^n])^2))/(2*
b^3*n^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 473, normalized size of antiderivative = 4.68

method	result
risch	$-\frac{2i\left(ix^4bn+2\pi b\,x^4\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)-2\pi b\,x^4\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2-2\pi b\,x^4\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2+2\pi b\,x^4\,\mathrm{csgn}(ic\,x^n)^3\right.}{\left(b\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)-b\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2-b\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2+b\pi\,\mathrm{csgn}(ic\,x^n)^3+2ib\ln(c)\right.}$

```
[In] int(x^3/(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(I*x^4*b*n+2*Pi*b*x^4*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-2*Pi*b*x^4*c
sgn(I*c)*csgn(I*c*x^n)^2-2*Pi*b*x^4*csgn(I*x^n)*csgn(I*c*x^n)^2+2*Pi*b*x^4*
csgn(I*c*x^n)^3+4*I*ln(c)*b*x^4+4*I*b*x^4*ln(x^n)+4*I*a*x^4)/(b*Pi*csgn(I*c
)*csgn(I*x^n)*csgn(I*c*x^n)-b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-b*Pi*csgn(I*x^n)
*csgn(I*c*x^n)^2+b*Pi*csgn(I*c*x^n)^3+2*I*b*ln(c)+2*I*b*ln(x^n)+2*I*a)^2/b^
2/n^2-8/b^3/n^3*x^4*c^(-4/n)*(x^n)^(-4/n)*exp(-2*(-I*b*Pi*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn
(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-4*ln(x)+2*I*(b*Pi*csgn(I
*c)*csgn(I*x^n)*csgn(I*c*x^n)-b*Pi*csgn(I*c)*csgn(I*c*x^n)^2-b*Pi*csgn(I*x^
n)*csgn(I*c*x^n)^2+b*Pi*csgn(I*c*x^n)^3+2*I*b*ln(c)+2*I*b*(ln(x^n)-n*ln(x))
+2*I*a)/b/n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(100) = 200.

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.09

$$\int \frac{x^3}{(a+b\log(cx^n))^3}\,dx = -\frac{\left(\left(4\,b^2n^2x^4\log(x)+4\,b^2nx^4\log(c)+(b^2n^2+4\,abn)x^4\right)e^{\left(\frac{4\,(b\log(c)+a)}{bn}\right)}-16\,\left(b^2n^2\log(x)^2+b^2\log(c)^2+\right.\right.}{2\,\left(b^5n^5\log(x)^2+b^5n^3\log(c)^2+2\,ab^4n^3\log(c)+\right.}$$

```
[In] integrate(x^3/(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] -1/2*((4*b^2*n^2*x^4*log(x) + 4*b^2*n*x^4*log(c) + (b^2*n^2 + 4*a*b*n)*x^4)
*e^(4*(b*log(c) + a)/(b*n)) - 16*(b^2*n^2*log(x)^2 + b^2*log(c)^2 + 2*a*b*l
```

```
og(c) + a^2 + 2*(b^2*n*log(c) + a*b*n)*log(x))*log_integral(x^4*e^(4*(b*log
(c) + a)/(b*n))))*e^(-4*(b*log(c) + a)/(b*n))/(b^5*n^5*log(x)^2 + b^5*n^3*l
og(c)^2 + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3 + 2*(b^5*n^4*log(c) + a*b^4*n^4)
*log(x))
```

Sympy [F]

$$\int \frac{x^3}{(a+b\log(cx^n))^3}\,dx = \int \frac{x^3}{(a+b\log(cx^n))^3}\,dx$$

```
[In] integrate(x**3/(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(x**3/(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int \frac{x^3}{(a+b\log(cx^n))^3}\,dx = \int \frac{x^3}{(b\log(cx^n)+a)^3}\,dx$$

```
[In] integrate(x^3/(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(4*b*x^4*log(x^n) + (b*(n + 4*log(c)) + 4*a)*x^4)/(b^4*n^2*log(c)^2 +
b^4*n^2*log(x^n)^2 + 2*a*b^3*n^2*log(c) + a^2*b^2*n^2 + 2*(b^4*n^2*log(c) +
 a*b^3*n^2)*log(x^n)) + 8*integrate(x^3/(b^3*n^2*log(c) + b^3*n^2*log(x^n)
+ a*b^2*n^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(100) = 200$.

Time = 0.36 (sec) , antiderivative size = 1029, normalized size of antiderivative = 10.19

$$\int \frac{x^3}{(a+b\log(cx^n))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] -2*b^2*n^2*x^4*log(x)/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3
*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) - 1/2*b^
2*n^2*x^4/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 +
2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) - 2*b^2*n*x^4*log(c)
/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n
^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) - 2*a*b*n*x^4/(b^5*n^5*log(x)
```

```
^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*
b^4*n^3*log(c) + a^2*b^3*n^3) + 8*b^2*n^2*Ei(4*log(c)/n + 4*a/(b*n) + 4*log
(x))*e^(-4*a/(b*n))*log(x)^2/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) +
 b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*
c^(4/n)) + 16*b^2*n*Ei(4*log(c)/n + 4*a/(b*n) + 4*log(x))*e^(-4*a/(b*n))*lo
g(c)*log(x)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2
 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(4/n)) + 8*b^2*
Ei(4*log(c)/n + 4*a/(b*n) + 4*log(x))*e^(-4*a/(b*n))*log(c)^2/((b^5*n^5*log
(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2
*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(4/n)) + 16*a*b*n*Ei(4*log(c)/n + 4*a/(b
*n) + 4*log(x))*e^(-4*a/(b*n))*log(x)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)
*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*
b^3*n^3)*c^(4/n)) + 16*a*b*Ei(4*log(c)/n + 4*a/(b*n) + 4*log(x))*e^(-4*a/(b
*n))*log(c)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2
 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(4/n)) + 8*a^2*
Ei(4*log(c)/n + 4*a/(b*n) + 4*log(x))*e^(-4*a/(b*n))/((b^5*n^5*log(x)^2 + 2
*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^
3*log(c) + a^2*b^3*n^3)*c^(4/n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+b\log(cx^n))^3}\,dx=\int \frac{x^3}{(a+b\,\ln(c\,x^n))^3}\,dx$$

```
[In] int(x^3/(a + b*log(c*x^n))^3,x)
[Out] int(x^3/(a + b*log(c*x^n))^3, x)
```

3.82 $\int \frac{x^2}{(a+b\log(cx^n))^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{x^2}{(a+b\log(cx^n))^3}\,dx = \frac{9e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{bn}\right)}{2b^3n^3} - \frac{x^3}{2bn(a+b\log(cx^n))^2} - \frac{3x^3}{2b^2n^2(a+b\log(cx^n))}$$

```
[Out] 9/2*x^3*Ei(3*(a+b*ln(c*x^n))/b/n)/b^3/exp(3*a/b/n)/n^3/((c*x^n)^(3/n))-1/2*
x^3/b/n/(a+b*ln(c*x^n))^2-3/2*x^3/b^2/n^2/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2343, 2347, 2209}

$$\int \frac{x^2}{(a+b\log(cx^n))^3}\,dx = \frac{9x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{bn}\right)}{2b^3n^3} - \frac{3x^3}{2b^2n^2(a+b\log(cx^n))} - \frac{x^3}{2bn(a+b\log(cx^n))^2}$$

```
[In] Int[x^2/(a + b*Log[c*x^n])^3,x]
```

```
[Out] (9*x^3*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(b*n)])/(2*b^3*E^((3*a)/(b*n))*
n^3*(c*x^n)^(3/n)) - x^3/(2*b*n*(a + b*Log[c*x^n])^2) - (3*x^3)/(2*b^2*n^2*
(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3}{2bn\left(a+b\log\left(cx^n\right)\right)^2}+\frac{3\int\frac{x^2}{(a+b\log(cx^n))^2}\,dx}{2bn}\\
&= -\frac{x^3}{2bn\left(a+b\log\left(cx^n\right)\right)^2}-\frac{3x^3}{2b^2n^2\left(a+b\log\left(cx^n\right)\right)}+\frac{9\int\frac{x^2}{a+b\log(cx^n)}\,dx}{2b^2n^2}\\
&= -\frac{x^3}{2bn\left(a+b\log\left(cx^n\right)\right)^2}-\frac{3x^3}{2b^2n^2\left(a+b\log\left(cx^n\right)\right)}\\
&\quad+\frac{\left(9x^3(cx^n)^{-3/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{3x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{2b^2n^3}\\
&= \frac{9e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\operatorname{Ei}\left(\frac{3(a+b\log(cx^n))}{bn}\right)}{2b^3n^3}-\frac{x^3}{2bn\left(a+b\log\left(cx^n\right)\right)^2}-\frac{3x^3}{2b^2n^2\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int\frac{x^2}{\left(a+b\log\left(cx^n\right)\right)^3}\,dx = \frac{x^3\left(9e^{-\frac{3a}{bn}}(cx^n)^{-3/n}\operatorname{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{bn}\right)-\frac{bn(3a+bn+3b\log(cx^n))}{(a+b\log(cx^n))^2}\right)}{2b^3n^3}$$

```
[In] Integrate[x^2/(a + b*Log[c*x^n])^3,x]
```

```
[Out] (x^3*((9*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(b*n)])/(E^((3*a)/(b*n))*(c*x
^n)^(3/n)) - (b*n*(3*a + b*n + 3*b*Log[c*x^n]))/(a + b*Log[c*x^n])^2))/(2*b
^3*n^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.54

method	result
risch	$-\frac{2bn\,x^3-3i\pi b\,x^3\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+3i\pi b\,x^3\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+3i\pi b\,x^3\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-3i\pi b\,x^3\,\mathrm{csgn}(ic\,x^n)^3-}{\left(-ib\pi\;\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\;\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\;\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2b\ln(c)+\right.}$

```
[In] int(x^2/(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(2*b*n*x^3-3*I*Pi*b*x^3*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+3*I*Pi*b*x^3*c
sgn(I*c)*csgn(I*c*x^n)^2+3*I*Pi*b*x^3*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*Pi*b*
x^3*csgn(I*c*x^n)^3+6*ln(c)*b*x^3+6*b*x^3*ln(x^n)+6*x^3*a)/(-I*b*Pi*csgn(I*
c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I
*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*ln(x^n)*b+2*a)^2/b
^2/n^2-9/2/b^3/n^3*x^3*c^(-3/n)*(x^n)^(-3/n)*exp(-3/2*(-I*b*Pi*csgn(I*c)*cs
gn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)
*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-3*ln(x)-3/2*(-I*b*P
i*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*
Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n
)-n*ln(x))+2*a)/b/n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(100) = 200.

Time = 0.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.01

$$\int \frac{x^2}{\left(a+b\log\left(cx^n\right)\right)^3}\,dx = -\frac{\left(\left(3\,b^2n^2x^3\log\left(x\right)+3\,b^2nx^3\log\left(c\right)+\left(b^2n^2+3\,abn\right)x^3\right)e^{\left(\frac{3\,\left(b\log(c)+a\right)}{bn}\right)}-9\,\left(b^2n^2\log\left(x\right)^2+b^2\log\left(c\right)^2+2\,a\right.\right.}{2\,\left(b^5n^5\log\left(x\right)^2+b^5n^3\log\left(c\right)^2+2\,ab^4n^3\log\left(c\right)+a^2\right.}$$

```
[In] integrate(x^2/(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] -1/2*((3*b^2*n^2*x^3*log(x) + 3*b^2*n*x^3*log(c) + (b^2*n^2 + 3*a*b*n)*x^3)
*e^(3*(b*log(c) + a)/(b*n)) - 9*(b^2*n^2*log(x)^2 + b^2*log(c)^2 + 2*a*b*lo
```

```
g(c) + a^2 + 2*(b^2*n*log(c) + a*b*n)*log(x))*log_integral(x^3*e^(3*(b*log(
c) + a)/(b*n))))*e^(-3*(b*log(c) + a)/(b*n))/(b^5*n^5*log(x)^2 + b^5*n^3*lo
g(c)^2 + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3 + 2*(b^5*n^4*log(c) + a*b^4*n^4)*
log(x))
```

Sympy [F]

$$\int \frac{x^2}{(a+b\log(cx^n))^3}\,dx = \int \frac{x^2}{(a+b\log(cx^n))^3}\,dx$$

```
[In] integrate(x**2/(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(x**2/(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\log(cx^n))^3}\,dx = \int \frac{x^2}{(b\log(cx^n)+a)^3}\,dx$$

```
[In] integrate(x^2/(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^3*log(x^n) + (b*(n + 3*log(c)) + 3*a)*x^3)/(b^4*n^2*log(c)^2 +
b^4*n^2*log(x^n)^2 + 2*a*b^3*n^2*log(c) + a^2*b^2*n^2 + 2*(b^4*n^2*log(c) +
 a*b^3*n^2)*log(x^n)) + 9*integrate(1/2*x^2/(b^3*n^2*log(c) + b^3*n^2*log(x
^n) + a*b^2*n^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. 2(100) = 200.

Time = 0.36 (sec) , antiderivative size = 1029, normalized size of antiderivative = 9.80

$$\int \frac{x^2}{(a+b\log(cx^n))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] -3/2*b^2*n^2*x^3*log(x)/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n
^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) - 1/2*
b^2*n^2*x^3/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2
+ 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) - 3/2*b^2*n*x^3*lo
g(c)/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b
^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) + 9/2*b^2*n^2*Ei(3*log(c)
```

```
/n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))*log(x)^2/((b^5*n^5*log(x)^2 + 2*b
^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*
log(c) + a^2*b^3*n^3)*c^(3/n)) - 3/2*a*b*n*x^3/(b^5*n^5*log(x)^2 + 2*b^5*n^
4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c
) + a^2*b^3*n^3) + 9*b^2*n*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b
*n))*log(c)*log(x)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*l
og(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(3/n)) +
 9/2*b^2*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))*log(c)^2/((b^
5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*l
og(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(3/n)) + 9*a*b*n*Ei(3*log(c)/n
+ 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))*log(x)/((b^5*n^5*log(x)^2 + 2*b^5*n^
4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c
) + a^2*b^3*n^3)*c^(3/n)) + 9*a*b*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^(
-3*a/(b*n))*log(c)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*l
og(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(3/n)) +
 9/2*a^2*Ei(3*log(c)/n + 3*a/(b*n) + 3*log(x))*e^(-3*a/(b*n))/((b^5*n^5*log
(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2
*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(3/n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\log(cx^n))^3}\,dx=\int \frac{x^2}{(a+b\,\ln(c\,x^n))^3}\,dx$$

```
[In] int(x^2/(a + b*log(c*x^n))^3,x)
[Out] int(x^2/(a + b*log(c*x^n))^3, x)
```

3.83 $\int \frac{x}{(a+b\log(cx^n))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 101

$$\int \frac{x}{(a+b\log(cx^n))^3}\,dx = \frac{2e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(cx^n))}{bn}\right)}{b^3n^3} - \frac{x^2}{2bn(a+b\log(cx^n))^2} - \frac{x^2}{b^2n^2(a+b\log(cx^n))}$$

```
[Out] 2*x^2*Ei(2*(a+b*ln(c*x^n))/b/n)/b^3/exp(2*a/b/n)/n^3/((c*x^n)^(2/n))-1/2*x^
2/b/n/(a+b*ln(c*x^n))^2-x^2/b^2/n^2/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2343, 2347, 2209}

$$\int \frac{x}{(a+b\log(cx^n))^3}\,dx = \frac{2x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\operatorname{ExpIntegralEi}\left(\frac{2(a+b\log(cx^n))}{bn}\right)}{b^3n^3} - \frac{x^2}{b^2n^2(a+b\log(cx^n))} - \frac{x^2}{2bn(a+b\log(cx^n))^2}$$

```
[In] Int[x/(a + b*Log[c*x^n])^3,x]
```

```
[Out] (2*x^2*ExpIntegralEi[(2*(a + b*Log[c*x^n]))/(b*n)])/(b^3*E^((2*a)/(b*n))*n^
3*(c*x^n)^(2/n)) - x^2/(2*b*n*(a + b*Log[c*x^n])^2) - x^2/(b^2*n^2*(a + b*L
og[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^2}{2bn\,(a+b\log(cx^n))^2}+\frac{\int \frac{x}{(a+b\log(cx^n))^2}\,dx}{bn}\\
&= -\frac{x^2}{2bn\,(a+b\log(cx^n))^2}-\frac{x^2}{b^2n^2\,(a+b\log(cx^n))}+\frac{2\int \frac{x}{a+b\log(cx^n)}\,dx}{b^2n^2}\\
&= -\frac{x^2}{2bn\,(a+b\log(cx^n))^2}-\frac{x^2}{b^2n^2\,(a+b\log(cx^n))}\\
&\quad+\frac{\left(2x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{a+bx}\,dx, x, \log(cx^n)\right)}{b^2n^3}\\
&= \frac{2e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\,\text{Ei}\left(\frac{2(a+b\log(cx^n))}{bn}\right)}{b^3n^3}-\frac{x^2}{2bn\,(a+b\log(cx^n))^2}-\frac{x^2}{b^2n^2\,(a+b\log(cx^n))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$
\begin{aligned}
&\int \frac{x}{(a+b\log(cx^n))^3}\,dx\\
&= \frac{x^2\left(4e^{-\frac{2a}{bn}}(cx^n)^{-2/n}\,\text{ExpIntegralEi}\left(\frac{2(a+b\log(cx^n))}{bn}\right)-\frac{bn(2a+bn+2b\log(cx^n))}{(a+b\log(cx^n))^2}\right)}{2b^3n^3}
\end{aligned}
$$

```
[In] Integrate[x/(a + b*Log[c*x^n])^3,x]
```

```
[Out] (x^2*((4*ExpIntegralEi[(2*(a + b*Log[c*x^n]))/(b*n)])/(E^((2*a)/(b*n))*(c*x
^n)^(2/n)) - (b*n*(2*a + b*n + 2*b*Log[c*x^n]))/(a + b*Log[c*x^n])^2))/(2*b
^3*n^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 476, normalized size of antiderivative = 4.71

method	result
risch	$-\frac{2\left(bn\,x^2-i\pi b\,x^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+i\pi b\,x^2\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+i\pi b\,x^2\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-i\pi b\,x^2\,\mathrm{csgn}(ic\,x^n)^3+}{\left(-ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2b\ln(c)\right.}$

```
[In] int(x/(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*n*x^2-I*Pi*b*x^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*Pi*b*x^2*csgn(
I*c)*csgn(I*c*x^n)^2+I*Pi*b*x^2*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*x^2*csgn
(I*c*x^n)^3+2*ln(c)*b*x^2+2*b*x^2*ln(x^n)+2*x^2*a)/(-I*b*Pi*csgn(I*c)*csgn(
I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*cs
gn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*ln(x^n)*b+2*a)^2/b^2/n^2-2
/b^3/n^3*x^2*c^(-2/n)*(x^n)^(-2/n)*exp(-(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn
(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)
^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-2*ln(x)-(-I*b*Pi*csgn(I*c)*csgn(I
*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csg
n(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/b/
n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(100) = 200$.

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.09

$$\int \frac{x}{(a+b\log{(cx^n)})^3}\,dx = -\frac{\left((2\,b^2n^2x^2\log{(x)}+2\,b^2nx^2\log{(c)}+(b^2n^2+2\,abn)x^2)e^{\left(\frac{2\,(b\log(c)+a)}{bn}\right)}-4\left(b^2n^2\log{(x)}^2+b^2\log{(c)}^2+2\right.}{2\left(b^5n^5\log{(x)}^2+b^5n^3\log{(c)}^2+2\,ab^4n^3\log{(c)}+\right.}$$

```
[In] integrate(x/(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] -1/2*((2*b^2*n^2*x^2*log(x) + 2*b^2*n*x^2*log(c) + (b^2*n^2 + 2*a*b*n)*x^2)
*e^(2*(b*log(c) + a)/(b*n)) - 4*(b^2*n^2*log(x)^2 + b^2*log(c)^2 + 2*a*b*lo
g(c) + a^2 + 2*(b^2*n*log(c) + a*b*n)*log(x))*log_integral(x^2*e^(2*(b*log(
```

```
c) + a)/(b*n))))*e^(-2*(b*log(c) + a)/(b*n))/(b^5*n^5*log(x)^2 + b^5*n^3*lo
g(c)^2 + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3 + 2*(b^5*n^4*log(c) + a*b^4*n^4)*
log(x))
```

Sympy [F]

$$\int \frac{x}{(a+b\log(cx^n))^3}\,dx=\int \frac{x}{(a+b\log(cx^n))^3}\,dx$$

```
[In] integrate(x/(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(x/(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int \frac{x}{(a+b\log(cx^n))^3}\,dx=\int \frac{x}{(b\log(cx^n)+a)^3}\,dx$$

```
[In] integrate(x/(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2*log(x^n) + (b*(n + 2*log(c)) + 2*a)*x^2)/(b^4*n^2*log(c)^2 +
b^4*n^2*log(x^n)^2 + 2*a*b^3*n^2*log(c) + a^2*b^2*n^2 + 2*(b^4*n^2*log(c) +
 a*b^3*n^2)*log(x^n)) + 2*integrate(x/(b^3*n^2*log(c) + b^3*n^2*log(x^n) +
a*b^2*n^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1029 vs. $2(100) = 200$.

Time = 0.46 (sec) , antiderivative size = 1029, normalized size of antiderivative = 10.19

$$\int \frac{x}{(a+b\log(cx^n))^3}\,dx=\text{Too large to display}$$

```
[In] integrate(x/(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] -b^2*n^2*x^2*log(x)/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*l
og(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) + 2*b^2*n^
2*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(x))*e^(-2*a/(b*n))*log(x)^2/((b^5*n^5*l
og(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) +
 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(2/n)) - 1/2*b^2*n^2*x^2/(b^5*n^5*log(
x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*
a*b^4*n^3*log(c) + a^2*b^3*n^3) - b^2*n*x^2*log(c)/(b^5*n^5*log(x)^2 + 2*b^
```

```
5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*l
og(c) + a^2*b^3*n^3) + 4*b^2*n*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(x))*e^(-2*
a/(b*n))*log(c)*log(x)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n
^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(2/n
)) - a*b*n*x^2/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)
^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) + 2*b^2*Ei(2*lo
g(c)/n + 2*a/(b*n) + 2*log(x))*e^(-2*a/(b*n))*log(c)^2/((b^5*n^5*log(x)^2 +
 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*
n^3*log(c) + a^2*b^3*n^3)*c^(2/n)) + 4*a*b*n*Ei(2*log(c)/n + 2*a/(b*n) + 2*
log(x))*e^(-2*a/(b*n))*log(x)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x)
+ b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)
*c^(2/n)) + 4*a*b*Ei(2*log(c)/n + 2*a/(b*n) + 2*log(x))*e^(-2*a/(b*n))*log(
c)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^
4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(2/n)) + 2*a^2*Ei(2*log(
c)/n + 2*a/(b*n) + 2*log(x))*e^(-2*a/(b*n))/((b^5*n^5*log(x)^2 + 2*b^5*n^4*
log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c)
+ a^2*b^3*n^3)*c^(2/n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\log(cx^n))^3}\,dx = \int \frac{x}{(a+b\,\ln(c\,x^n))^3}\,dx$$

```
[In] int(x/(a + b*log(c*x^n))^3,x)
[Out] int(x/(a + b*log(c*x^n))^3, x)
```

3.84 $\int \frac{1}{(a+b\log(cx^n))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 98

$$\int \frac{1}{(a+b\log(cx^n))^3}\,dx = \frac{e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{bn}\right)}{2b^3n^3} - \frac{x}{2bn(a+b\log(cx^n))^2} - \frac{x}{2b^2n^2(a+b\log(cx^n))}$$

```
[Out] 1/2*x*Ei((a+b*ln(c*x^n))/b/n)/b^3/exp(a/b/n)/n^3/((c*x^n)^(1/n))-1/2*x/b/n/
(a+b*ln(c*x^n))^2-1/2*x/b^2/n^2/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2334, 2337, 2209}

$$\int \frac{1}{(a+b\log(cx^n))^3}\,dx = \frac{xe^{-\frac{a}{bn}}(cx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{bn}\right)}{2b^3n^3} - \frac{x}{2b^2n^2(a+b\log(cx^n))} - \frac{x}{2bn(a+b\log(cx^n))^2}$$

```
[In] Int[(a + b*Log[c*x^n])^(-3),x]
```

```
[Out] (x*ExpIntegralEi[(a + b*Log[c*x^n])/(b*n)])/(2*b^3*E^(a/(b*n))*n^3*(c*x^n)^
n^(-1)) - x/(2*b*n*(a + b*Log[c*x^n])^2) - x/(2*b^2*n^2*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
```

```
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x}{2bn\,(a+b\log{(cx^n)})^2}+\frac{\int \frac{1}{(a+b\log(cx^n))^2}\,dx}{2bn}\\
&= -\frac{x}{2bn\,(a+b\log{(cx^n)})^2}-\frac{x}{2b^2n^2\,(a+b\log{(cx^n)})}+\frac{\int \frac{1}{a+b\log(cx^n)}\,dx}{2b^2n^2}\\
&= -\frac{x}{2bn\,(a+b\log{(cx^n)})^2}-\frac{x}{2b^2n^2\,(a+b\log{(cx^n)})}\\
&\quad+\frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{a+bx}\,dx,x,\log{(cx^n)}\right)}{2b^2n^3}\\
&= \frac{e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\,\text{Ei}\left(\frac{a+b\log(cx^n)}{bn}\right)}{2b^3n^3}-\frac{x}{2bn\,(a+b\log{(cx^n)})^2}-\frac{x}{2b^2n^2\,(a+b\log{(cx^n)})}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\log{(cx^n)})^3}\,dx=\frac{x\left(e^{-\frac{a}{bn}}(cx^n)^{-1/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{bn}\right)-\frac{bn(a+bn+b\log(cx^n))}{(a+b\log(cx^n))^2}\right)}{2b^3n^3}$$

```
[In] Integrate[(a + b*Log[c*x^n])^(-3),x]
[Out] (x*(ExpIntegralEi[(a + b*Log[c*x^n])/(b*n)]/(E^(a/(b*n))*(c*x^n)^n^(-1)) -
(b*n*(a + b*n + b*Log[c*x^n]))/(a + b*Log[c*x^n])^2))/(2*b^3*n^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 459, normalized size of antiderivative = 4.68

method	result
risch	$-\frac{-i\pi bx\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+i\pi bx\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+i\pi bx\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-i\pi bx\,\mathrm{csgn}(ic\,x^n)^3+2\ln(c)bx+2bx\,\mathrm{l}}{b^2n^2\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2b\ln(c)+2\right.}$

```
[In] int(1/(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-I*Pi*b*x*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*Pi*b*x*csgn(I*c)*csgn(I*c
*x^n)^2+I*Pi*b*x*csgn(I*x^n)*csgn(I*c*x^n)^2-I*Pi*b*x*csgn(I*c*x^n)^3+2*ln(
c)*b*x+2*b*x*ln(x^n)+2*a*x+2*b*n*x)/b^2/n^2/(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*
csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*
x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*ln(x^n)*b+2*a)^2-1/2/b^3/n^3*x*c^
(-1/n)*(x^n)^(-1/n)*exp(-1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I
*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*c
sgn(I*c*x^n)^3+2*a)/b/n)*Ei(1,-ln(x)-1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csg
n(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n
)^2-I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(91) = 182.

Time = 0.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.02

$$\int \frac{1}{\left(a+b\log\left(cx^n\right)\right)^3}\,dx = -\frac{\left(\left(b^2n^2x\log\left(x\right)+b^2nx\log\left(c\right)+\left(b^2n^2+abn\right)x\right)e^{\left(\frac{b\log(c)+a}{bn}\right)}-\left(b^2n^2\log\left(x\right)^2+b^2\log\left(c\right)^2+2\,ab\log\left(c\right)+a\right.\right.}{2\left(b^5n^5\log\left(x\right)^2+b^5n^3\log\left(c\right)^2+2\,ab^4n^3\log\left(c\right)+a^2b^3n^3+2\right.}$$

```
[In] integrate(1/(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] -1/2*((b^2*n^2*x*log(x) + b^2*n*x*log(c) + (b^2*n^2 + a*b*n)*x)*e^((b*log(c
) + a)/(b*n)) - (b^2*n^2*log(x)^2 + b^2*log(c)^2 + 2*a*b*log(c) + a^2 + 2*(
b^2*n*log(c) + a*b*n)*log(x))*log_integral(x*e^((b*log(c) + a)/(b*n))))*e^(
-(b*log(c) + a)/(b*n))/(b^5*n^5*log(x)^2 + b^5*n^3*log(c)^2 + 2*a*b^4*n^3*l
og(c) + a^2*b^3*n^3 + 2*(b^5*n^4*log(c) + a*b^4*n^4)*log(x))
```

Sympy [F]

$$\int \frac{1}{(a+b\log(cx^n))^3}\,dx = \int \frac{1}{(a+b\log(cx^n))^3}\,dx$$

```
[In] integrate(1/(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral((a + b*log(c*x**n))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\log(cx^n))^3}\,dx = \int \frac{1}{(b\log(cx^n)+a)^3}\,dx$$

```
[In] integrate(1/(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*x*log(x^n) + (b*(n + log(c)) + a)*x)/(b^4*n^2*log(c)^2 + b^4*n^2*lo
g(x^n)^2 + 2*a*b^3*n^2*log(c) + a^2*b^2*n^2 + 2*(b^4*n^2*log(c) + a*b^3*n^2
)*log(x^n)) + integrate(1/2/(b^3*n^2*log(c) + b^3*n^2*log(x^n) + a*b^2*n^2)
, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 982 vs. $2(91) = 182$.

Time = 0.42 (sec) , antiderivative size = 982, normalized size of antiderivative = 10.02

$$\int \frac{1}{(a+b\log(cx^n))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*n^2*Ei(log(c)/n + a/(b*n) + log(x))*e^(-a/(b*n))*log(x)^2/((b^5*n^5
*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x)
 + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(1/n)) - 1/2*b^2*n^2*x*log(x)/(b^5*n
^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(
x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) + b^2*n*Ei(log(c)/n + a/(b*n) + log(
x))*e^(-a/(b*n))*log(c)*log(x)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x)
 + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3
)*c^(1/n)) - 1/2*b^2*n^2*x/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^
5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) - 1
/2*b^2*n*x*log(c)/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log
(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) + 1/2*b^2*Ei
```

```
(log(c)/n + a/(b*n) + log(x))*e^(-a/(b*n))*log(c)^2/((b^5*n^5*log(x)^2 + 2*
b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3
*log(c) + a^2*b^3*n^3)*c^(1/n)) + a*b*n*Ei(log(c)/n + a/(b*n) + log(x))*e^(
-a/(b*n))*log(x)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log
(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(1/n)) - 1
/2*a*b*n*x/(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 +
 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3) + a*b*Ei(log(c)/n +
 a/(b*n) + log(x))*e^(-a/(b*n))*log(c)/((b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c
)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^4*log(x) + 2*a*b^4*n^3*log(c) + a^2
*b^3*n^3)*c^(1/n)) + 1/2*a^2*Ei(log(c)/n + a/(b*n) + log(x))*e^(-a/(b*n))/(
(b^5*n^5*log(x)^2 + 2*b^5*n^4*log(c)*log(x) + b^5*n^3*log(c)^2 + 2*a*b^4*n^
4*log(x) + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3)*c^(1/n))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\log(cx^n))^3}\,dx = \int \frac{1}{(a+b\,\ln(c\,x^n))^3}\,dx$$

```
[In] int(1/(a + b*log(c*x^n))^3,x)
[Out] int(1/(a + b*log(c*x^n))^3, x)
```

3.85 $\int \frac{1}{x(a+b\log(cx^n))^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 22

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2bn\left(a+b\log\left(cx^n\right)\right)^2}$$

```
[Out] -1/2/b/n/(a+b*ln(c*x^n))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2339, 30}

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)^3}\,dx=-\frac{1}{2bn\left(a+b\log\left(cx^n\right)\right)^2}$$

```
[In] Int[1/(x*(a + b*Log[c*x^n])^3),x]
[Out] -1/2*1/(b*n*(a + b*Log[c*x^n])^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{x^3}\,dx, x, a + b\log\left(cx^n\right)\right)}{bn}\\ &= -\frac{1}{2bn\left(a + b\log\left(cx^n\right)\right)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a + b\log\left(cx^n\right)\right)^3}\,dx = -\frac{1}{2bn\left(a + b\log\left(cx^n\right)\right)^2}$$

```
[In] Integrate[1/(x*(a + b*Log[c*x^n])^3),x]
[Out] -1/2*1/(b*n*(a + b*Log[c*x^n])^2)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result
derivativedivides	$-\frac{1}{2bn(a+b\ln(c\,x^n))^2}$
default	$-\frac{1}{2bn(a+b\ln(c\,x^n))^2}$
parallelrisch	$-\frac{1}{2bn(a+b\ln(c\,x^n))^2}$
risch	$-\frac{2}{bn\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2b\,\mathrm{l}\right.}$

```
[In] int(1/x/(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b/n/(a+b*ln(c*x^n))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(20) = 40$.

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)^3}\,dx = -\frac{1}{2\left(b^3n^3\log\left(x\right)^2+b^3n\log\left(c\right)^2+2\,ab^2n\log\left(c\right)+a^2bn+2\left(b^3n^2\log\left(c\right)+ab^2n^2\right)\log\left(x\right)\right)}$$

```
[In] integrate(1/x/(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*n^3*log(x)^2 + b^3*n*log(c)^2 + 2*a*b^2*n*log(c) + a^2*b*n + 2*(b
^3*n^2*log(c) + a*b^2*n^2)*log(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

Time = 1.87 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)^3}\,dx = \begin{cases} \frac{\log\left(x\right)}{a^3} & \text{for}\ b=0 \wedge \left(b=0 \vee n=0\right) \\ \frac{\log\left(x\right)}{\left(a+b\log\left(c\right)\right)^3} & \text{for}\ n=0 \\ -\frac{1}{2a^2bn+4ab^2n\log\left(cx^n\right)+2b^3n\log\left(cx^n\right)^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((log(x)/a**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)/(a + b*l
og(c))**3, Eq(n, 0)), (-1/(2*a**2*b*n + 4*a*b**2*n*log(c*x**n) + 2*b**3*n*l
og(c*x**n)**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)^3}\,dx = -\frac{1}{2\left(b\log\left(cx^n\right)+a\right)^2bn}$$

```
[In] integrate(1/x/(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -1/2/((b*log(c*x^n) + a)^2*b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)^3}\,dx = -\frac{1}{2\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2bn}$$

```
[In] integrate(1/x/(a+b*log(c*x^n))^3,x, algorithm="giac")
[Out] -1/2/((b*n*log(x) + b*log(c) + a)^2*b*n)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{x\left(a+b\log\left(cx^n\right)\right)^3}\,dx = -\frac{1}{2\,n\,a^2\,b+4\,n\,a\,b^2\,\ln\left(c\,x^n\right)+2\,n\,b^3\,\ln\left(c\,x^n\right)^2}$$

```
[In] int(1/(x*(a + b*log(c*x^n))^3),x)
[Out] -1/(2*b^3*n*log(c*x^n)^2 + 2*a^2*b*n + 4*a*b^2*n*log(c*x^n))
```

3.86 $\int \frac{1}{x^2(a+b\log(cx^n))^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 102

$$\int \frac{1}{x^2\left(a+b\log\left(cx^n\right)\right)^3}\,dx=\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{bn}\right)}{2b^3n^3x}-\frac{1}{2bnx\left(a+b\log\left(cx^n\right)\right)^2}+\frac{1}{2b^2n^2x\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 1/2*exp(a/b/n)*(c*x^n)^(1/n)*Ei((-a-b*ln(c*x^n))/b/n)/b^3/n^3/x-1/2/b/n/x/(
a+b*ln(c*x^n))^2+1/2/b^2/n^2/x/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2343, 2347, 2209}

$$\int \frac{1}{x^2\left(a+b\log\left(cx^n\right)\right)^3}\,dx=\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{bn}\right)}{2b^3n^3x}+\frac{1}{2b^2n^2x\left(a+b\log\left(cx^n\right)\right)}-\frac{1}{2bnx\left(a+b\log\left(cx^n\right)\right)^2}$$

```
[In] Int[1/(x^2*(a + b*Log[c*x^n])^3),x]
```

```
[Out] (E^(a/(b*n))*(c*x^n)^n^(-1)*ExpIntegralEi[-((a + b*Log[c*x^n])/(b*n))])/(2*
b^3*n^3*x) - 1/(2*b*n*x*(a + b*Log[c*x^n])^2) + 1/(2*b^2*n^2*x*(a + b*Log[c
*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2bnx\left(a+b\log\left(cx^n\right)\right)^2}-\frac{\int\frac{1}{x^2(a+b\log(cx^n))^2}\,dx}{2bn}\\
&= -\frac{1}{2bnx\left(a+b\log\left(cx^n\right)\right)^2}+\frac{1}{2b^2n^2x\left(a+b\log\left(cx^n\right)\right)}+\frac{\int\frac{1}{x^2(a+b\log(cx^n))}\,dx}{2b^2n^2}\\
&= -\frac{1}{2bnx\left(a+b\log\left(cx^n\right)\right)^2}+\frac{1}{2b^2n^2x\left(a+b\log\left(cx^n\right)\right)}+\frac{\left(cx^n\right)^{\frac{1}{n}}\operatorname{Subst}\left(\int\frac{e^{-\frac{x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{2b^2n^3x}\\
&= \frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{Ei}\left(-\frac{a+b\log(cx^n)}{bn}\right)}{2b^3n^3x}-\frac{1}{2bnx\left(a+b\log\left(cx^n\right)\right)^2}+\frac{1}{2b^2n^2x\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.92

$$\int\frac{1}{x^2\left(a+b\log\left(cx^n\right)\right)^3}\,dx$$
$$=\frac{e^{\frac{a}{bn}}\left(cx^n\right)^{\frac{1}{n}}\operatorname{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^2+bn(a-bn+b\log\left(cx^n\right))}{2b^3n^3x\left(a+b\log\left(cx^n\right)\right)^2}$$

```
[In] Integrate[1/(x^2*(a + b*Log[c*x^n])^3),x]
```

```
[Out] (E^(a/(b*n))*(c*x^n)^n^(-1)*ExpIntegralEi[-((a + b*Log[c*x^n])/(b*n))]*(a +
 b*Log[c*x^n])^2 + b*n*(a - b*n + b*Log[c*x^n]))/(2*b^3*n^3*x*(a + b*Log[c*
x^n])^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 449, normalized size of antiderivative = 4.40

method	result
risch	$\frac{-2bn+2a+2b\ln(c)+2\ln(x^n)b-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\ \mathrm{csg}}{\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2b\ln(c)+2\ln(x^n)b+\right.}$

```
[In] int(1/x^2/(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*b*n+2*a+2*b*ln(c)+2*ln(x^n)*b-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n
)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*P
i*csgn(I*c*x^n)^3)/(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn
(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^
n)^3+2*b*ln(c)+2*ln(x^n)*b+2*a)^2/b^2/n^2/x-1/2/b^3/n^3/x*c^(1/n)*(x^n)^(1/
n)*exp(1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*cs
gn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a
)/b/n)*Ei(1,ln(x)+1/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*c
sgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c
*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(95) = 190.

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.88

$$\int \frac{1}{x^2\left(a+b\log\left(cx^n\right)\right)^3}\,dx$$

$$= \frac{b^2n^2\log\left(x\right)-b^2n^2+b^2n\log\left(c\right)+abn+\left(b^2n^2x\log\left(x\right)^2+b^2x\log\left(c\right)^2+2\,abx\log\left(c\right)+a^2x+2\left(b^2nx\log\right.\right.}{2\left(b^5n^5x\log\left(x\right)^2+b^5n^3x\log\left(c\right)^2+2\,ab^4n^3x\log\left(c\right)+a^2b^3n^3x+2\left(b^5n^4x\right.\right.}$$

```
[In] integrate(1/x^2/(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*n^2*log(x) - b^2*n^2 + b^2*n*log(c) + a*b*n + (b^2*n^2*x*log(x)^2
+ b^2*x*log(c)^2 + 2*a*b*x*log(c) + a^2*x + 2*(b^2*n*x*log(c) + a*b*n*x)*lo
g(x))*e^((b*log(c) + a)/(b*n))*log_integral(e^(-(b*log(c) + a)/(b*n))/x))/(
b^5*n^5*x*log(x)^2 + b^5*n^3*x*log(c)^2 + 2*a*b^4*n^3*x*log(c) + a^2*b^3*n^
3*x + 2*(b^5*n^4*x*log(c) + a*b^4*n^4*x)*log(x))
```

Sympy [F]

$$\int \frac{1}{x^2 \left(a+b\log\left(cx^n\right)\right)^3}\,dx = \int \frac{1}{x^2 \left(a+b\log\left(cx^n\right)\right)^3}\,dx$$

```
[In] integrate(1/x**2/(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(1/(x**2*(a + b*log(c*x**n))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(a+b\log\left(cx^n\right)\right)^3}\,dx = \int \frac{1}{\left(b\log\left(cx^n\right)+a\right)^3 x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*(n - log(c)) - b*log(x^n) - a)/(b^4*n^2*x*log(x^n)^2 + 2*(b^4*n^2*l
og(c) + a*b^3*n^2)*x*log(x^n) + (b^4*n^2*log(c)^2 + 2*a*b^3*n^2*log(c) + a^
2*b^2*n^2)*x) + integrate(1/2/(b^3*n^2*x^2*log(x^n) + (b^3*n^2*log(c) + a*b
^2*n^2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(a+b\log\left(cx^n\right)\right)^3}\,dx = \int \frac{1}{\left(b\log\left(cx^n\right)+a\right)^3 x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*log(c*x^n) + a)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(a+b\log\left(cx^n\right)\right)^3}\,dx = \int \frac{1}{x^2 \left(a+b\,\ln\left(c\,x^n\right)\right)^3}\,dx$$

```
[In] int(1/(x^2*(a + b*log(c*x^n))^3),x)
```

```
[Out] int(1/(x^2*(a + b*log(c*x^n))^3), x)
```

3.87 $\int \frac{1}{x^3(a+b\log(cx^n))^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{1}{x^3\left(a+b\log\left(cx^n\right)\right)^3}\,dx=\frac{2e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2(a+b\log(cx^n))}{bn}\right)}{b^3n^3x^2}-\frac{1}{2bnx^2\left(a+b\log\left(cx^n\right)\right)^2}+\frac{1}{b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] 2*exp(2*a/b/n)*(c*x^n)^(2/n)*Ei(-2*(a+b*ln(c*x^n))/b/n)/b^3/n^3/x^2-1/2/b/n
/x^2/(a+b*ln(c*x^n))^2+1/b^2/n^2/x^2/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2343, 2347, 2209}

$$\int \frac{1}{x^3\left(a+b\log\left(cx^n\right)\right)^3}\,dx=\frac{2e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2(a+b\log(cx^n))}{bn}\right)}{b^3n^3x^2}+\frac{1}{b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}-\frac{1}{2bnx^2\left(a+b\log\left(cx^n\right)\right)^2}$$

```
[In] Int[1/(x^3*(a + b*Log[c*x^n])^3),x]
```

```
[Out] (2*E^((2*a)/(b*n))*(c*x^n)^(2/n)*ExpIntegralEi[(-2*(a + b*Log[c*x^n]))/(b*n
)])/(b^3*n^3*x^2) - 1/(2*b*n*x^2*(a + b*Log[c*x^n])^2) + 1/(b^2*n^2*x^2*(a
+ b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2bnx^2\left(a+b\log\left(cx^n\right)\right)^2}-\frac{\int \frac{1}{x^3\left(a+b\log\left(cx^n\right)\right)^2}\,dx}{bn}\\
&= -\frac{1}{2bnx^2\left(a+b\log\left(cx^n\right)\right)^2}+\frac{1}{b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}+\frac{2\int \frac{1}{x^3\left(a+b\log\left(cx^n\right)\right)}\,dx}{b^2n^2}\\
&= -\frac{1}{2bnx^2\left(a+b\log\left(cx^n\right)\right)^2}+\frac{1}{b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}\\
&\quad+\frac{\left(2\left(cx^n\right)^{2/n}\right)\text{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{b^2n^3x^2}\\
&= \frac{2e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{Ei}\left(-\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{b^3n^3x^2}-\frac{1}{2bnx^2\left(a+b\log\left(cx^n\right)\right)^2}+\frac{1}{b^2n^2x^2\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a+b\log\left(cx^n\right)\right)^3}\,dx = \frac{4e^{\frac{2a}{bn}}\left(cx^n\right)^{2/n}\text{ExpIntegralEi}\left(-\frac{2\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\frac{bn\left(2a-bn+2b\log\left(cx^n\right)\right)}{\left(a+b\log\left(cx^n\right)\right)^2}}{2b^3n^3x^2}$$

```
[In] Integrate[1/(x^3*(a + b*Log[c*x^n])^3),x]
```

```
[Out] (4*E^((2*a)/(b*n))*(c*x^n)^(2/n)*ExpIntegralEi[(-2*(a + b*Log[c*x^n]))/(b*n
)] + (b*n*(2*a - b*n + 2*b*Log[c*x^n]))/(a + b*Log[c*x^n])^2)/(2*b^3*n^3*x^
2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.62 (sec) , antiderivative size = 454, normalized size of antiderivative = 4.54

method	result
risch	$\frac{-2bn+4a+4b\ln(c)+4\ln(x^n)b-2ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+2ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+2ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-2ib\pi}{\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2b\ln(c)+2\ln(x^n)b+\right.}$

```
[In] int(1/x^3/(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-b*n+2*a+2*b*ln(c)+2*ln(x^n)*b-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n
)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*P
i*csgn(I*c*x^n)^3)/(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn
(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^
n)^3+2*b*ln(c)+2*ln(x^n)*b+2*a)^2/b^2/n^2/x^2-2/b^3/n^3/x^2*c^(2/n)*(x^n)^(
2/n)*exp((-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(I*c)*csgn
(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n)^3+2*a)/
b/n)*Ei(1,2*ln(x)+(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*csgn(
I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c*x^n
)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(97) = 194.

Time = 0.33 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.21

$$\int \frac{1}{x^3 \left(a+b\log\left(cx^n\right)\right)^3}\,dx$$

$$= \frac{2\,b^2n^2\log\left(x\right)-b^2n^2+2\,b^2n\log\left(c\right)+2\,abn+4\,\left(b^2n^2x^2\log\left(x\right)^2+b^2x^2\log\left(c\right)^2+2\,abx^2\log\left(c\right)+a^2x^2+2\right.}{2\,\left(b^5n^5x^2\log\left(x\right)^2+b^5n^3x^2\log\left(c\right)^2+2\,ab^4n^3x^2\log\left(c\right)+a^2b^3n^3x^2+\right.}$$

```
[In] integrate(1/x^3/(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*n^2*log(x) - b^2*n^2 + 2*b^2*n*log(c) + 2*a*b*n + 4*(b^2*n^2*x^2
*log(x)^2 + b^2*x^2*log(c)^2 + 2*a*b*x^2*log(c) + a^2*x^2 + 2*(b^2*n*x^2*lo
g(c) + a*b*n*x^2)*log(x))*e^(2*(b*log(c) + a)/(b*n))*log_integral(e^(-2*(b*
```

```
log(c) + a)/(b*n))/x^2))/(b^5*n^5*x^2*log(x)^2 + b^5*n^3*x^2*log(c)^2 + 2*a
*b^4*n^3*x^2*log(c) + a^2*b^3*n^3*x^2 + 2*(b^5*n^4*x^2*log(c) + a*b^4*n^4*x
^2)*log(x))
```

Sympy [F]

$$\int \frac{1}{x^3 (a + b \log(cx^n))^3} dx = \int \frac{1}{x^3 (a + b \log(cx^n))^3} dx$$

```
[In] integrate(1/x**3/(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral(1/(x**3*(a + b*log(c*x**n))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^3 (a + b \log(cx^n))^3} dx = \int \frac{1}{(b \log(cx^n) + a)^3 x^3} dx$$

```
[In] integrate(1/x^3/(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b*(n - 2*log(c)) - 2*b*log(x^n) - 2*a)/(b^4*n^2*x^2*log(x^n)^2 + 2*(b
^4*n^2*log(c) + a*b^3*n^2)*x^2*log(x^n) + (b^4*n^2*log(c)^2 + 2*a*b^3*n^2*l
og(c) + a^2*b^2*n^2)*x^2) + 2*integrate(1/(b^3*n^2*x^3*log(x^n) + (b^3*n^2*
log(c) + a*b^2*n^2)*x^3), x)
```

Giac [F]

$$\int \frac{1}{x^3 (a + b \log(cx^n))^3} dx = \int \frac{1}{(b \log(cx^n) + a)^3 x^3} dx$$

```
[In] integrate(1/x^3/(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*log(c*x^n) + a)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (a + b\log(cx^n))^3}\,dx = \int \frac{1}{x^3 (a + b\,\ln(c\,x^n))^3}\,dx$$

```
[In] int(1/(x^3*(a + b*log(c*x^n))^3),x)
[Out] int(1/(x^3*(a + b*log(c*x^n))^3), x)
```

3.88 $\int \frac{1}{x^4(a+b\log(cx^n))^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{1}{x^4 (a+b\log(cx^n))^3}\,dx = \frac{9e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{bn}\right)}{2b^3n^3x^3} - \frac{1}{2bnx^3(a+b\log(cx^n))^2} + \frac{3}{2b^2n^2x^3(a+b\log(cx^n))}$$

```
[Out] 9/2*exp(3*a/b/n)*(c*x^n)^(3/n)*Ei(-3*(a+b*ln(c*x^n))/b/n)/b^3/n^3/x^3-1/2/b
/n/x^3/(a+b*ln(c*x^n))^2+3/2/b^2/n^2/x^3/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec), antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2343, 2347, 2209}

$$\int \frac{1}{x^4 (a+b\log(cx^n))^3}\,dx = \frac{9e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{bn}\right)}{2b^3n^3x^3} + \frac{3}{2b^2n^2x^3(a+b\log(cx^n))} - \frac{1}{2bnx^3(a+b\log(cx^n))^2}$$

```
[In] Int[1/(x^4*(a + b*Log[c*x^n])^3),x]
```

```
[Out] (9*E^((3*a)/(b*n))*(c*x^n)^(3/n)*ExpIntegralEi[(-3*(a + b*Log[c*x^n]))/(b*n
)])/(2*b^3*n^3*x^3) - 1/(2*b*n*x^3*(a + b*Log[c*x^n])^2) + 3/(2*b^2*n^2*x^3
*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2bnx^3\left(a+b\log\left(cx^n\right)\right)^2}-\frac{3\int\frac{1}{x^4\left(a+b\log\left(cx^n\right)\right)^2}\,dx}{2bn}\\
&= -\frac{1}{2bnx^3\left(a+b\log\left(cx^n\right)\right)^2}+\frac{3}{2b^2n^2x^3\left(a+b\log\left(cx^n\right)\right)}+\frac{9\int\frac{1}{x^4\left(a+b\log\left(cx^n\right)\right)}\,dx}{2b^2n^2}\\
&= -\frac{1}{2bnx^3\left(a+b\log\left(cx^n\right)\right)^2}+\frac{3}{2b^2n^2x^3\left(a+b\log\left(cx^n\right)\right)}\\
&\quad+\frac{\left(9\left(cx^n\right)^{3/n}\right)\text{Subst}\left(\int\frac{e^{-\frac{3x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{2b^2n^3x^3}\\
&= \frac{9e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\text{Ei}\left(-\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2b^3n^3x^3}-\frac{1}{2bnx^3\left(a+b\log\left(cx^n\right)\right)^2}+\frac{3}{2b^2n^2x^3\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int\frac{1}{x^4\left(a+b\log\left(cx^n\right)\right)^3}\,dx = \frac{9e^{\frac{3a}{bn}}\left(cx^n\right)^{3/n}\text{ExpIntegralEi}\left(-\frac{3\left(a+b\log\left(cx^n\right)\right)}{bn}\right)+\frac{bn\left(3a-bn+3b\log\left(cx^n\right)\right)}{\left(a+b\log\left(cx^n\right)\right)^2}}{2b^3n^3x^3}$$

```
[In] Integrate[1/(x^4*(a + b*Log[c*x^n])^3),x]
```

```
[Out] (9*E^((3*a)/(b*n))*(c*x^n)^(3/n)*ExpIntegralEi[(-3*(a + b*Log[c*x^n]))/(b*n
)] + (b*n*(3*a - b*n + 3*b*Log[c*x^n]))/(a + b*Log[c*x^n])^2)/(2*b^3*n^3*x^
3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.85 (sec) , antiderivative size = 455, normalized size of antiderivative = 4.33

method	result
risch	$\frac{6a+6b\ln(c)+6\ln(x^n)b-3ib\pi\ \text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)+3ib\pi\ \text{csgn}(ic)\,\text{csgn}(ic\,x^n)^2+3ib\pi\ \text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)^2-3ib\pi\,\text{csgn}(ic\,x}{b^2n^2\left(-ib\pi\ \text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)+ib\pi\ \text{csgn}(ic)\,\text{csgn}(ic\,x^n)^2+ib\pi\ \text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)^2-ib\pi\,\text{csgn}(ic\,x^n)^3+2b\ln(c)+2\ln(x^n)\right.}$

```
[In] int(1/x^4/(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (6*a+6*b*ln(c)+6*ln(x^n)*b-3*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+3*I
*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+3*I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-3*I*b*
Pi*csgn(I*c*x^n)^3-2*b*n)/b^2/n^2/(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x
^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b
*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*ln(x^n)*b+2*a)^2/x^3-9/2/b^3/n^3/x^3*c^(3/n
)*(x^n)^(3/n)*exp(3/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+I*b*Pi*c
sgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi*csgn(I*c
*x^n)^3+2*a)/b/n)*Ei(1,3*ln(x)+3/2*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*
x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*
b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+2*b*(ln(x^n)-n*ln(x))+2*a)/b/n)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(98) = 196.

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.10

$$\int \frac{1}{x^4\left(a+b\log\left(cx^n\right)\right)^3}\,dx$$
$$=\frac{3\,b^2n^2\log\left(x\right)-b^2n^2+3\,b^2n\log\left(c\right)+3\,abn+9\left(b^2n^2x^3\log\left(x\right)^2+b^2x^3\log\left(c\right)^2+2\,abx^3\log\left(c\right)+a^2x^3+2\,(\right.}{2\left(b^5n^5x^3\log\left(x\right)^2+b^5n^3x^3\log\left(c\right)^2+2\,ab^4n^3x^3\log\left(c\right)+a^2b^3n^3x^3+\right.}$$

```
[In] integrate(1/x^4/(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*n^2*log(x) - b^2*n^2 + 3*b^2*n*log(c) + 3*a*b*n + 9*(b^2*n^2*x^3
*log(x)^2 + b^2*x^3*log(c)^2 + 2*a*b*x^3*log(c) + a^2*x^3 + 2*(b^2*n*x^3*lo
g(c) + a*b*n*x^3)*log(x))*e^(3*(b*log(c) + a)/(b*n))*log_integral(e^(-3*(b*
```

```
log(c) + a)/(b*n))/x^3))/(b^5*n^5*x^3*log(x)^2 + b^5*n^3*x^3*log(c)^2 + 2*a
*b^4*n^3*x^3*log(c) + a^2*b^3*n^3*x^3 + 2*(b^5*n^4*x^3*log(c) + a*b^4*n^4*x
^3)*log(x))
```

Sympy [F]

$$\int \frac{1}{x^4 (a + b\log (cx^n))^3}\, dx = \int \frac{1}{x^4 (a + b\log (cx^n))^3}\, dx$$

```
[In] integrate(1/x**4/(a+b*ln(c*x**n))**3,x)
[Out] Integral(1/(x**4*(a + b*log(c*x**n))**3), x)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + b\log (cx^n))^3}\, dx = \int \frac{1}{(b\log (cx^n) + a)^3 x^4}\, dx$$

```
[In] integrate(1/x^4/(a+b*log(c*x^n))^3,x, algorithm="maxima")
[Out] -1/2*(b*(n - 3*log(c)) - 3*b*log(x^n) - 3*a)/(b^4*n^2*x^3*log(x^n)^2 + 2*(b
^4*n^2*log(c) + a*b^3*n^2)*x^3*log(x^n) + (b^4*n^2*log(c)^2 + 2*a*b^3*n^2*l
og(c) + a^2*b^2*n^2)*x^3) + 9*integrate(1/2/(b^3*n^2*x^4*log(x^n) + (b^3*n^
2*log(c) + a*b^2*n^2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a + b\log (cx^n))^3}\, dx = \int \frac{1}{(b\log (cx^n) + a)^3 x^4}\, dx$$

```
[In] integrate(1/x^4/(a+b*log(c*x^n))^3,x, algorithm="giac")
[Out] integrate(1/((b*log(c*x^n) + a)^3*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a + b \log(cx^n))^3} \, dx = \int \frac{1}{x^4 (a + b \ln(c x^n))^3} \, dx$$

```
[In] int(1/(x^4*(a + b*log(c*x^n))^3),x)
[Out] int(1/(x^4*(a + b*log(c*x^n))^3), x)
```

3.89 $\int (dx)^{5/2} (a + b \log (cx^n)) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int (dx)^{5/2} (a + b \log (cx^n)) \, dx = -\frac{4bn(dx)^{7/2}}{49d} + \frac{2(dx)^{7/2} (a + b \log (cx^n))}{7d}$$

```
[Out] -4/49*b*n*(d*x)^(7/2)/d+2/7*(d*x)^(7/2)*(a+b*ln(c*x^n))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2341}

$$\int (dx)^{5/2} (a + b \log (cx^n)) \, dx = \frac{2(dx)^{7/2} (a + b \log (cx^n))}{7d} - \frac{4bn(dx)^{7/2}}{49d}$$

```
[In] Int[(d*x)^(5/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-4*b*n*(d*x)^(7/2))/(49*d) + (2*(d*x)^(7/2)*(a + b*Log[c*x^n]))/(7*d)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{4bn(dx)^{7/2}}{49d} + \frac{2(dx)^{7/2} (a + b \log (cx^n))}{7d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int (dx)^{5/2}(a+b\log(cx^n))\,dx=\frac{2}{49}x(dx)^{5/2}(7a-2bn+7b\log(cx^n))$$

```
[In] Integrate[(d*x)^(5/2)*(a + b*Log[c*x^n]),x]
[Out] (2*x*(d*x)^(5/2)*(7*a - 2*b*n + 7*b*Log[c*x^n]))/49
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.12

method	result
risch	$\frac{2d^3b\,x^4\ln(x^n)}{7\sqrt{dx}}+\frac{d^3\left(-7ib\pi\ \text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)+7ib\pi\ \text{csgn}(ic)\,\text{csgn}(ic\,x^n)^2+7ib\pi\ \text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)^2-7ib\pi\,\text{csgn}(ic\,x^n$ ... $}{49\sqrt{dx}}$

```
[In] int((d*x)^(5/2)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] 2/7*d^3*b*x^4/(d*x)^(1/2)*ln(x^n)+1/49*d^3*(-7*I*b*Pi*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)+7*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+7*I*b*Pi*csgn(I*x^n)*csgn
(I*c*x^n)^2-7*I*b*Pi*csgn(I*c*x^n)^3+14*b*ln(c)-4*b*n+14*a)*x^4/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int (dx)^{5/2}(a+b\log(cx^n))\,dx=\frac{2}{49}\left(7\,bd^2nx^3\log(x)+7\,bd^2x^3\log(c)-\left(2\,bd^2n-7\,ad^2\right)x^3\right)\sqrt{dx}$$

```
[In] integrate((d*x)^(5/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] 2/49*(7*b*d^2*n*x^3*log(x) + 7*b*d^2*x^3*log(c) - (2*b*d^2*n - 7*a*d^2)*x^3
)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 13.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int (dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{2ax(dx)^{\frac{5}{2}}}{7}-\frac{4bnx(dx)^{\frac{5}{2}}}{49}+\frac{2bx(dx)^{\frac{5}{2}}\log\left(cx^n\right)}{7}$$

```
[In] integrate((d*x)**(5/2)*(a+b*ln(c*x**n)),x)
[Out] 2*a*x*(d*x)**(5/2)/7 - 4*b*n*x*(d*x)**(5/2)/49 + 2*b*x*(d*x)**(5/2)*log(c*x
**n)/7
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int (dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)\,dx=-\frac{4\,(dx)^{\frac{7}{2}}\,bn}{49\,d}+\frac{2\,(dx)^{\frac{7}{2}}\,b\log\left(cx^n\right)}{7\,d}+\frac{2\,(dx)^{\frac{7}{2}}\,a}{7\,d}$$

```
[In] integrate((d*x)^(5/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] -4/49*(d*x)^(7/2)*b*n/d + 2/7*(d*x)^(7/2)*b*log(c*x^n)/d + 2/7*(d*x)^(7/2)*
a/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.85

$$\begin{aligned}\int (dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)\,dx&=\left(\frac{1}{7}i+\frac{1}{7}\right)\sqrt{2}bd^2nx^{\frac{7}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)\log\left(x\right)\\&-\left(\frac{1}{7}i-\frac{1}{7}\right)\sqrt{2}bd^2nx^{\frac{7}{2}}\sqrt{|d|}\log\left(x\right)\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)\\&-\left(\frac{2}{49}i+\frac{2}{49}\right)\sqrt{2}bd^2nx^{\frac{7}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)\\&+\left(\frac{2}{49}i-\frac{2}{49}\right)\sqrt{2}bd^2nx^{\frac{7}{2}}\sqrt{|d|}\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)+\frac{2}{7}bd^{\frac{5}{2}}x^{\frac{7}{2}}\log\left(c\right)+\frac{2}{7}ad^{\frac{5}{2}}x^{\frac{7}{2}}\end{aligned}$$

```
[In] integrate((d*x)^(5/2)*(a+b*log(c*x^n)),x, algorithm="giac")
[Out] (1/7*I + 1/7)*sqrt(2)*b*d^2*n*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x
) - (1/7*I - 1/7)*sqrt(2)*b*d^2*n*x^(7/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sg
n(d)) - (2/49*I + 2/49)*sqrt(2)*b*d^2*n*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn
(d)) + (2/49*I - 2/49)*sqrt(2)*b*d^2*n*x^(7/2)*sqrt(abs(d))*sin(1/4*pi*sgn(
d)) + 2/7*b*d^(5/2)*x^(7/2)*log(c) + 2/7*a*d^(5/2)*x^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2} (a + b \log (cx^n)) \, dx = \int (d\,x)^{5/2} (a + b\, \ln (c\,x^n)) \, dx$$

```
[In] int((d*x)^(5/2)*(a + b*log(c*x^n)),x)
[Out] int((d*x)^(5/2)*(a + b*log(c*x^n)), x)
```

3.90 $\int (dx)^{3/2} (a+b\log(cx^n)) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int (dx)^{3/2} (a+b\log(cx^n)) \, dx = -\frac{4bn(dx)^{5/2}}{25d} + \frac{2(dx)^{5/2}(a+b\log(cx^n))}{5d}$$

```
[Out] -4/25*b*n*(d*x)^(5/2)/d+2/5*(d*x)^(5/2)*(a+b*ln(c*x^n))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2341}

$$\int (dx)^{3/2} (a+b\log(cx^n)) \, dx = \frac{2(dx)^{5/2}(a+b\log(cx^n))}{5d} - \frac{4bn(dx)^{5/2}}{25d}$$

```
[In] Int[(d*x)^(3/2)*(a + b*Log[c*x^n]),x]
```

```
[Out] (-4*b*n*(d*x)^(5/2))/(25*d) + (2*(d*x)^(5/2)*(a + b*Log[c*x^n]))/(5*d)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{4bn(dx)^{5/2}}{25d} + \frac{2(dx)^{5/2}(a+b\log(cx^n))}{5d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int (dx)^{3/2} (a + b\log(cx^n))\, dx = \frac{2}{25}x(dx)^{3/2}(5a - 2bn + 5b\log(cx^n))$$

```
[In] Integrate[(d*x)^(3/2)*(a + b*Log[c*x^n]),x]
[Out] (2*x*(d*x)^(3/2)*(5*a - 2*b*n + 5*b*Log[c*x^n]))/25
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.12

method	result
risch	$\frac{2d^2 b\, x^3 \ln(x^n)}{5\sqrt{dx}} + \frac{d^2\left(-5ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+5ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+5ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-5ib\pi\,\mathrm{csgn}(ic\,x^n\right.}{25\sqrt{dx}}$

```
[In] int((d*x)^(3/2)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] 2/5*d^2*b*x^3/(d*x)^(1/2)*ln(x^n)+1/25*d^2*(-5*I*b*Pi*csgn(I*c)*csgn(I*x^n)
*csgn(I*c*x^n)+5*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+5*I*b*Pi*csgn(I*x^n)*csgn
(I*c*x^n)^2-5*I*b*Pi*csgn(I*c*x^n)^3+10*b*ln(c)-4*b*n+10*a)*x^3/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int (dx)^{3/2} (a + b\log(cx^n))\, dx = \frac{2}{25}\left(5\,bdnx^2\log(x) + 5\,bdx^2\log(c) - (2\,bdn - 5\,ad)x^2\right)\sqrt{dx}$$

```
[In] integrate((d*x)^(3/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] 2/25*(5*b*d*n*x^2*log(x) + 5*b*d*x^2*log(c) - (2*b*d*n - 5*a*d)*x^2)*sqrt(d
*x)
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int (dx)^{3/2} \left(a + b \log \left(cx^{n}\right)\right) \, dx = \frac{2ax(dx)^{\frac{3}{2}}}{5} - \frac{4bnx(dx)^{\frac{3}{2}}}{25} + \frac{2bx(dx)^{\frac{3}{2}} \log \left(cx^{n}\right)}{5}$$

```
[In] integrate((d*x)**(3/2)*(a+b*ln(c*x**n)),x)
[Out] 2*a*x*(d*x)**(3/2)/5 - 4*b*n*x*(d*x)**(3/2)/25 + 2*b*x*(d*x)**(3/2)*log(c*x
**n)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int (dx)^{3/2} \left(a + b \log \left(cx^{n}\right)\right) \, dx = -\frac{4 \, (dx)^{\frac{5}{2}} \, bn}{25 \, d} + \frac{2 \, (dx)^{\frac{5}{2}} \, b \log \left(cx^{n}\right)}{5 \, d} + \frac{2 \, (dx)^{\frac{5}{2}} \, a}{5 \, d}$$

```
[In] integrate((d*x)^(3/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] -4/25*(d*x)^(5/2)*b*n/d + 2/5*(d*x)^(5/2)*b*log(c*x^n)/d + 2/5*(d*x)^(5/2)*
a/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.63

$$\int (dx)^{3/2} \left(a + b \log \left(cx^{n}\right)\right) \, dx = -\frac{1}{25} \left(-(5i + 5) \sqrt{2} bnx^{\frac{5}{2}} \sqrt{|d|} \cos \left(\frac{1}{4} \pi \mathrm{sgn}(d)\right) \log (x) + (5i - 5) \sqrt{2} bnx^{\frac{5}{2}} \sqrt{|d|} \log (x) \sin \left(\frac{1}{4} \pi \mathrm{sgn}(d)\right) + \right.$$

```
[In] integrate((d*x)^(3/2)*(a+b*log(c*x^n)),x, algorithm="giac")
[Out] -1/25*(-(5*I + 5)*sqrt(2)*b*n*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x
) + (5*I - 5)*sqrt(2)*b*n*x^(5/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d)) +
(2*I + 2)*sqrt(2)*b*n*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) - (2*I - 2)*s
qrt(2)*b*n*x^(5/2)*sqrt(abs(d))*sin(1/4*pi*sgn(d)) - 10*b*sqrt(d)*x^(5/2)*l
og(c) - 10*a*sqrt(d)*x^(5/2))*d
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} (a + b\log(cx^n)) \, dx = \int (d\,x)^{3/2} \left(a + b\,\ln(c\,x^n)\right) \, dx$$

```
[In] int((d*x)^(3/2)*(a + b*log(c*x^n)),x)
[Out] int((d*x)^(3/2)*(a + b*log(c*x^n)), x)
```

3.91 $\int \sqrt{dx}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \sqrt{dx}(a+b\log(cx^n))\,dx = -\frac{4bn(dx)^{3/2}}{9d}+\frac{2(dx)^{3/2}(a+b\log(cx^n))}{3d}$$

```
[Out] -4/9*b*n*(d*x)^(3/2)/d+2/3*(d*x)^(3/2)*(a+b*ln(c*x^n))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2341}

$$\int \sqrt{dx}(a+b\log(cx^n))\,dx = \frac{2(dx)^{3/2}(a+b\log(cx^n))}{3d}-\frac{4bn(dx)^{3/2}}{9d}$$

```
[In] Int[Sqrt[d*x]*(a + b*Log[c*x^n]),x]
[Out] (-4*b*n*(d*x)^(3/2))/(9*d) + (2*(d*x)^(3/2)*(a + b*Log[c*x^n]))/(3*d)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{4bn(dx)^{3/2}}{9d}+\frac{2(dx)^{3/2}(a+b\log(cx^n))}{3d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \sqrt{dx}(a+b\log(cx^n))\,dx=\frac{2}{9}x\sqrt{dx}(3a-2bn+3b\log(cx^n))$$

```
[In] Integrate[Sqrt[d*x]*(a + b*Log[c*x^n]),x]
```

```
[Out] (2*x*Sqrt[d*x]*(3*a - 2*b*n + 3*b*Log[c*x^n]))/9
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.02

method	result
risch	$\frac{2db\,x^2\ln(x^n)}{3\sqrt{dx}}+\frac{d\left(-3ib\pi\ \operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)+3ib\pi\ \operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2+3ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2-3ib\pi\operatorname{csgn}(ic\,x^n)^3 \cdots\right.}{9\sqrt{dx}}$

```
[In] int((d*x)^(1/2)*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*d*b*x^2/(d*x)^(1/2)*ln(x^n)+1/9*d*(-3*I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn
(I*c*x^n)+3*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+3*I*b*Pi*csgn(I*x^n)*csgn(I*c*
x^n)^2-3*I*b*Pi*csgn(I*c*x^n)^3+6*b*ln(c)-4*b*n+6*a)*x^2/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \sqrt{dx}(a+b\log(cx^n))\,dx=\frac{2}{9}\left(3\,bnx\log(x)+3\,bx\log(c)-(2\,bn-3\,a)x\right)\sqrt{dx}$$

```
[In] integrate((d*x)^(1/2)*(a+b*log(c*x^n)),x, algorithm="fricas")
```

```
[Out] 2/9*(3*b*n*x*log(x) + 3*b*x*log(c) - (2*b*n - 3*a)*x)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int \sqrt{dx}(a+b\log{(cx^n)})\,dx = \frac{2ax\sqrt{dx}}{3} - \frac{4bnx\sqrt{dx}}{9} + \frac{2bx\sqrt{dx}\log{(cx^n)}}{3}$$

```
[In] integrate((d*x)**(1/2)*(a+b*ln(c*x**n)),x)
[Out] 2*a*x*sqrt(d*x)/3 - 4*b*n*x*sqrt(d*x)/9 + 2*b*x*sqrt(d*x)*log(c*x**n)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sqrt{dx}(a+b\log{(cx^n)})\,dx = -\frac{4\,(dx)^{\frac{3}{2}}\,bn}{9\,d} + \frac{2\,(dx)^{\frac{3}{2}}\,b\log{(cx^n)}}{3\,d} + \frac{2\,(dx)^{\frac{3}{2}}\,a}{3\,d}$$

```
[In] integrate((d*x)^(1/2)*(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] -4/9*(d*x)^(3/2)*b*n/d + 2/3*(d*x)^(3/2)*b*log(c*x^n)/d + 2/3*(d*x)^(3/2)*a
/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.56

$$\begin{aligned}\int \sqrt{dx}(a+b\log{(cx^n)})\,dx ={}& \left(\frac{1}{3}i+\frac{1}{3}\right)\sqrt{2}bnx^{\frac{3}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)\log{(x)} \\ &-\left(\frac{1}{3}i-\frac{1}{3}\right)\sqrt{2}bnx^{\frac{3}{2}}\sqrt{|d|}\log{(x)}\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) \\ &-\left(\frac{2}{9}i+\frac{2}{9}\right)\sqrt{2}bnx^{\frac{3}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) \\ &+\left(\frac{2}{9}i-\frac{2}{9}\right)\sqrt{2}bnx^{\frac{3}{2}}\sqrt{|d|}\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) \\ &+\frac{2}{3}b\sqrt{d}x^{\frac{3}{2}}\log{(c)}+\frac{2}{3}a\sqrt{d}x^{\frac{3}{2}}\end{aligned}$$

```
[In] integrate((d*x)^(1/2)*(a+b*log(c*x^n)),x, algorithm="giac")
[Out] (1/3*I + 1/3)*sqrt(2)*b*n*x^(3/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x) -
(1/3*I - 1/3)*sqrt(2)*b*n*x^(3/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d)) -
(2/9*I + 2/9)*sqrt(2)*b*n*x^(3/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) + (2/9*I
- 2/9)*sqrt(2)*b*n*x^(3/2)*sqrt(abs(d))*sin(1/4*pi*sgn(d)) + 2/3*b*sqrt(d)*
x^(3/2)*log(c) + 2/3*a*sqrt(d)*x^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a+b\log(cx^n)) \, dx = \int \sqrt{d\,x}\,(a+b\,\ln(c\,x^n)) \, dx$$

```
[In] int((d*x)^(1/2)*(a + b*log(c*x^n)),x)
[Out] int((d*x)^(1/2)*(a + b*log(c*x^n)), x)
```

3.92 $\int \frac{a+b\log(cx^n)}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{a+b\log(cx^n)}{\sqrt{dx}}\,dx=-\frac{4bn\sqrt{dx}}{d}+\frac{2\sqrt{dx}(a+b\log(cx^n))}{d}$$

```
[Out] -4*b*n*(d*x)^(1/2)/d+2*(a+b*ln(c*x^n))*(d*x)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2341}

$$\int \frac{a+b\log(cx^n)}{\sqrt{dx}}\,dx=\frac{2\sqrt{dx}(a+b\log(cx^n))}{d}-\frac{4bn\sqrt{dx}}{d}$$

```
[In] Int[(a + b*Log[c*x^n])/Sqrt[d*x],x]
```

```
[Out] (-4*b*n*Sqrt[d*x])/d + (2*Sqrt[d*x]*(a + b*Log[c*x^n]))/d
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=-\frac{4bn\sqrt{dx}}{d}+\frac{2\sqrt{dx}(a+b\log(cx^n))}{d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{a+b\log\left(cx^n\right)}{\sqrt{dx}}\,dx=\frac{2x(a-2bn+b\log\left(cx^n\right))}{\sqrt{dx}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/Sqrt[d*x],x]
[Out] (2*x*(a - 2*b*n + b*Log[c*x^n]))/Sqrt[d*x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{2\sqrt{dx}\,a+2\sqrt{dx}\,b\ln(c\,x^n)-4bn\sqrt{dx}}{d}$
default	$\frac{2\sqrt{dx}\,a+2\sqrt{dx}\,b\ln(c\,x^n)-4bn\sqrt{dx}}{d}$
parts	$\frac{2a\sqrt{dx}}{d}+\frac{2b\sqrt{dx}\,\ln(c\,x^n)}{d}-\frac{4bn\sqrt{dx}}{d}$
risch	$\frac{2bx\ln(x^n)}{\sqrt{dx}}+\frac{\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\ \mathrm{csgn}(ic\right.}{\sqrt{dx}}$

```
[In] int((a+b*ln(c*x^n))/(d*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*((d*x)^(1/2)*a+(d*x)^(1/2)*b*ln(c*x^n)-2*b*n*(d*x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{a+b\log\left(cx^n\right)}{\sqrt{dx}}\,dx=\frac{2\left(bn\log\left(x\right)-2\,bn+b\log\left(c\right)+a\right)\sqrt{dx}}{d}$$

```
[In] integrate((a+b*log(c*x^n))/(d*x)^(1/2),x, algorithm="fricas")
[Out] 2*(b*n*log(x) - 2*b*n + b*log(c) + a)*sqrt(d*x)/d
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{a+b\log\left(cx^n\right)}{\sqrt{dx}}\,dx = \frac{2ax}{\sqrt{dx}} - \frac{4bnx}{\sqrt{dx}} + \frac{2bx\log\left(cx^n\right)}{\sqrt{dx}}$$

```
[In] integrate((a+b*ln(c*x**n))/(d*x)**(1/2),x)
[Out] 2*a*x/sqrt(d*x) - 4*b*n*x/sqrt(d*x) + 2*b*x*log(c*x**n)/sqrt(d*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{a+b\log\left(cx^n\right)}{\sqrt{dx}}\,dx = -\frac{4\sqrt{dx}bn}{d} + \frac{2\sqrt{dx}b\log\left(cx^n\right)}{d} + \frac{2\sqrt{dx}a}{d}$$

```
[In] integrate((a+b*log(c*x^n))/(d*x)^(1/2),x, algorithm="maxima")
[Out] -4*sqrt(d*x)*b*n/d + 2*sqrt(d*x)*b*log(c*x^n)/d + 2*sqrt(d*x)*a/d
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{a+b\log\left(cx^n\right)}{\sqrt{dx}}\,dx = \frac{2\left(\left(\sqrt{dx}\log\left(x\right) - 2\sqrt{dx}\right)bn + \sqrt{dx}b\log\left(c\right) + \sqrt{dx}a\right)}{d}$$

```
[In] integrate((a+b*log(c*x^n))/(d*x)^(1/2),x, algorithm="giac")
[Out] 2*((sqrt(d*x)*log(x) - 2*sqrt(d*x))*b*n + sqrt(d*x)*b*log(c) + sqrt(d*x)*a)
/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{\sqrt{dx}}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{\sqrt{d\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d*x)^(1/2),x)
[Out] int((a + b*log(c*x^n))/(d*x)^(1/2), x)
```

3.93 $\int \frac{a+b\log(cx^n)}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{a+b\log\left(cx^n\right)}{(dx)^{3/2}}\,dx=-\frac{4bn}{d\sqrt{dx}}-\frac{2(a+b\log\left(cx^n\right))}{d\sqrt{dx}}$$

```
[Out] -4*b*n/d/(d*x)^(1/2)-2*(a+b*ln(c*x^n))/d/(d*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2341}

$$\int \frac{a+b\log\left(cx^n\right)}{(dx)^{3/2}}\,dx=-\frac{2(a+b\log\left(cx^n\right))}{d\sqrt{dx}}-\frac{4bn}{d\sqrt{dx}}$$

```
[In] Int[(a + b*Log[c*x^n])/(d*x)^(3/2),x]
```

```
[Out] (-4*b*n)/(d*Sqrt[d*x]) - (2*(a + b*Log[c*x^n]))/(d*Sqrt[d*x])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=-\frac{4bn}{d\sqrt{dx}}-\frac{2(a+b\log\left(cx^n\right))}{d\sqrt{dx}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{a+b\log(cx^n)}{(dx)^{3/2}}\,dx=-\frac{2x(a+2bn+b\log(cx^n))}{(dx)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*x^n])/(d*x)^(3/2),x]
[Out] (-2*x*(a + 2*b*n + b*Log[c*x^n]))/(d*x)^(3/2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.30

method	result
risch	$-\frac{2b\ln(x^n)}{d\sqrt{dx}}-\frac{-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\,\mathrm{csgn}(ic\,x^n)^3+2b\ln(c}{d\sqrt{dx}}$

```
[In] int((a+b*ln(c*x^n))/(d*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/d*b/(d*x)^(1/2)*ln(x^n)-1/d*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)
+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-I*b*Pi
*csgn(I*c*x^n)^3+2*b*ln(c)+4*b*n+2*a)/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{a+b\log(cx^n)}{(dx)^{3/2}}\,dx=-\frac{2\,(bn\log(x)+2\,bn+b\log(c)+a)\sqrt{dx}}{d^2x}$$

```
[In] integrate((a+b*log(c*x^n))/(d*x)^(3/2),x, algorithm="fricas")
[Out] -2*(b*n*log(x) + 2*b*n + b*log(c) + a)*sqrt(d*x)/(d^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{a+b\log\left(cx^n\right)}{(dx)^{3/2}}\,dx = -\frac{2ax}{(dx)^{\frac{3}{2}}} - \frac{4bnx}{(dx)^{\frac{3}{2}}} - \frac{2bx\log\left(cx^n\right)}{(dx)^{\frac{3}{2}}}$$

```
[In] integrate((a+b*ln(c*x**n))/(d*x)**(3/2),x)
[Out] -2*a*x/(d*x)**(3/2) - 4*b*n*x/(d*x)**(3/2) - 2*b*x*log(c*x**n)/(d*x)**(3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{a+b\log\left(cx^n\right)}{(dx)^{3/2}}\,dx = -\frac{4\,bn}{\sqrt{dx}d} - \frac{2\,b\log\left(cx^n\right)}{\sqrt{dx}d} - \frac{2\,a}{\sqrt{dx}d}$$

```
[In] integrate((a+b*log(c*x^n))/(d*x)^(3/2),x, algorithm="maxima")
[Out] -4*b*n/(sqrt(d*x)*d) - 2*b*log(c*x^n)/(sqrt(d*x)*d) - 2*a/(sqrt(d*x)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \frac{a+b\log\left(cx^n\right)}{(dx)^{3/2}}\,dx = -\frac{2\left(\frac{bn\log(dx)}{\sqrt{dx}} - \frac{bn\log(d)-2\,bn-b\log(c)-a}{\sqrt{dx}}\right)}{d}$$

```
[In] integrate((a+b*log(c*x^n))/(d*x)^(3/2),x, algorithm="giac")
[Out] -2*(b*n*log(d*x)/sqrt(d*x) - (b*n*log(d) - 2*b*n - b*log(c) - a)/sqrt(d*x))
/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log\left(cx^n\right)}{(dx)^{3/2}}\,dx=\int \frac{a+b\,\ln\left(c\,x^n\right)}{\left(d\,x\right)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d*x)^(3/2),x)
[Out] int((a + b*log(c*x^n))/(d*x)^(3/2), x)
```

3.94 $\int \frac{a+b\log(cx^n)}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{a+b\log(cx^n)}{(dx)^{5/2}}\,dx = -\frac{4bn}{9d(dx)^{3/2}} - \frac{2(a+b\log(cx^n))}{3d(dx)^{3/2}}$$

```
[Out] -4/9*b*n/d/(d*x)^(3/2)-2/3*(a+b*ln(c*x^n))/d/(d*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2341}

$$\int \frac{a+b\log(cx^n)}{(dx)^{5/2}}\,dx = -\frac{2(a+b\log(cx^n))}{3d(dx)^{3/2}} - \frac{4bn}{9d(dx)^{3/2}}$$

```
[In] Int[(a + b*Log[c*x^n])/(d*x)^(5/2),x]
[Out] (-4*b*n)/(9*d*(d*x)^(3/2)) - (2*(a + b*Log[c*x^n]))/(3*d*(d*x)^(3/2))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{4bn}{9d(dx)^{3/2}} - \frac{2(a+b\log(cx^n))}{3d(dx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int \frac{a+b\log(cx^n)}{(dx)^{5/2}}\,dx = -\frac{2x(3a+2bn+3b\log(cx^n))}{9(dx)^{5/2}}$$

[In] Integrate[(a + b*Log[c*x^n])/(d*x)^(5/2),x]

[Out] (-2*x*(3*a + 2*b*n + 3*b*Log[c*x^n]))/(9*(d*x)^(5/2))

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.12

method	result
risch	$-\frac{2b\ln(x^n)}{3d^2x\sqrt{dx}}-\frac{-3ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)+3ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2+3ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2-3ib\pi\,\mathrm{csgn}(icx^n)^3+6b}{9d^2x\sqrt{dx}}$

[In] int((a+b*ln(c*x^n))/(d*x)^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/3/d^2*b/x/(d*x)^(1/2)*ln(x^n)-1/9/d^2*(-3*I*b*Pi*csgn(I*c)*csgn(I*x^n)*c sgn(I*c*x^n)+3*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+3*I*b*Pi*csgn(I*x^n)*csgn(I *c*x^n)^2-3*I*b*Pi*csgn(I*c*x^n)^3+6*b*ln(c)+4*b*n+6*a)/x/(d*x)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{a+b\log(cx^n)}{(dx)^{5/2}}\,dx = -\frac{2\,(3\,bn\log(x)+2\,bn+3\,b\log(c)+3\,a)\sqrt{dx}}{9\,d^3x^2}$$

[In] integrate((a+b*log(c*x^n))/(d*x)^(5/2),x, algorithm="fricas")

[Out] -2/9*(3*b*n*log(x) + 2*b*n + 3*b*log(c) + 3*a)*sqrt(d*x)/(d^3*x^2)

Sympy [A] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{a+b\log\left(cx^{n}\right)}{(dx)^{5/2}}\,dx = -\frac{2ax}{3\left(dx\right)^{\frac{5}{2}}} - \frac{4bnx}{9\left(dx\right)^{\frac{5}{2}}} - \frac{2bx\log\left(cx^{n}\right)}{3\left(dx\right)^{\frac{5}{2}}}$$

```
[In] integrate((a+b*ln(c*x**n))/(d*x)**(5/2),x)
[Out] -2*a*x/(3*(d*x)**(5/2)) - 4*b*n*x/(9*(d*x)**(5/2)) - 2*b*x*log(c*x**n)/(3*(
d*x)**(5/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{a+b\log\left(cx^{n}\right)}{(dx)^{5/2}}\,dx = -\frac{4\,bn}{9\,\left(dx\right)^{\frac{3}{2}}\,d} - \frac{2\,b\log\left(cx^{n}\right)}{3\,\left(dx\right)^{\frac{3}{2}}\,d} - \frac{2\,a}{3\,\left(dx\right)^{\frac{3}{2}}\,d}$$

```
[In] integrate((a+b*log(c*x^n))/(d*x)^(5/2),x, algorithm="maxima")
[Out] -4/9*b*n/((d*x)^(3/2)*d) - 2/3*b*log(c*x^n)/((d*x)^(3/2)*d) - 2/3*a/((d*x)^
(3/2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(33) = 66.

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.63

$$\int \frac{a+b\log\left(cx^{n}\right)}{(dx)^{5/2}}\,dx = -\frac{2\left(\frac{3\,bdn\log\left(dx\right)}{\sqrt{dx}x} - \frac{3\,bd^{2}n\log\left(d\right)-2\,bd^{2}n-3\,bd^{2}\log\left(c\right)-3\,ad^{2}}{\sqrt{dx}dx}\right)}{9\,d^{3}}$$

```
[In] integrate((a+b*log(c*x^n))/(d*x)^(5/2),x, algorithm="giac")
[Out] -2/9*(3*b*d*n*log(d*x)/(sqrt(d*x)*x) - (3*b*d^2*n*log(d) - 2*b*d^2*n - 3*b*
d^2*log(c) - 3*a*d^2)/(sqrt(d*x)*d*x))/d^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\log(cx^n)}{(dx)^{5/2}}\,dx = \int \frac{a+b\,\ln(c\,x^n)}{(d\,x)^{5/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))/(d*x)^(5/2),x)
[Out] int((a + b*log(c*x^n))/(d*x)^(5/2), x)
```

3.95 $\int (dx)^{5/2} (a + b\log(cx^n))^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int (dx)^{5/2} (a + b\log(cx^n))^2 \, dx = \frac{16b^2n^2(dx)^{7/2}}{343d} - \frac{8bn(dx)^{7/2}(a + b\log(cx^n))}{49d} + \frac{2(dx)^{7/2}(a + b\log(cx^n))^2}{7d}$$

```
[Out] 16/343*b^2*n^2*(d*x)^(7/2)/d-8/49*b*n*(d*x)^(7/2)*(a+b*ln(c*x^n))/d+2/7*(d*
x)^(7/2)*(a+b*ln(c*x^n))^2/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2342, 2341}

$$\int (dx)^{5/2} (a + b\log(cx^n))^2 \, dx = \frac{2(dx)^{7/2}(a + b\log(cx^n))^2}{7d} - \frac{8bn(dx)^{7/2}(a + b\log(cx^n))}{49d} + \frac{16b^2n^2(dx)^{7/2}}{343d}$$

```
[In] Int[(d*x)^(5/2)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (16*b^2*n^2*(d*x)^(7/2))/(343*d) - (8*b*n*(d*x)^(7/2)*(a + b*Log[c*x^n]))/(
49*d) + (2*(d*x)^(7/2)*(a + b*Log[c*x^n])^2)/(7*d)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2(dx)^{7/2}\left(a+b\log\left(cx^n\right)\right)^2}{7d} - \frac{1}{7}(4bn)\int (dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)\,dx \\ &= \frac{16b^2n^2(dx)^{7/2}}{343d} - \frac{8bn(dx)^{7/2}\left(a+b\log\left(cx^n\right)\right)}{49d} + \frac{2(dx)^{7/2}\left(a+b\log\left(cx^n\right)\right)^2}{7d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{2}{343}x(dx)^{5/2}\left(49a^2-28abn+8b^2n^2+14b(7a-2bn)\log\left(cx^n\right)+49b^2\log^2\left(cx^n\right)\right)$$

```
[In] Integrate[(d*x)^(5/2)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(49*a^2 - 28*a*b*n + 8*b^2*n^2 + 14*b*(7*a - 2*b*n)*Log[c*
x^n] + 49*b^2*Log[c*x^n]^2))/343
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 716, normalized size of antiderivative = 9.81

method	result
risch	$\frac{2d^3x^4b^2\ln(x^n)^2}{7\sqrt{dx}} + \frac{2d^3bx^4\left(-7ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)+7ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2+7ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2-7ib\pi\ \mathrm{csg}\right.}{49\sqrt{dx}}$

```
[In] int((d*x)^(5/2)*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/7*d^3*x^4*b^2/(d*x)^(1/2)*ln(x^n)^2+2/49*d^3*b*x^4*(-7*I*b*Pi*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)+7*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+7*I*b*Pi*csgn(I
*x^n)*csgn(I*c*x^n)^2-7*I*b*Pi*csgn(I*c*x^n)^3+14*b*ln(c)-4*b*n+14*a)/(d*x)
^(1/2)*ln(x^n)+1/686*d^3*(196*a^2+56*I*Pi*b^2*n*csgn(I*c*x^n)^3+98*Pi^2*b^2
*csgn(I*c)*csgn(I*c*x^n)^5+32*b^2*n^2-196*I*ln(c)*Pi*b^2*csgn(I*c)*csgn(I*x
```

```
^n)*csgn(I*c*x^n)-196*I*Pi*a*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+56*I*Pi*
b^2*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+392*ln(c)*a*b+196*ln(c)^2*b^2-112
*b^2*ln(c)*n-112*a*b*n-56*I*Pi*b^2*n*csgn(I*c)*csgn(I*c*x^n)^2-56*I*Pi*b^2*
n*csgn(I*x^n)*csgn(I*c*x^n)^2+196*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2-49*P
i^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+98*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)
^5+98*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-196*Pi^2*b^2*csgn(I*
c)*csgn(I*x^n)*csgn(I*c*x^n)^4+196*I*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)
^2-196*I*ln(c)*Pi*b^2*csgn(I*c*x^n)^3-196*I*Pi*a*b*csgn(I*c*x^n)^3+196*I*Pi
*a*b*csgn(I*c)*csgn(I*c*x^n)^2+98*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c
*x^n)^3-49*Pi^2*b^2*csgn(I*c*x^n)^6+196*I*ln(c)*Pi*b^2*csgn(I*c)*csgn(I*c*x
^n)^2-49*Pi^2*b^2*csgn(I*c)^2*csgn(I*c*x^n)^4-49*Pi^2*b^2*csgn(I*c)^2*csgn(
I*x^n)^2*csgn(I*c*x^n)^2)*x^4/(d*x)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(61) = 122$.

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.93

$$\int (dx)^{5/2} (a + b\log(cx^n))^2 \, dx = \frac{2}{343} \left(49 b^2 d^2 n^2 x^3 \log(x)^2 + 49 b^2 d^2 x^3 \log(c)^2 - 14 \left(2 b^2 d^2 n - 7 abd^2\right) x^3 \log(c) + \left(8 b^2 d^2 n\right.\right.$$

```
[In] integrate((d*x)^(5/2)*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 2/343*(49*b^2*d^2*n^2*x^3*log(x)^2 + 49*b^2*d^2*x^3*log(c)^2 - 14*(2*b^2*d^
2*n - 7*a*b*d^2)*x^3*log(c) + (8*b^2*d^2*n^2 - 28*a*b*d^2*n + 49*a^2*d^2)*x
^3 + 14*(7*b^2*d^2*n*x^3*log(c) - (2*b^2*d^2*n^2 - 7*a*b*d^2*n)*x^3)*log(x)
)*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 22.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int (dx)^{5/2} (a + b\log(cx^n))^2 \, dx = \frac{2a^2x(dx)^{\frac{5}{2}}}{7} - \frac{8abnx(dx)^{\frac{5}{2}}}{49} + \frac{4abx(dx)^{\frac{5}{2}}\log(cx^n)}{7} + \frac{16b^2n^2x(dx)^{\frac{5}{2}}}{343} - \frac{8b^2nx(dx)^{\frac{5}{2}}\log(cx^n)}{49} + \frac{2b^2x(dx)^{\frac{5}{2}}\log(cx^n)^2}{7}$$

```
[In] integrate((d*x)**(5/2)*(a+b*ln(c*x**n))**2,x)
```

```
[Out] 2*a**2*x*(d*x)**(5/2)/7 - 8*a*b*n*x*(d*x)**(5/2)/49 + 4*a*b*x*(d*x)**(5/2)*
log(c*x**n)/7 + 16*b**2*n**2*x*(d*x)**(5/2)/343 - 8*b**2*n*x*(d*x)**(5/2)*l
og(c*x**n)/49 + 2*b**2*x*(d*x)**(5/2)*log(c*x**n)**2/7
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int (dx)^{5/2} (a + b\log(cx^n))^2 \, dx = \frac{2 (dx)^{\frac{7}{2}} b^2 \log(cx^n)^2}{7 d} - \frac{8 (dx)^{\frac{7}{2}} abn}{49 d} + \frac{4 (dx)^{\frac{7}{2}} ab \log(cx^n)}{7 d} + \frac{2 (dx)^{\frac{7}{2}} a^2}{7 d} + \frac{8}{343} \left(\frac{2 (dx)^{\frac{7}{2}} n^2}{d} - \frac{7 (dx)^{\frac{7}{2}} n \log(cx^n)}{d} \right) b^2$$

```
[In] integrate((d*x)^(5/2)*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 2/7*(d*x)^(7/2)*b^2*log(c*x^n)^2/d - 8/49*(d*x)^(7/2)*a*b*n/d + 4/7*(d*x)^(
7/2)*a*b*log(c*x^n)/d + 2/7*(d*x)^(7/2)*a^2/d + 8/343*(2*(d*x)^(7/2)*n^2/d
- 7*(d*x)^(7/2)*n*log(c*x^n)/d)*b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 425, normalized size of antiderivative = 5.82

$$\int (dx)^{5/2} (a+b\log(cx^n))^2 \, dx = \left(\frac{1}{7}i + \frac{1}{7}\right)\sqrt{2}b^2d^2n^2x^{\frac{7}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)\log(x)^2 - \left(\frac{1}{7}i - \frac{1}{7}\right)\sqrt{2}b^2d^2n^2x^{\frac{7}{2}}\sqrt{|d|}\log(x)^2\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) - \left(\frac{4}{49}i + \frac{4}{49}\right)\sqrt{2}b^2d^2n^2x^{\frac{7}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)\log(x) + \left(\frac{2}{7}i + \frac{2}{7}\right)\sqrt{2}b^2d^2nx^{\frac{7}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)\log(c)\log(x) + \left(\frac{4}{49}i - \frac{4}{49}\right)\sqrt{2}b^2d^2n^2x^{\frac{7}{2}}\sqrt{|d|}\log(x)\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) - \left(\frac{2}{7}i - \frac{2}{7}\right)\sqrt{2}b^2d^2nx^{\frac{7}{2}}\sqrt{|d|}\log(c)\log(x)\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) + \left(\frac{8}{343}i + \frac{8}{343}\right)\sqrt{2}b^2d^2n^2x^{\frac{7}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) - \left(\frac{4}{49}i + \frac{4}{49}\right)\sqrt{2}b^2d^2nx^{\frac{7}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)\log(c) + \left(\frac{2}{7}i + \frac{2}{7}\right)\sqrt{2}abd^2nx^{\frac{7}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)\log(x) - \left(\frac{8}{343}i - \frac{8}{343}\right)\sqrt{2}b^2d^2n^2x^{\frac{7}{2}}\sqrt{|d|}\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) + \left(\frac{4}{49}i - \frac{4}{49}\right)\sqrt{2}b^2d^2nx^{\frac{7}{2}}\sqrt{|d|}\log(c)\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) - \left(\frac{2}{7}i - \frac{2}{7}\right)\sqrt{2}abd^2nx^{\frac{7}{2}}\sqrt{|d|}\log(x)\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) - \left(\frac{4}{49}i + \frac{4}{49}\right)\sqrt{2}abd^2nx^{\frac{7}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) + \left(\frac{4}{49}i - \frac{4}{49}\right)\sqrt{2}abd^2nx^{\frac{7}{2}}\sqrt{|d|}\sin\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right) + \frac{2}{7}b^2d^{\frac{5}{2}}x^{\frac{7}{2}}\log(c)^2 + \frac{4}{7}abd^{\frac{5}{2}}x^{\frac{7}{2}}\log(c) + \frac{2}{7}a^2d^{\frac{5}{2}}x^{\frac{7}{2}}$$

```
[In] integrate((d*x)^(5/2)*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] (1/7*I + 1/7)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*l
og(x)^2 - (1/7*I - 1/7)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d))*log(x)^2*s
in(1/4*pi*sgn(d)) - (4/49*I + 4/49)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d)
)*cos(1/4*pi*sgn(d))*log(x) + (2/7*I + 2/7)*sqrt(2)*b^2*d^2*n*x^(7/2)*sqrt(
abs(d))*cos(1/4*pi*sgn(d))*log(c)*log(x) + (4/49*I - 4/49)*sqrt(2)*b^2*d^2*
```

```
n^2*x^(7/2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d)) - (2/7*I - 2/7)*sqrt(2)*
b^2*d^2*n*x^(7/2)*sqrt(abs(d))*log(c)*log(x)*sin(1/4*pi*sgn(d)) + (8/343*I
+ 8/343)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) - (4/4
9*I + 4/49)*sqrt(2)*b^2*d^2*n*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(c
) + (2/7*I + 2/7)*sqrt(2)*a*b*d^2*n*x^(7/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))
*log(x) - (8/343*I - 8/343)*sqrt(2)*b^2*d^2*n^2*x^(7/2)*sqrt(abs(d))*sin(1/
4*pi*sgn(d)) + (4/49*I - 4/49)*sqrt(2)*b^2*d^2*n*x^(7/2)*sqrt(abs(d))*log(c
)*sin(1/4*pi*sgn(d)) - (2/7*I - 2/7)*sqrt(2)*a*b*d^2*n*x^(7/2)*sqrt(abs(d))
*log(x)*sin(1/4*pi*sgn(d)) - (4/49*I + 4/49)*sqrt(2)*a*b*d^2*n*x^(7/2)*sqrt
(abs(d))*cos(1/4*pi*sgn(d)) + (4/49*I - 4/49)*sqrt(2)*a*b*d^2*n*x^(7/2)*sqr
t(abs(d))*sin(1/4*pi*sgn(d)) + 2/7*b^2*d^(5/2)*x^(7/2)*log(c)^2 + 4/7*a*b*d
^(5/2)*x^(7/2)*log(c) + 2/7*a^2*d^(5/2)*x^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2} \left(a+b\log\left(cx^n\right)\right)^2\,dx=\int {\left(d\,x\right)}^{5/2}\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2 \,d x$$

```
[In] int((d*x)^(5/2)*(a + b*log(c*x^n))^2,x)
[Out] int((d*x)^(5/2)*(a + b*log(c*x^n))^2, x)
```

3.96 $\int (dx)^{3/2} (a + b\log(cx^n))^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int (dx)^{3/2} (a + b\log(cx^n))^2 \, dx = \frac{16b^2n^2(dx)^{5/2}}{125d} - \frac{8bn(dx)^{5/2}(a + b\log(cx^n))}{25d} + \frac{2(dx)^{5/2}(a + b\log(cx^n))^2}{5d}$$

```
[Out] 16/125*b^2*n^2*(d*x)^(5/2)/d-8/25*b*n*(d*x)^(5/2)*(a+b*ln(c*x^n))/d+2/5*(d*
x)^(5/2)*(a+b*ln(c*x^n))^2/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2342, 2341}

$$\int (dx)^{3/2} (a + b\log(cx^n))^2 \, dx = \frac{2(dx)^{5/2}(a + b\log(cx^n))^2}{5d} - \frac{8bn(dx)^{5/2}(a + b\log(cx^n))}{25d} + \frac{16b^2n^2(dx)^{5/2}}{125d}$$

```
[In] Int[(d*x)^(3/2)*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (16*b^2*n^2*(d*x)^(5/2))/(125*d) - (8*b*n*(d*x)^(5/2)*(a + b*Log[c*x^n]))/(
25*d) + (2*(d*x)^(5/2)*(a + b*Log[c*x^n])^2)/(5*d)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{2(dx)^{5/2}(a+b\log(cx^n))^2}{5d} - \frac{1}{5}(4bn)\int (dx)^{3/2}(a+b\log(cx^n))\,dx$$
$$= \frac{16b^2n^2(dx)^{5/2}}{125d} - \frac{8bn(dx)^{5/2}(a+b\log(cx^n))}{25d} + \frac{2(dx)^{5/2}(a+b\log(cx^n))^2}{5d}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int (dx)^{3/2}(a+b\log(cx^n))^2\,dx = \frac{2}{125}x(dx)^{3/2}\left(25a^2-20abn+8b^2n^2+10b(5a-2bn)\log(cx^n)+25b^2\log^2(cx^n)\right)$$

```
[In] Integrate[(d*x)^(3/2)*(a + b*Log[c*x^n])^2,x]
[Out] (2*x*(d*x)^(3/2)*(25*a^2 - 20*a*b*n + 8*b^2*n^2 + 10*b*(5*a - 2*b*n)*Log[c*
x^n] + 25*b^2*Log[c*x^n]^2))/125
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 716, normalized size of antiderivative = 9.81

method	result
risch	$\frac{2d^2b^2x^3\ln(x^n)^2}{5\sqrt{dx}} + \frac{2d^2bx^3\left(-5ib\pi\,\text{csgn}(ic)\,\text{csgn}(ix^n)\,\text{csgn}(icx^n)+5ib\pi\,\text{csgn}(ic)\,\text{csgn}(icx^n)^2+5ib\pi\,\text{csgn}(ix^n)\,\text{csgn}(icx^n)^2-5ib\pi\,\text{csg}\right.}{25\sqrt{dx}}$

```
[In] int((d*x)^(3/2)*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
[Out] 2/5*d^2*b^2*x^3/(d*x)^(1/2)*ln(x^n)^2+2/25*d^2*b*x^3*(-5*I*b*Pi*csgn(I*c)*c
sgn(I*x^n)*csgn(I*c*x^n)+5*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+5*I*b*Pi*csgn(I
*x^n)*csgn(I*c*x^n)^2-5*I*b*Pi*csgn(I*c*x^n)^3+10*b*ln(c)-4*b*n+10*a)/(d*x)
^(1/2)*ln(x^n)+1/250*d^2*(100*a^2+50*Pi^2*b^2*csgn(I*c)*csgn(I*c*x^n)^5+40*
I*Pi*b^2*n*csgn(I*c*x^n)^3+32*b^2*n^2+40*I*Pi*b^2*n*csgn(I*c)*csgn(I*x^n)*c
```

```
sgn(I*c*x^n)-100*I*ln(c)*Pi*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-100*I*P
i*a*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+200*ln(c)*a*b+100*ln(c)^2*b^2-80*
b^2*ln(c)*n-80*a*b*n-25*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+50*Pi^2*b^2*
csgn(I*x^n)*csgn(I*c*x^n)^5+50*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^
n)^3-100*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4+100*I*ln(c)*Pi*b^2*
csgn(I*c)*csgn(I*c*x^n)^2+100*I*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+10
0*I*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2+100*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)
^2+50*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-25*Pi^2*b^2*csgn(I*c
*x^n)^6-25*Pi^2*b^2*csgn(I*c)^2*csgn(I*c*x^n)^4-25*Pi^2*b^2*csgn(I*c)^2*csg
n(I*x^n)^2*csgn(I*c*x^n)^2-40*I*Pi*b^2*n*csgn(I*c)*csgn(I*c*x^n)^2-40*I*Pi*
b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-100*I*ln(c)*Pi*b^2*csgn(I*c*x^n)^3-100*I*
Pi*a*b*csgn(I*c*x^n)^3)*x^3/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.66

$$\int (dx)^{3/2} (a +b\log (cx^n))^2 \, dx = \frac{2}{125} \left(25 \, b^2 d n^2 x^2 \log (x)^2 + 25 \, b^2 d x^2 \log (c)^2 - 10 \left(2 \, b^2 d n - 5 \, a b d\right) x^2 \log (c) + \left(8 \, b^2 d n^2 - \right.\right.$$

```
[In] integrate((d*x)^(3/2)*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 2/125*(25*b^2*d*n^2*x^2*log(x)^2 + 25*b^2*d*x^2*log(c)^2 - 10*(2*b^2*d*n -
5*a*b*d)*x^2*log(c) + (8*b^2*d*n^2 - 20*a*b*d*n + 25*a^2*d)*x^2 + 10*(5*b^2
*d*n*x^2*log(c) - (2*b^2*d*n^2 - 5*a*b*d*n)*x^2)*log(x))*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int (dx)^{3/2} (a + b \log (cx^n))^2 \, dx = \frac{2a^2 x (dx)^{\frac{3}{2}}}{5} - \frac{8abnx(dx)^{\frac{3}{2}}}{25} + \frac{4abx(dx)^{\frac{3}{2}} \log (cx^n)}{5} + \frac{16b^2 n^2 x(dx)^{\frac{3}{2}}}{125} - \frac{8b^2 nx(dx)^{\frac{3}{2}} \log (cx^n)}{25} + \frac{2b^2 x(dx)^{\frac{3}{2}} \log (cx^n)^2}{5}$$

```
[In] integrate((d*x)**(3/2)*(a+b*ln(c*x**n))**2,x)
```

```
[Out] 2*a**2*x*(d*x)**(3/2)/5 - 8*a*b*n*x*(d*x)**(3/2)/25 + 4*a*b*x*(d*x)**(3/2)*
log(c*x**n)/5 + 16*b**2*n**2*x*(d*x)**(3/2)/125 - 8*b**2*n*x*(d*x)**(3/2)*l
og(c*x**n)/25 + 2*b**2*x*(d*x)**(3/2)*log(c*x**n)**2/5
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int (dx)^{3/2} (a+b\log(cx^n))^2\,dx = \frac{2(dx)^{\frac{5}{2}}b^2\log(cx^n)^2}{5d} - \frac{8(dx)^{\frac{5}{2}}abn}{25d} + \frac{4(dx)^{\frac{5}{2}}ab\log(cx^n)}{5d} + \frac{2(dx)^{\frac{5}{2}}a^2}{5d} + \frac{8}{125}\left(\frac{2(dx)^{\frac{5}{2}}n^2}{d} - \frac{5(dx)^{\frac{5}{2}}n\log(cx^n)}{d}\right)b^2$$

```
[In] integrate((d*x)^(3/2)*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 2/5*(d*x)^(5/2)*b^2*log(c*x^n)^2/d - 8/25*(d*x)^(5/2)*a*b*n/d + 4/5*(d*x)^(
5/2)*a*b*log(c*x^n)/d + 2/5*(d*x)^(5/2)*a^2/d + 8/125*(2*(d*x)^(5/2)*n^2/d
- 5*(d*x)^(5/2)*n*log(c*x^n)/d)*b^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 386, normalized size of antiderivative = 5.29

$$\int (dx)^{3/2} (a+b\log(cx^n))^2\,dx = -\frac{1}{125}\left(-(25i+25)\sqrt{2}b^2n^2x^{\frac{5}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\mathrm{sgn}(d)\right)\log(x)^2 + (25i-25)\sqrt{2}b^2n^2x^{\frac{5}{2}}\sqrt{|d|}\log(x)^2\sin\left(\frac{1}{4}\pi\mathrm{sg}\right.\right.$$

```
[In] integrate((d*x)^(3/2)*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] -1/125*(-(25*I + 25)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)
)*log(x)^2 + (25*I - 25)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*log(x)^2*sin(
1/4*pi*sgn(d)) + (20*I + 20)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*cos(1/4*p
i*sgn(d))*log(x) - (50*I + 50)*sqrt(2)*b^2*n*x^(5/2)*sqrt(abs(d))*cos(1/4*p
i*sgn(d))*log(c)*log(x) - (20*I - 20)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*
log(x)*sin(1/4*pi*sgn(d)) + (50*I - 50)*sqrt(2)*b^2*n*x^(5/2)*sqrt(abs(d))*
log(c)*log(x)*sin(1/4*pi*sgn(d)) - (8*I + 8)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(a
bs(d))*cos(1/4*pi*sgn(d)) + (20*I + 20)*sqrt(2)*b^2*n*x^(5/2)*sqrt(abs(d))*
cos(1/4*pi*sgn(d))*log(c) - (50*I + 50)*sqrt(2)*a*b*n*x^(5/2)*sqrt(abs(d))*
cos(1/4*pi*sgn(d))*log(x) + (8*I - 8)*sqrt(2)*b^2*n^2*x^(5/2)*sqrt(abs(d))*
sin(1/4*pi*sgn(d)) - (20*I - 20)*sqrt(2)*b^2*n*x^(5/2)*sqrt(abs(d))*log(c)*
sin(1/4*pi*sgn(d)) + (50*I - 50)*sqrt(2)*a*b*n*x^(5/2)*sqrt(abs(d))*log(x)*
sin(1/4*pi*sgn(d)) + (20*I + 20)*sqrt(2)*a*b*n*x^(5/2)*sqrt(abs(d))*cos(1/4
*pi*sgn(d)) - (20*I - 20)*sqrt(2)*a*b*n*x^(5/2)*sqrt(abs(d))*sin(1/4*pi*sgn
(d)) - 50*b^2*sqrt(d)*x^(5/2)*log(c)^2 - 100*a*b*sqrt(d)*x^(5/2)*log(c) - 5
0*a^2*sqrt(d)*x^(5/2))*d
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} (a + b \log (cx^n))^2 \, dx = \int (d\,x)^{3/2} \left(a + b\,\ln\left(c\,x^n\right)\right)^2 dx$$

```
[In] int((d*x)^(3/2)*(a + b*log(c*x^n))^2,x)
[Out] int((d*x)^(3/2)*(a + b*log(c*x^n))^2, x)
```

3.97 $\int \sqrt{dx}(a+b\log(cx^n))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \sqrt{dx}(a+b\log(cx^n))^2\,dx = \frac{16b^2n^2(dx)^{3/2}}{27d} - \frac{8bn(dx)^{3/2}(a+b\log(cx^n))}{9d} + \frac{2(dx)^{3/2}(a+b\log(cx^n))^2}{3d}$$

```
[Out] 16/27*b^2*n^2*(d*x)^(3/2)/d-8/9*b*n*(d*x)^(3/2)*(a+b*ln(c*x^n))/d+2/3*(d*x)
^(3/2)*(a+b*ln(c*x^n))^2/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2342, 2341}

$$\int \sqrt{dx}(a+b\log(cx^n))^2\,dx = -\frac{8bn(dx)^{3/2}(a+b\log(cx^n))}{9d} + \frac{2(dx)^{3/2}(a+b\log(cx^n))^2}{3d} + \frac{16b^2n^2(dx)^{3/2}}{27d}$$

```
[In] Int[Sqrt[d*x]*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (16*b^2*n^2*(d*x)^(3/2))/(27*d) - (8*b*n*(d*x)^(3/2)*(a + b*Log[c*x^n]))/(9
*d) + (2*(d*x)^(3/2)*(a + b*Log[c*x^n])^2)/(3*d)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)^2}{3d}-\frac{1}{3}(4bn)\int\sqrt{dx}\left(a+b\log\left(cx^n\right)\right)\,dx\\ &= \frac{16b^2n^2(dx)^{3/2}}{27d}-\frac{8bn(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)}{9d}+\frac{2(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)^2}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int\sqrt{dx}\left(a+b\log\left(cx^n\right)\right)^2\,dx=\frac{2}{27}x\sqrt{dx}\left(9a^2-12abn+8b^2n^2+6b(3a-2bn)\log\left(cx^n\right)+9b^2\log^2\left(cx^n\right)\right)$$

```
[In] Integrate[Sqrt[d*x]*(a + b*Log[c*x^n])^2,x]
[Out] (2*x*Sqrt[d*x]*(9*a^2 - 12*a*b*n + 8*b^2*n^2 + 6*b*(3*a - 2*b*n)*Log[c*x^n]
 + 9*b^2*Log[c*x^n]^2))/27
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 710, normalized size of antiderivative = 9.73

method	result
risch	$\frac{2d\,b^2x^2\ln(x^n)^2}{3\sqrt{dx}}+\frac{2db\,x^2\left(-3ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+3ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+3ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-3ib\pi\ \mathrm{csg}}{9\sqrt{dx}}$

```
[In] int((d*x)^(1/2)*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
[Out] 2/3*d*b^2*x^2/(d*x)^(1/2)*ln(x^n)^2+2/9*d*b*x^2*(-3*I*b*Pi*csgn(I*c)*csgn(I
*x^n)*csgn(I*c*x^n)+3*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+3*I*b*Pi*csgn(I*x^n)
*csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*c*x^n)^3+6*b*ln(c)-4*b*n+6*a)/(d*x)^(1/2)*
```

```
ln(x^n)+1/54*d*(36*a^2+18*Pi^2*b^2*csgn(I*c)*csgn(I*c*x^n)^5+24*I*Pi*b^2*n*
csgn(I*c*x^n)^3+32*b^2*n^2+36*I*ln(c)*Pi*b^2*csgn(I*c)*csgn(I*c*x^n)^2+36*I
*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*c*x^n)^2+36*I*Pi*a*b*csgn(I*c)*csgn(I*c*x^
n)^2+72*ln(c)*a*b+36*ln(c)^2*b^2-48*b^2*ln(c)*n-48*a*b*n-36*I*ln(c)*Pi*b^2*
csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-36*I*Pi*a*b*csgn(I*c)*csgn(I*x^n)*csgn(
I*c*x^n)-9*Pi^2*b^2*csgn(I*x^n)^2*csgn(I*c*x^n)^4+18*Pi^2*b^2*csgn(I*x^n)*c
sgn(I*c*x^n)^5+18*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-36*Pi^2*
b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^4-24*I*Pi*b^2*n*csgn(I*c)*csgn(I*c*
x^n)^2-24*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2+18*Pi^2*b^2*csgn(I*c)^2*cs
gn(I*x^n)*csgn(I*c*x^n)^3-9*Pi^2*b^2*csgn(I*c*x^n)^6-36*I*ln(c)*Pi*b^2*csgn
(I*c*x^n)^3-36*I*Pi*a*b*csgn(I*c*x^n)^3-9*Pi^2*b^2*csgn(I*c)^2*csgn(I*c*x^n
)^4-9*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)^2*csgn(I*c*x^n)^2+24*I*Pi*b^2*n*csgn
(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+36*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2)*x^
2/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int \sqrt{dx}(a+b\log(cx^n))^2\,dx$$
$$=\frac{2}{27}\left(9\,b^2n^2x\log(x)^2+9\,b^2x\log(c)^2-6\left(2\,b^2n-3\,ab\right)x\log(c)+\left(8\,b^2n^2-12\,abn+9\,a^2\right)x+6\left(3\,b^2nx\log(\right.\right.$$

```
[In] integrate((d*x)^(1/2)*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 2/27*(9*b^2*n^2*x*log(x)^2 + 9*b^2*x*log(c)^2 - 6*(2*b^2*n - 3*a*b)*x*log(c
) + (8*b^2*n^2 - 12*a*b*n + 9*a^2)*x + 6*(3*b^2*n*x*log(c) - (2*b^2*n^2 - 3
*a*b*n)*x)*log(x))*sqrt(d*x)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \sqrt{dx}(a+b\log(cx^n))^2\,dx=\frac{2a^2x\sqrt{dx}}{3}-\frac{8abnx\sqrt{dx}}{9}+\frac{4abx\sqrt{dx}\log(cx^n)}{3}+\frac{16b^2n^2x\sqrt{dx}}{27}-\frac{8b^2nx\sqrt{dx}\log(cx^n)}{9}+\frac{2b^2x\sqrt{dx}\log(cx^n)^2}{3}$$

```
[In] integrate((d*x)**(1/2)*(a+b*ln(c*x**n))**2,x)
```

```
[Out] 2*a**2*x*sqrt(d*x)/3 - 8*a*b*n*x*sqrt(d*x)/9 + 4*a*b*x*sqrt(d*x)*log(c*x**n
)/3 + 16*b**2*n**2*x*sqrt(d*x)/27 - 8*b**2*n*x*sqrt(d*x)*log(c*x**n)/9 + 2*
b**2*x*sqrt(d*x)*log(c*x**n)**2/3
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \sqrt{dx}(a+b\log(cx^n))^2\,dx = \frac{2(dx)^{\frac{3}{2}}b^2\log(cx^n)^2}{3d} - \frac{8(dx)^{\frac{3}{2}}abn}{9d} + \frac{4(dx)^{\frac{3}{2}}ab\log(cx^n)}{3d} + \frac{8}{27}\left(\frac{2(dx)^{\frac{3}{2}}n^2}{d} - \frac{3(dx)^{\frac{3}{2}}n\log(cx^n)}{d}\right)b^2 + \frac{2(dx)^{\frac{3}{2}}a^2}{3d}$$

```
[In] integrate((d*x)^(1/2)*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(d*x)^(3/2)*b^2*log(c*x^n)^2/d - 8/9*(d*x)^(3/2)*a*b*n/d + 4/3*(d*x)^(3
/2)*a*b*log(c*x^n)/d + 8/27*(2*(d*x)^(3/2)*n^2/d - 3*(d*x)^(3/2)*n*log(c*x^
n)/d)*b^2 + 2/3*(d*x)^(3/2)*a^2/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 383, normalized size of antiderivative = 5.25

$$\int \sqrt{dx}(a+b\log(cx^n))^2\,dx = \left(\frac{1}{3}i+\frac{1}{3}\right)\sqrt{2}b^2n^2x^{\frac{3}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\text{sgn}(d)\right)\log(x)^2 - \left(\frac{1}{3}i-\frac{1}{3}\right)\sqrt{2}b^2n^2x^{\frac{3}{2}}\sqrt{|d|}\log(x)^2\sin\left(\frac{1}{4}\pi\text{sgn}(d)\right) - \left(\frac{4}{9}i+\frac{4}{9}\right)\sqrt{2}b^2n^2x^{\frac{3}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\text{sgn}(d)\right)\log(x) + \left(\frac{2}{3}i+\frac{2}{3}\right)\sqrt{2}b^2nx^{\frac{3}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\text{sgn}(d)\right)\log(c)\log(x) + \left(\frac{4}{9}i-\frac{4}{9}\right)\sqrt{2}b^2n^2x^{\frac{3}{2}}\sqrt{|d|}\log(x)\sin\left(\frac{1}{4}\pi\text{sgn}(d)\right) - \left(\frac{2}{3}i-\frac{2}{3}\right)\sqrt{2}b^2nx^{\frac{3}{2}}\sqrt{|d|}\log(c)\log(x)\sin\left(\frac{1}{4}\pi\text{sgn}(d)\right) + \left(\frac{8}{27}i+\frac{8}{27}\right)\sqrt{2}b^2n^2x^{\frac{3}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\text{sgn}(d)\right) - \left(\frac{4}{9}i+\frac{4}{9}\right)\sqrt{2}b^2nx^{\frac{3}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\text{sgn}(d)\right)\log(c) + \left(\frac{2}{3}i+\frac{2}{3}\right)\sqrt{2}abnx^{\frac{3}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\text{sgn}(d)\right)\log(x) - \left(\frac{8}{27}i-\frac{8}{27}\right)\sqrt{2}b^2n^2x^{\frac{3}{2}}\sqrt{|d|}\sin\left(\frac{1}{4}\pi\text{sgn}(d)\right) + \left(\frac{4}{9}i-\frac{4}{9}\right)\sqrt{2}b^2nx^{\frac{3}{2}}\sqrt{|d|}\log(c)\sin\left(\frac{1}{4}\pi\text{sgn}(d)\right) - \left(\frac{2}{3}i-\frac{2}{3}\right)\sqrt{2}abnx^{\frac{3}{2}}\sqrt{|d|}\log(x)\sin\left(\frac{1}{4}\pi\text{sgn}(d)\right) - \left(\frac{4}{9}i+\frac{4}{9}\right)\sqrt{2}abnx^{\frac{3}{2}}\sqrt{|d|}\cos\left(\frac{1}{4}\pi\text{sgn}(d)\right) + \left(\frac{4}{9}i-\frac{4}{9}\right)\sqrt{2}abnx^{\frac{3}{2}}\sqrt{|d|}\sin\left(\frac{1}{4}\pi\text{sgn}(d)\right) + \frac{2}{3}b^2\sqrt{d}x^{\frac{3}{2}}\log(c)^2 + \frac{4}{3}ab\sqrt{d}x^{\frac{3}{2}}\log(c) + \frac{2}{3}a^2\sqrt{d}x^{\frac{3}{2}}$$

```
[In] integrate((d*x)^(1/2)*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] (1/3*I + 1/3)*sqrt(2)*b^2*n^2*x^(3/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x
)^2 - (1/3*I - 1/3)*sqrt(2)*b^2*n^2*x^(3/2)*sqrt(abs(d))*log(x)^2*sin(1/4*p
i*sgn(d)) - (4/9*I + 4/9)*sqrt(2)*b^2*n^2*x^(3/2)*sqrt(abs(d))*cos(1/4*pi*s
gn(d))*log(x) + (2/3*I + 2/3)*sqrt(2)*b^2*n*x^(3/2)*sqrt(abs(d))*cos(1/4*pi
*sgn(d))*log(c)*log(x) + (4/9*I - 4/9)*sqrt(2)*b^2*n^2*x^(3/2)*sqrt(abs(d))
*log(x)*sin(1/4*pi*sgn(d)) - (2/3*I - 2/3)*sqrt(2)*b^2*n*x^(3/2)*sqrt(abs(d
))*log(c)*log(x)*sin(1/4*pi*sgn(d)) + (8/27*I + 8/27)*sqrt(2)*b^2*n^2*x^(3/
```

```
2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) - (4/9*I + 4/9)*sqrt(2)*b^2*n*x^(3/2)*sq
rt(abs(d))*cos(1/4*pi*sgn(d))*log(c) + (2/3*I + 2/3)*sqrt(2)*a*b*n*x^(3/2)*
sqrt(abs(d))*cos(1/4*pi*sgn(d))*log(x) - (8/27*I - 8/27)*sqrt(2)*b^2*n^2*x^
(3/2)*sqrt(abs(d))*sin(1/4*pi*sgn(d)) + (4/9*I - 4/9)*sqrt(2)*b^2*n*x^(3/2)
*sqrt(abs(d))*log(c)*sin(1/4*pi*sgn(d)) - (2/3*I - 2/3)*sqrt(2)*a*b*n*x^(3/
2)*sqrt(abs(d))*log(x)*sin(1/4*pi*sgn(d)) - (4/9*I + 4/9)*sqrt(2)*a*b*n*x^(
3/2)*sqrt(abs(d))*cos(1/4*pi*sgn(d)) + (4/9*I - 4/9)*sqrt(2)*a*b*n*x^(3/2)*
sqrt(abs(d))*sin(1/4*pi*sgn(d)) + 2/3*b^2*sqrt(d)*x^(3/2)*log(c)^2 + 4/3*a*
b*sqrt(d)*x^(3/2)*log(c) + 2/3*a^2*sqrt(d)*x^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a+b\log(cx^n))^2\,dx = \int \sqrt{d\,x}\,(a+b\,\ln(c\,x^n))^2\,dx$$

```
[In] int((d*x)^(1/2)*(a + b*log(c*x^n))^2,x)
```

```
[Out] int((d*x)^(1/2)*(a + b*log(c*x^n))^2, x)
```

3.98 $\int \frac{(a+b\log(cx^n))^2}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{(a+b\log(cx^n))^2}{\sqrt{dx}}\,dx = \frac{16b^2n^2\sqrt{dx}}{d} - \frac{8bn\sqrt{dx}(a+b\log(cx^n))}{d} + \frac{2\sqrt{dx}(a+b\log(cx^n))^2}{d}$$

```
[Out] 16*b^2*n^2*(d*x)^(1/2)/d-8*b*n*(a+b*ln(c*x^n))*(d*x)^(1/2)/d+2*(a+b*ln(c*x^
n))^2*(d*x)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2342, 2341}

$$\int \frac{(a+b\log(cx^n))^2}{\sqrt{dx}}\,dx = -\frac{8bn\sqrt{dx}(a+b\log(cx^n))}{d} + \frac{2\sqrt{dx}(a+b\log(cx^n))^2}{d} + \frac{16b^2n^2\sqrt{dx}}{d}$$

```
[In] Int[(a + b*Log[c*x^n])^2/Sqrt[d*x],x]
```

```
[Out] (16*b^2*n^2*Sqrt[d*x])/d - (8*b*n*Sqrt[d*x]*(a + b*Log[c*x^n]))/d + (2*Sqrt
[d*x]*(a + b*Log[c*x^n])^2)/d
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{dx}(a+b\log(cx^n))^2}{d} - (4bn)\int \frac{a+b\log(cx^n)}{\sqrt{dx}}\,dx \\ &= \frac{16b^2n^2\sqrt{dx}}{d} - \frac{8bn\sqrt{dx}(a+b\log(cx^n))}{d} + \frac{2\sqrt{dx}(a+b\log(cx^n))^2}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{(a+b\log(cx^n))^2}{\sqrt{dx}}\,dx = \frac{2x\left(a^2-4abn+8b^2n^2+2b(a-2bn)\log(cx^n)+b^2\log^2(cx^n)\right)}{\sqrt{dx}}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/Sqrt[d*x],x]
[Out] (2*x*(a^2 - 4*a*b*n + 8*b^2*n^2 + 2*b*(a - 2*b*n)*Log[c*x^n] + b^2*Log[c*x^
n]^2))/Sqrt[d*x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{2\sqrt{dx}\,a^2+2b^2\sqrt{dx}\,\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2+16b^2n^2\sqrt{dx}-8b^2n\sqrt{dx}\,\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+4\sqrt{dx}\,ab\ln(c\,x^n)-8abn\sqrt{dx}}{d}$
default	$\frac{2\sqrt{dx}\,a^2+2b^2\sqrt{dx}\,\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)^2+16b^2n^2\sqrt{dx}-8b^2n\sqrt{dx}\,\ln\left(c\,\mathrm{e}^{n\ln(x)}\right)+4\sqrt{dx}\,ab\ln(c\,x^n)-8abn\sqrt{dx}}{d}$
risch	$\frac{2b^2x\ln(x^n)^2}{\sqrt{dx}}+\frac{2b\left(-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ic\,x^n)^2+ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2-ib\pi\ \mathrm{cs}\right.}{\sqrt{dx}}$

```
[In] int((a+b*ln(c*x^n))^2/(d*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/d*((d*x)^(1/2)*a^2+b^2*(d*x)^(1/2)*ln(c*exp(n*ln(x)))^2+8*b^2*n^2*(d*x)^(
1/2)-4*b^2*n*(d*x)^(1/2)*ln(c*exp(n*ln(x)))+2*(d*x)^(1/2)*a*b*ln(c*x^n)-4*a
*b*n*(d*x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

$$\int \frac{(a + b \log (cx^n))^2}{\sqrt{dx}} \, dx$$
$$= \frac{2 \left(b^2 n^2 \log (x)^2 + 8 \, b^2 n^2 + b^2 \log (c)^2 - 4 \, abn + a^2 - 2 \, (2 \, b^2 n - ab) \log (c) - 2 \, (2 \, b^2 n^2 - b^2 n \log (c) - abn) \log\right.}{d}$$

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(b^2*n^2*log(x)^2 + 8*b^2*n^2 + b^2*log(c)^2 - 4*a*b*n + a^2 - 2*(2*b^2*n
 - a*b)*log(c) - 2*(2*b^2*n^2 - b^2*n*log(c) - a*b*n)*log(x))*sqrt(d*x)/d
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.63

$$\int \frac{(a + b \log (cx^n))^2}{\sqrt{dx}} \, dx = \frac{2a^2x}{\sqrt{dx}} - \frac{8abnx}{\sqrt{dx}} + \frac{4abx \log (cx^n)}{\sqrt{dx}} + \frac{16b^2n^2x}{\sqrt{dx}} - \frac{8b^2nx \log (cx^n)}{\sqrt{dx}} + \frac{2b^2x \log (cx^n)^2}{\sqrt{dx}}$$

```
[In] integrate((a+b*ln(c*x**n))**2/(d*x)**(1/2),x)
```

```
[Out] 2*a**2*x/sqrt(d*x) - 8*a*b*n*x/sqrt(d*x) + 4*a*b*x*log(c*x**n)/sqrt(d*x) +
16*b**2*n**2*x/sqrt(d*x) - 8*b**2*n*x*log(c*x**n)/sqrt(d*x) + 2*b**2*x*log(
c*x**n)**2/sqrt(d*x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int \frac{(a + b \log (cx^n))^2}{\sqrt{dx}} \, dx = \frac{2 \sqrt{dx} b^2 \log (cx^n)^2}{d} + 8 \left(\frac{2 \sqrt{dx} n^2}{d} - \frac{\sqrt{dx} n \log (cx^n)}{d}\right) b^2 - \frac{8 \sqrt{dx} abn}{d} + \frac{4 \sqrt{dx} ab \log (cx^n)}{d} + \frac{2 \sqrt{dx} a^2}{d}$$

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(d*x)*b^2*log(c*x^n)^2/d + 8*(2*sqrt(d*x)*n^2/d - sqrt(d*x)*n*log(c*x
^n)/d)*b^2 - 8*sqrt(d*x)*a*b*n/d + 4*sqrt(d*x)*a*b*log(c*x^n)/d + 2*sqrt(d*
x)*a^2/d
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \frac{(a+b\log(cx^n))^2}{\sqrt{dx}}\,dx = \frac{2\left(\left(\sqrt{dx}\log(x)^2-4\sqrt{dx}\log(x)+8\sqrt{dx}\right)b^2n^2+2\left(\sqrt{dx}\log(x)-2\sqrt{dx}\right)b^2n\log(c)+\sqrt{dx}b^2\log(c)^2+\cdots\right)}{d}$$

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x)*log(x)^2 - 4*sqrt(d*x)*log(x) + 8*sqrt(d*x))*b^2*n^2 + 2*(sqr
t(d*x)*log(x) - 2*sqrt(d*x))*b^2*n*log(c) + sqrt(d*x)*b^2*log(c)^2 + 2*(sqr
t(d*x)*log(x) - 2*sqrt(d*x))*a*b*n + 2*sqrt(d*x)*a*b*log(c) + sqrt(d*x)*a^2
)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{\sqrt{dx}}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{\sqrt{d\,x}}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(d*x)^(1/2),x)
```

```
[Out] int((a + b*log(c*x^n))^2/(d*x)^(1/2), x)
```

3.99 $\int \frac{(a+b\log(cx^n))^2}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{3/2}}\,dx = -\frac{16b^2n^2}{d\sqrt{dx}} - \frac{8bn(a+b\log(cx^n))}{d\sqrt{dx}} - \frac{2(a+b\log(cx^n))^2}{d\sqrt{dx}}$$

```
[Out] -16*b^2*n^2/d/(d*x)^(1/2)-8*b*n*(a+b*ln(c*x^n))/d/(d*x)^(1/2)-2*(a+b*ln(c*x
^n))^2/d/(d*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2342, 2341}

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{3/2}}\,dx = -\frac{8bn(a+b\log(cx^n))}{d\sqrt{dx}} - \frac{2(a+b\log(cx^n))^2}{d\sqrt{dx}} - \frac{16b^2n^2}{d\sqrt{dx}}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(d*x)^(3/2),x]
```

```
[Out] (-16*b^2*n^2)/(d*Sqrt[d*x]) - (8*b*n*(a + b*Log[c*x^n]))/(d*Sqrt[d*x]) - (2
*(a + b*Log[c*x^n])^2)/(d*Sqrt[d*x])
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(a+b\log(cx^n))^2}{d\sqrt{dx}} + (4bn)\int\frac{a+b\log(cx^n)}{(dx)^{3/2}}\,dx\\ &= -\frac{16b^2n^2}{d\sqrt{dx}} - \frac{8bn(a+b\log(cx^n))}{d\sqrt{dx}} - \frac{2(a+b\log(cx^n))^2}{d\sqrt{dx}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int\frac{(a+b\log(cx^n))^2}{(dx)^{3/2}}\,dx = -\frac{2x\left(a^2+4abn+8b^2n^2+2b(a+2bn)\log(cx^n)+b^2\log^2(cx^n)\right)}{(dx)^{3/2}}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(d*x)^(3/2),x]
```

```
[Out] (-2*x*(a^2 + 4*a*b*n + 8*b^2*n^2 + 2*b*(a + 2*b*n)*Log[c*x^n] + b^2*Log[c*x
^n]^2))/(d*x)^(3/2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 707, normalized size of antiderivative = 10.55

method	result
risch	$-\frac{2b^2\ln(x^n)^2}{d\sqrt{dx}} - \frac{2b\left(-ib\pi\ \operatorname{csgn}(ic)\operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)+ib\pi\ \operatorname{csgn}(ic)\operatorname{csgn}(ic\,x^n)^2+ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2-ib\pi\operatorname{csgn}(ic\,x^n)^3 \cdots\right.}{d\sqrt{dx}}$

```
[In] int((a+b*ln(c*x^n))^2/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*b^2/(d*x)^(1/2)*ln(x^n)^2-2/d*b*(-I*b*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*
c*x^n)+I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+I*b*Pi*csgn(I*x^n)*csgn(I*c*x^n)^2-
I*b*Pi*csgn(I*c*x^n)^3+2*b*ln(c)+4*b*n+2*a)/(d*x)^(1/2)*ln(x^n)-1/2/d*(4*a^
2+2*Pi^2*b^2*csgn(I*c)*csgn(I*c*x^n)^5+4*I*ln(c)*Pi*b^2*csgn(I*x^n)*csgn(I*
c*x^n)^2+4*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2+32*b^2*n^2+8*I*Pi*b^2*n*csg
n(I*c)*csgn(I*c*x^n)^2+8*I*Pi*b^2*n*csgn(I*x^n)*csgn(I*c*x^n)^2-4*I*ln(c)*P
i*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+8*ln(c)*a*b+4*ln(c)^2*b^2+16*b^2*
ln(c)*n+16*a*b*n-4*I*Pi*a*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-4*I*ln(c)*P
```

```
i*b^2*csgn(I*c*x^n)^3-4*I*Pi*a*b*csgn(I*c*x^n)^3-Pi^2*b^2*csgn(I*x^n)^2*csg
n(I*c*x^n)^4+2*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5+2*Pi^2*b^2*csgn(I*c)*cs
gn(I*x^n)^2*csgn(I*c*x^n)^3-4*Pi^2*b^2*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)^
4-8*I*Pi*b^2*n*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)+2*Pi^2*b^2*csgn(I*c)^2*c
sgn(I*x^n)*csgn(I*c*x^n)^3-Pi^2*b^2*csgn(I*c*x^n)^6-Pi^2*b^2*csgn(I*c)^2*cs
gn(I*c*x^n)^4+4*I*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2-Pi^2*b^2*csgn(I*c)^2*csg
n(I*x^n)^2*csgn(I*c*x^n)^2-8*I*Pi*b^2*n*csgn(I*c*x^n)^3+4*I*ln(c)*Pi*b^2*cs
gn(I*c)*csgn(I*c*x^n)^2)/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.30

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{3/2}}\,dx = -\frac{2\left(b^2n^2\log(x)^2+8\,b^2n^2+b^2\log(c)^2+4\,abn+a^2+2\,(2\,b^2n+ab)\log(c)+2\,(2\,b^2n^2+b^2n\log(c)+abn)\,\mathrm{lo}\right.}{d^2x}$$

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(b^2*n^2*log(x)^2 + 8*b^2*n^2 + b^2*log(c)^2 + 4*a*b*n + a^2 + 2*(2*b^2*
n + a*b)*log(c) + 2*(2*b^2*n^2 + b^2*n*log(c) + a*b*n)*log(x))*sqrt(d*x)/(d
^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.64

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{3/2}}\,dx = -\frac{2a^2x}{(dx)^{\frac{3}{2}}} - \frac{8abnx}{(dx)^{\frac{3}{2}}} - \frac{4abx\log(cx^n)}{(dx)^{\frac{3}{2}}} - \frac{16b^2n^2x}{(dx)^{\frac{3}{2}}} - \frac{8b^2nx\log(cx^n)}{(dx)^{\frac{3}{2}}} - \frac{2b^2x\log(cx^n)^2}{(dx)^{\frac{3}{2}}}$$

```
[In] integrate((a+b*ln(c*x**n))**2/(d*x)**(3/2),x)
```

```
[Out] -2*a**2*x/(d*x)**(3/2) - 8*a*b*n*x/(d*x)**(3/2) - 4*a*b*x*log(c*x**n)/(d*x)
**(3/2) - 16*b**2*n**2*x/(d*x)**(3/2) - 8*b**2*n*x*log(c*x**n)/(d*x)**(3/2)
 - 2*b**2*x*log(c*x**n)**2/(d*x)**(3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.51

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{3/2}}\,dx = -8\,b^2\left(\frac{2\,n^2}{\sqrt{dx}d}+\frac{n\log(cx^n)}{\sqrt{dx}d}\right) - \frac{2\,b^2\log(cx^n)^2}{\sqrt{dx}d} - \frac{8\,abn}{\sqrt{dx}d} - \frac{4\,ab\log(cx^n)}{\sqrt{dx}d} - \frac{2\,a^2}{\sqrt{dx}d}$$

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -8*b^2*(2*n^2/(sqrt(d*x)*d) + n*log(c*x^n)/(sqrt(d*x)*d)) - 2*b^2*log(c*x^n
)^2/(sqrt(d*x)*d) - 8*a*b*n/(sqrt(d*x)*d) - 4*a*b*log(c*x^n)/(sqrt(d*x)*d)
- 2*a^2/(sqrt(d*x)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(61) = 122.

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.22

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{3/2}}\,dx = -\frac{2\left(\frac{b^2n^2\log(dx)^2}{\sqrt{dx}} - \frac{2\left(b^2n^2\log(d)-2\,b^2n^2-b^2n\log(c)-abn\right)\log(dx)}{\sqrt{dx}} + \frac{b^2n^2\log(d)^2-4\,b^2n^2\log(d)-2\,b^2n\log(c)\log(d)+8\,b^2n^2+4\,b^2n\log(c)+}{\sqrt{dx}}\right.}{d}$$

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(b^2*n^2*log(d*x)^2/sqrt(d*x) - 2*(b^2*n^2*log(d) - 2*b^2*n^2 - b^2*n*lo
g(c) - a*b*n)*log(d*x)/sqrt(d*x) + (b^2*n^2*log(d)^2 - 4*b^2*n^2*log(d) - 2
*b^2*n*log(c)*log(d) + 8*b^2*n^2 + 4*b^2*n*log(c) + b^2*log(c)^2 - 2*a*b*n*
log(d) + 4*a*b*n + 2*a*b*log(c) + a^2)/sqrt(d*x))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{3/2}}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{(d\,x)^{3/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(d*x)^(3/2),x)
```

```
[Out] int((a + b*log(c*x^n))^2/(d*x)^(3/2), x)
```

3.100 $\int \frac{(a+b\log(cx^n))^2}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 73

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{5/2}}\,dx = -\frac{16b^2n^2}{27d(dx)^{3/2}} - \frac{8bn(a+b\log(cx^n))}{9d(dx)^{3/2}} - \frac{2(a+b\log(cx^n))^2}{3d(dx)^{3/2}}$$

```
[Out] -16/27*b^2*n^2/d/(d*x)^(3/2)-8/9*b*n*(a+b*ln(c*x^n))/d/(d*x)^(3/2)-2/3*(a+b
*ln(c*x^n))^2/d/(d*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2342, 2341}

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{5/2}}\,dx = -\frac{8bn(a+b\log(cx^n))}{9d(dx)^{3/2}} - \frac{2(a+b\log(cx^n))^2}{3d(dx)^{3/2}} - \frac{16b^2n^2}{27d(dx)^{3/2}}$$

```
[In] Int[(a + b*Log[c*x^n])^2/(d*x)^(5/2),x]
```

```
[Out] (-16*b^2*n^2)/(27*d*(d*x)^(3/2)) - (8*b*n*(a + b*Log[c*x^n]))/(9*d*(d*x)^(3
/2)) - (2*(a + b*Log[c*x^n])^2)/(3*d*(d*x)^(3/2))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\log(cx^n))^2}{3d(dx)^{3/2}} + \frac{1}{3}(4bn)\int \frac{a+b\log(cx^n)}{(dx)^{5/2}}\,dx$$
$$= -\frac{16b^2n^2}{27d(dx)^{3/2}} - \frac{8bn(a+b\log(cx^n))}{9d(dx)^{3/2}} - \frac{2(a+b\log(cx^n))^2}{3d(dx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{5/2}}\,dx = -\frac{2x(9a^2+12abn+8b^2n^2+6b(3a+2bn)\log(cx^n)+9b^2\log^2(cx^n))}{27(dx)^{5/2}}$$

```
[In] Integrate[(a + b*Log[c*x^n])^2/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(9*a^2 + 12*a*b*n + 8*b^2*n^2 + 6*b*(3*a + 2*b*n)*Log[c*x^n] + 9*b^2*
Log[c*x^n]^2))/(27*(d*x)^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 716, normalized size of antiderivative = 9.81

method	result
risch	$-\frac{2b^2\ln(x^n)^2}{3d^2x\sqrt{dx}} - \frac{2b\left(-3ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)+3ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(icx^n)^2+3ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2-3ib\pi\,\mathrm{csgn}(ic\right.}{9d^2x\sqrt{dx}}$

```
[In] int((a+b*ln(c*x^n))^2/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d^2*b^2/x/(d*x)^(1/2)*ln(x^n)^2-2/9/d^2*b*(-3*I*b*Pi*csgn(I*c)*csgn(I*
x^n)*csgn(I*c*x^n)+3*I*b*Pi*csgn(I*c)*csgn(I*c*x^n)^2+3*I*b*Pi*csgn(I*x^n)*
csgn(I*c*x^n)^2-3*I*b*Pi*csgn(I*c*x^n)^3+6*b*ln(c)+4*b*n+6*a)/x/(d*x)^(1/2)
*ln(x^n)-1/54/d^2*(36*a^2+18*Pi^2*b^2*csgn(I*c)*csgn(I*c*x^n)^5+32*b^2*n^2+
36*I*ln(c)*Pi*b^2*csgn(I*c)*csgn(I*c*x^n)^2+36*I*ln(c)*Pi*b^2*csgn(I*x^n)*c
sgn(I*c*x^n)^2+36*I*Pi*a*b*csgn(I*c)*csgn(I*c*x^n)^2+24*I*Pi*b^2*n*csgn(I*x
^n)*csgn(I*c*x^n)^2+24*I*Pi*b^2*n*csgn(I*c)*csgn(I*c*x^n)^2+72*ln(c)*a*b+36
*ln(c)^2*b^2+48*b^2*ln(c)*n+48*a*b*n-36*I*ln(c)*Pi*b^2*csgn(I*c)*csgn(I*x^n
```

```
)*csgn(I*c*x^n)-36*I*Pi*a*b*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)-9*Pi^2*b^2*
csgn(I*x^n)^2*csgn(I*c*x^n)^4+18*Pi^2*b^2*csgn(I*x^n)*csgn(I*c*x^n)^5+18*Pi
^2*b^2*csgn(I*c)*csgn(I*x^n)^2*csgn(I*c*x^n)^3-36*Pi^2*b^2*csgn(I*c)*csgn(I
*x^n)*csgn(I*c*x^n)^4+18*Pi^2*b^2*csgn(I*c)^2*csgn(I*x^n)*csgn(I*c*x^n)^3-9
*Pi^2*b^2*csgn(I*c*x^n)^6-36*I*ln(c)*Pi*b^2*csgn(I*c*x^n)^3-36*I*Pi*a*b*csg
n(I*c*x^n)^3-9*Pi^2*b^2*csgn(I*c)^2*csgn(I*c*x^n)^4-9*Pi^2*b^2*csgn(I*c)^2*
csgn(I*x^n)^2*csgn(I*c*x^n)^2-24*I*Pi*csgn(I*c)*csgn(I*x^n)*csgn(I*c*x^n)*b
^2*n-24*I*Pi*b^2*n*csgn(I*c*x^n)^3+36*I*Pi*a*b*csgn(I*x^n)*csgn(I*c*x^n)^2)
/x/(d*x)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{5/2}}\,dx = -\frac{2\,\left(9\,b^2n^2\log(x)^2+8\,b^2n^2+9\,b^2\log(c)^2+12\,abn+9\,a^2+6\,(2\,b^2n+3\,ab)\log(c)+6\,(2\,b^2n^2+3\,b^2n\log(c)\cdots}{27\,d^3x^2}$$

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(9*b^2*n^2*log(x)^2 + 8*b^2*n^2 + 9*b^2*log(c)^2 + 12*a*b*n + 9*a^2 +
 6*(2*b^2*n + 3*a*b)*log(c) + 6*(2*b^2*n^2 + 3*b^2*n*log(c) + 3*a*b*n)*log(
x))*sqrt(d*x)/(d^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.66

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{5/2}}\,dx = -\frac{2a^2x}{3\,(dx)^{\frac{5}{2}}}-\frac{8abnx}{9\,(dx)^{\frac{5}{2}}}-\frac{4abx\log(cx^n)}{3\,(dx)^{\frac{5}{2}}} - \frac{16b^2n^2x}{27\,(dx)^{\frac{5}{2}}}-\frac{8b^2nx\log(cx^n)}{9\,(dx)^{\frac{5}{2}}}-\frac{2b^2x\log(cx^n)^2}{3\,(dx)^{\frac{5}{2}}}$$

```
[In] integrate((a+b*ln(c*x**n))**2/(d*x)**(5/2),x)
```

```
[Out] -2*a**2*x/(3*(d*x)**(5/2)) - 8*a*b*n*x/(9*(d*x)**(5/2)) - 4*a*b*x*log(c*x**
n)/(3*(d*x)**(5/2)) - 16*b**2*n**2*x/(27*(d*x)**(5/2)) - 8*b**2*n*x*log(c*x
**n)/(9*(d*x)**(5/2)) - 2*b**2*x*log(c*x**n)**2/(3*(d*x)**(5/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{5/2}}\,dx = -\frac{8}{27}\,b^2\left(\frac{2\,n^2}{(dx)^{\frac{3}{2}}\,d}+\frac{3\,n\log(cx^n)}{(dx)^{\frac{3}{2}}\,d}\right) - \frac{2\,b^2\log(cx^n)^2}{3\,(dx)^{\frac{3}{2}}\,d} - \frac{8\,abn}{9\,(dx)^{\frac{3}{2}}\,d} - \frac{4\,ab\log(cx^n)}{3\,(dx)^{\frac{3}{2}}\,d} - \frac{2\,a^2}{3\,(dx)^{\frac{3}{2}}\,d}$$

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/27*b^2*(2*n^2/((d*x)^(3/2)*d) + 3*n*log(c*x^n)/((d*x)^(3/2)*d)) - 2/3*b^
2*log(c*x^n)^2/((d*x)^(3/2)*d) - 8/9*a*b*n/((d*x)^(3/2)*d) - 4/3*a*b*log(c*
x^n)/((d*x)^(3/2)*d) - 2/3*a^2/((d*x)^(3/2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(61) = 122.

Time = 0.33 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.92

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{5/2}}\,dx = -\frac{2\left(\frac{9\,b^2dn^2\log(dx)^2}{\sqrt{dx}x} - \frac{6\,(3\,b^2d^2n^2\log(d)-2\,b^2d^2n^2-3\,b^2d^2n\log(c)-3\,abd^2n)\log(dx)}{\sqrt{dx}dx} + \frac{9\,b^2d^2n^2\log(d)^2-12\,b^2d^2n^2\log(d)-18\,b^2d^2n\log(c)\,\mathrm{l}}{}\right.}{27\,d^3}$$

```
[In] integrate((a+b*log(c*x^n))^2/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/27*(9*b^2*d*n^2*log(d*x)^2/(sqrt(d*x)*x) - 6*(3*b^2*d^2*n^2*log(d) - 2*b
^2*d^2*n^2 - 3*b^2*d^2*n*log(c) - 3*a*b*d^2*n)*log(d*x)/(sqrt(d*x)*d*x) + (
9*b^2*d^2*n^2*log(d)^2 - 12*b^2*d^2*n^2*log(d) - 18*b^2*d^2*n*log(c)*log(d)
 + 8*b^2*d^2*n^2 + 12*b^2*d^2*n*log(c) + 9*b^2*d^2*log(c)^2 - 18*a*b*d^2*n*
log(d) + 12*a*b*d^2*n + 18*a*b*d^2*log(c) + 9*a^2*d^2)/(sqrt(d*x)*d*x))/d^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^2}{(dx)^{5/2}}\,dx = \int \frac{(a+b\,\ln(c\,x^n))^2}{(d\,x)^{5/2}}\,dx$$

```
[In] int((a + b*log(c*x^n))^2/(d*x)^(5/2),x)
[Out] int((a + b*log(c*x^n))^2/(d*x)^(5/2), x)
```

3.101 $\int \frac{(dx)^{5/2}}{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{(dx)^{5/2}}{a+b\log(cx^n)}\,dx=\frac{e^{-\frac{7a}{2bn}}(dx)^{7/2}\,(cx^n)^{-\frac{7}{2}/n}\,\text{ExpIntegralEi}\left(\frac{7(a+b\log(cx^n))}{2bn}\right)}{bdn}$$

```
[Out] (d*x)^(7/2)*Ei(7/2*(a+b*ln(c*x^n))/b/n)/b/d/exp(7/2*a/b/n)/n/((c*x^n)^(7/2/
n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2347, 2209}

$$\int \frac{(dx)^{5/2}}{a+b\log(cx^n)}\,dx=\frac{(dx)^{7/2}e^{-\frac{7a}{2bn}}\,(cx^n)^{-\frac{7}{2}/n}\,\text{ExpIntegralEi}\left(\frac{7(a+b\log(cx^n))}{2bn}\right)}{bdn}$$

```
[In] Int[(d*x)^(5/2)/(a + b*Log[c*x^n]),x]
```

```
[Out] ((d*x)^(7/2)*ExpIntegralEi[(7*(a + b*Log[c*x^n]))/(2*b*n)])/(b*d*E^((7*a)/(
2*b*n))*n*(c*x^n)^(7/(2*n)))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left((dx)^{7/2}\left(cx^n\right)^{-\frac{7}{2}/n}\right)\text{Subst}\left(\int\frac{e^{\frac{7x}{2n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{dn}$$

$$= \frac{e^{-\frac{7a}{2bn}}(dx)^{7/2}\left(cx^n\right)^{-\frac{7}{2}/n}\text{Ei}\left(\frac{7(a+b\log(cx^n))}{2bn}\right)}{bdn}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int\frac{(dx)^{5/2}}{a+b\log\left(cx^n\right)}\,dx = \frac{e^{-\frac{7a}{2bn}}x(dx)^{5/2}\left(cx^n\right)^{-\frac{7}{2}/n}\text{ExpIntegralEi}\left(\frac{7(a+b\log(cx^n))}{2bn}\right)}{bn}$$

```
[In] Integrate[(d*x)^(5/2)/(a + b*Log[c*x^n]),x]
[Out] (x*(d*x)^(5/2)*ExpIntegralEi[(7*(a + b*Log[c*x^n]))/(2*b*n)])/(b*E^((7*a)/(
2*b*n))*n*(c*x^n)^(7/(2*n)))
```

Maple [F]

$$\int\frac{(dx)^{\frac{5}{2}}}{a+b\ln\left(c\,x^n\right)}dx$$

```
[In] int((d*x)^(5/2)/(a+b*ln(c*x^n)),x)
[Out] int((d*x)^(5/2)/(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int\frac{(dx)^{5/2}}{a+b\log\left(cx^n\right)}\,dx = \int\frac{(dx)^{\frac{5}{2}}}{b\log\left(cx^n\right)+a}\,dx$$

```
[In] integrate((d*x)^(5/2)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(sqrt(d*x)*d^2*x^2/(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int \frac{(dx)^{5/2}}{a+b\log\left(cx^n\right)}\,dx=\int \frac{(dx)^{\frac{5}{2}}}{a+b\log\left(cx^n\right)}\,dx$$

```
[In] integrate((d*x)**(5/2)/(a+b*ln(c*x**n)),x)
[Out] Integral((d*x)**(5/2)/(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \frac{(dx)^{5/2}}{a+b\log\left(cx^n\right)}\,dx=\int \frac{(dx)^{\frac{5}{2}}}{b\log\left(cx^n\right)+a}\,dx$$

```
[In] integrate((d*x)^(5/2)/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] 2*b*d^(5/2)*n*integrate(1/7*x^(5/2)/(b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*
log(c) + a^2 + 2*(b^2*log(c) + a*b)*log(x^n)), x) + 2/7*d^(5/2)*x^(7/2)/(b*
log(c) + b*log(x^n) + a)
```

Giac [F]

$$\int \frac{(dx)^{5/2}}{a+b\log\left(cx^n\right)}\,dx=\int \frac{(dx)^{\frac{5}{2}}}{b\log\left(cx^n\right)+a}\,dx$$

```
[In] integrate((d*x)^(5/2)/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate((d*x)^(5/2)/(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{5/2}}{a+b\log\left(cx^n\right)}\,dx=\int \frac{{\left(d\,x\right)}^{5/2}}{a+b\,\ln\left(c\,x^n\right)}\,dx$$

```
[In] int((d*x)^(5/2)/(a + b*log(c*x^n)),x)
[Out] int((d*x)^(5/2)/(a + b*log(c*x^n)), x)
```

3.102 $\int \frac{(dx)^{3/2}}{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{(dx)^{3/2}}{a+b\log(cx^n)}\,dx=\frac{e^{-\frac{5a}{2bn}}(dx)^{5/2}(cx^n)^{-\frac{5}{2}/n}\,\text{ExpIntegralEi}\left(\frac{5(a+b\log(cx^n))}{2bn}\right)}{bdn}$$

```
[Out] (d*x)^(5/2)*Ei(5/2*(a+b*ln(c*x^n))/b/n)/b/d/exp(5/2*a/b/n)/n/((c*x^n)^(5/2/
n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2347, 2209}

$$\int \frac{(dx)^{3/2}}{a+b\log(cx^n)}\,dx=\frac{(dx)^{5/2}e^{-\frac{5a}{2bn}}(cx^n)^{-\frac{5}{2}/n}\,\text{ExpIntegralEi}\left(\frac{5(a+b\log(cx^n))}{2bn}\right)}{bdn}$$

```
[In] Int[(d*x)^(3/2)/(a + b*Log[c*x^n]),x]
```

```
[Out] ((d*x)^(5/2)*ExpIntegralEi[(5*(a + b*Log[c*x^n]))/(2*b*n)])/(b*d*E^((5*a)/(
2*b*n))*n*(c*x^n)^(5/(2*n)))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left((dx)^{5/2}\left(cx^n\right)^{-\frac{5}{2}/n}\right)\text{Subst}\left(\int\frac{e^{\frac{5x}{2n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{dn}\\ &= \frac{e^{-\frac{5a}{2bn}}(dx)^{5/2}\left(cx^n\right)^{-\frac{5}{2}/n}\,\text{Ei}\left(\frac{5(a+b\log(cx^n))}{2bn}\right)}{bdn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int\frac{(dx)^{3/2}}{a+b\log\left(cx^n\right)}\,dx = \frac{e^{-\frac{5a}{2bn}}x(dx)^{3/2}\left(cx^n\right)^{-\frac{5}{2}/n}\,\text{ExpIntegralEi}\left(\frac{5(a+b\log(cx^n))}{2bn}\right)}{bn}$$

```
[In] Integrate[(d*x)^(3/2)/(a + b*Log[c*x^n]),x]
[Out] (x*(d*x)^(3/2)*ExpIntegralEi[(5*(a + b*Log[c*x^n]))/(2*b*n)])/(b*E^((5*a)/(
2*b*n))*n*(c*x^n)^(5/(2*n)))
```

Maple [F]

$$\int\frac{(dx)^{\frac{3}{2}}}{a+b\ln\left(c\,x^n\right)}dx$$

```
[In] int((d*x)^(3/2)/(a+b*ln(c*x^n)),x)
[Out] int((d*x)^(3/2)/(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int\frac{(dx)^{3/2}}{a+b\log\left(cx^n\right)}\,dx = \int\frac{(dx)^{\frac{3}{2}}}{b\log\left(cx^n\right)+a}\,dx$$

```
[In] integrate((d*x)^(3/2)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(sqrt(d*x)*d*x/(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int \frac{(dx)^{3/2}}{a + b\log(cx^n)}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{a + b\log(cx^n)}\,dx$$

```
[In] integrate((d*x)**(3/2)/(a+b*ln(c*x**n)),x)
[Out] Integral((d*x)**(3/2)/(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \frac{(dx)^{3/2}}{a + b\log(cx^n)}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{b\log(cx^n) + a}\,dx$$

```
[In] integrate((d*x)^(3/2)/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] 2*b*d^(3/2)*n*integrate(1/5*x^(3/2)/(b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*
log(c) + a^2 + 2*(b^2*log(c) + a*b)*log(x^n)), x) + 2/5*d^(3/2)*x^(5/2)/(b*
log(c) + b*log(x^n) + a)
```

Giac [F]

$$\int \frac{(dx)^{3/2}}{a + b\log(cx^n)}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{b\log(cx^n) + a}\,dx$$

```
[In] integrate((d*x)^(3/2)/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate((d*x)^(3/2)/(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{a + b\log(cx^n)}\,dx = \int \frac{(d\,x)^{3/2}}{a + b\,\ln(c\,x^n)}\,dx$$

```
[In] int((d*x)^(3/2)/(a + b*log(c*x^n)),x)
[Out] int((d*x)^(3/2)/(a + b*log(c*x^n)), x)
```

3.103 $\int \frac{\sqrt{dx}}{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{\sqrt{dx}}{a+b\log\left(cx^n\right)}\,dx=\frac{e^{-\frac{3a}{2bn}}(dx)^{3/2}\left(cx^n\right)^{-\frac{3}{2}/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{2bn}\right)}{bdn}$$

```
[Out] (d*x)^(3/2)*Ei(3/2*(a+b*ln(c*x^n))/b/n)/b/d/exp(3/2*a/b/n)/n/((c*x^n)^(3/2/
n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2347, 2209}

$$\int \frac{\sqrt{dx}}{a+b\log\left(cx^n\right)}\,dx=\frac{(dx)^{3/2}e^{-\frac{3a}{2bn}}\left(cx^n\right)^{-\frac{3}{2}/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{2bn}\right)}{bdn}$$

```
[In] Int[Sqrt[d*x]/(a + b*Log[c*x^n]),x]
```

```
[Out] ((d*x)^(3/2)*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(2*b*n)])/(b*d*E^((3*a)/(
2*b*n))*n*(c*x^n)^(3/(2*n)))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left((dx)^{3/2}\left(cx^n\right)^{-\frac{3}{2}/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{2n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{dn}\\ &= \frac{e^{-\frac{3a}{2bn}}(dx)^{3/2}\left(cx^n\right)^{-\frac{3}{2}/n}\text{Ei}\left(\frac{3(a+b\log(cx^n))}{2bn}\right)}{bdn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int\frac{\sqrt{dx}}{a+b\log\left(cx^n\right)}\,dx=\frac{e^{-\frac{3a}{2bn}}x\sqrt{dx}\left(cx^n\right)^{-\frac{3}{2}/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{2bn}\right)}{bn}$$

```
[In] Integrate[Sqrt[d*x]/(a + b*Log[c*x^n]),x]
[Out] (x*Sqrt[d*x]*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(2*b*n)])/(b*E^((3*a)/(2*
b*n))*n*(c*x^n)^(3/(2*n)))
```

Maple [F]

$$\int\frac{\sqrt{dx}}{a+b\ln\left(c\,x^n\right)}dx$$

```
[In] int((d*x)^(1/2)/(a+b*ln(c*x^n)),x)
[Out] int((d*x)^(1/2)/(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int\frac{\sqrt{dx}}{a+b\log\left(cx^n\right)}\,dx=\int\frac{\sqrt{dx}}{b\log\left(cx^n\right)+a}\,dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(sqrt(d*x)/(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{a+b\log\left(cx^n\right)}\,dx = \int \frac{\sqrt{dx}}{a+b\log\left(cx^n\right)}\,dx$$

```
[In] integrate((d*x)**(1/2)/(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(sqrt(d*x)/(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \frac{\sqrt{dx}}{a+b\log\left(cx^n\right)}\,dx = \int \frac{\sqrt{dx}}{b\log\left(cx^n\right)+a}\,dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(d)*n*integrate(1/3*sqrt(x)/(b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*
log(c) + a^2 + 2*(b^2*log(c) + a*b)*log(x^n)), x) + 2/3*sqrt(d)*x^(3/2)/(b*
log(c) + b*log(x^n) + a)
```

Giac [F]

$$\int \frac{\sqrt{dx}}{a+b\log\left(cx^n\right)}\,dx = \int \frac{\sqrt{dx}}{b\log\left(cx^n\right)+a}\,dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{a+b\log\left(cx^n\right)}\,dx = \int \frac{\sqrt{d\,x}}{a+b\,\ln\left(c\,x^n\right)}\,dx$$

```
[In] int((d*x)^(1/2)/(a + b*log(c*x^n)),x)
```

```
[Out] int((d*x)^(1/2)/(a + b*log(c*x^n)), x)
```

3.104 $\int \frac{1}{\sqrt{dx}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))}\,dx=\frac{e^{-\frac{a}{2bn}}\sqrt{dx}(cx^n)^{-\frac{1}{2}/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{2bn}\right)}{bdn}$$

```
[Out] Ei(1/2*(a+b*ln(c*x^n))/b/n)*(d*x)^(1/2)/b/d/exp(1/2*a/b/n)/n/((c*x^n)^(1/2/
n))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2347, 2209}

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))}\,dx=\frac{\sqrt{dx}e^{-\frac{a}{2bn}}(cx^n)^{-\frac{1}{2}/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{2bn}\right)}{bdn}$$

```
[In] Int[1/(Sqrt[d*x]*(a + b*Log[c*x^n])),x]
```

```
[Out] (Sqrt[d*x]*ExpIntegralEi[(a + b*Log[c*x^n])/(2*b*n)])/(b*d*E^(a/(2*b*n))*n*
(c*x^n)^(1/(2*n)))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{dx}(cx^n)^{-\frac{1}{2}/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{2n}}}{a+bx}\,dx, x, \log(cx^n)\right)}{dn}\\ &= \frac{e^{-\frac{a}{2bn}}\sqrt{dx}(cx^n)^{-\frac{1}{2}/n}\,\text{Ei}\left(\frac{a+b\log(cx^n)}{2bn}\right)}{bdn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))}\,dx = \frac{e^{-\frac{a}{2bn}}x(cx^n)^{-\frac{1}{2}/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{2bn}\right)}{bn\sqrt{dx}}$$

```
[In] Integrate[1/(Sqrt[d*x]*(a + b*Log[c*x^n])),x]
[Out] (x*ExpIntegralEi[(a + b*Log[c*x^n])/(2*b*n)])/(b*E^(a/(2*b*n))*n*Sqrt[d*x]*
(c*x^n)^(1/(2*n)))
```

Maple [F]

$$\int \frac{1}{\sqrt{dx}\,\left(a+b\ln\left(c\,x^{n}\right)\right)}dx$$

```
[In] int(1/(d*x)^(1/2)/(a+b*ln(c*x^n)),x)
[Out] int(1/(d*x)^(1/2)/(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))}\,dx = \int \frac{1}{\sqrt{dx}(b\log(cx^n)+a)}\,dx$$

```
[In] integrate(1/(d*x)^(1/2)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(sqrt(d*x)/(b*d*x*log(c*x^n) + a*d*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{\sqrt{dx}\,(a+b\log{(cx^n)})}\,dx$$

```
[In] integrate(1/(d*x)**(1/2)/(a+b*ln(c*x**n)),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{\sqrt{dx}(b\log{(cx^n)}+a)}\,dx$$

```
[In] integrate(1/(d*x)^(1/2)/(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 2*b*n*integrate(1/((b^2*sqrt(d)*log(c)^2 + b^2*sqrt(d)*log(x^n)^2 + 2*a*b*s
qrt(d)*log(c) + a^2*sqrt(d) + 2*(b^2*sqrt(d)*log(c) + a*b*sqrt(d))*log(x^n)
)*sqrt(x)), x) + 2*sqrt(x)/(b*sqrt(d)*log(c) + b*sqrt(d)*log(x^n) + a*sqrt(
d))
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{\sqrt{dx}(b\log{(cx^n)}+a)}\,dx$$

```
[In] integrate(1/(d*x)^(1/2)/(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x)*(b*log(c*x^n) + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{\sqrt{d\,x}\,\left(a+b\,\ln\left(c\,x^n\right)\right)}\,dx$$

```
[In] int(1/((d*x)^(1/2)*(a + b*log(c*x^n))),x)
```

```
[Out] int(1/((d*x)^(1/2)*(a + b*log(c*x^n))), x)
```

3.105 $\int \frac{1}{(dx)^{3/2}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 67

$$\int \frac{1}{(dx)^{3/2}\,(a+b\log(cx^n))}\,dx=\frac{e^{\frac{a}{2bn}}(cx^n)^{\frac{1}{2}/n}\,\text{ExpIntegralEi}\left(\frac{-a-b\log(cx^n)}{2bn}\right)}{bdn\sqrt{dx}}$$

```
[Out] exp(1/2*a/b/n)*(c*x^n)^(1/2/n)*Ei(1/2*(-a-b*ln(c*x^n))/b/n)/b/d/n/(d*x)^(1/
2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2347, 2209}

$$\int \frac{1}{(dx)^{3/2}\,(a+b\log(cx^n))}\,dx=\frac{e^{\frac{a}{2bn}}(cx^n)^{\frac{1}{2}/n}\,\text{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{2bn}\right)}{bdn\sqrt{dx}}$$

```
[In] Int[1/((d*x)^(3/2)*(a + b*Log[c*x^n])),x]
```

```
[Out] (E^(a/(2*b*n))*(c*x^n)^(1/(2*n))*ExpIntegralEi[-1/2*(a + b*Log[c*x^n])/(b*n
)])/(b*d*n*Sqrt[d*x])
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{(cx^n)^{\frac{1}{2}/n}\,\text{Subst}\left(\int \frac{e^{-\frac{x}{2n}}}{a+bx}\,dx, x, \log(cx^n)\right)}{dn\sqrt{dx}}$$

$$= \frac{e^{\frac{a}{2bn}}(cx^n)^{\frac{1}{2}/n}\,\text{Ei}\left(-\frac{a+b\log(cx^n)}{2bn}\right)}{bdn\sqrt{dx}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{1}{(dx)^{3/2}\,(a+b\log(cx^n))}\,dx = \frac{e^{\frac{a}{2bn}}x(cx^n)^{\frac{1}{2}/n}\,\text{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{2bn}\right)}{bn(dx)^{3/2}}$$

```
[In] Integrate[1/((d*x)^(3/2)*(a + b*Log[c*x^n])),x]
[Out] (E^(a/(2*b*n))*x*(c*x^n)^(1/(2*n))*ExpIntegralEi[-1/2*(a + b*Log[c*x^n])/(b
*n)])/(b*n*(d*x)^(3/2))
```

Maple [F]

$$\int \frac{1}{(dx)^{\frac{3}{2}}\,(a+b\ln(c\,x^n))}dx$$

```
[In] int(1/(d*x)^(3/2)/(a+b*ln(c*x^n)),x)
[Out] int(1/(d*x)^(3/2)/(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \frac{1}{(dx)^{3/2}\,(a+b\log(cx^n))}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\log(cx^n)+a)}\,dx$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(sqrt(d*x)/(b*d^2*x^2*log(c*x^n) + a*d^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx$$

```
[In] integrate(1/(d*x)**(3/2)/(a+b*ln(c*x**n)),x)
[Out] Integral(1/((d*x)**(3/2)*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\left(b\log\left(cx^n\right)+a\right)}\,dx$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] -2*b*n*integrate(1/((b^2*d^(3/2)*log(c)^2 + b^2*d^(3/2)*log(x^n)^2 + 2*a*b*
d^(3/2)*log(c) + a^2*d^(3/2) + 2*(b^2*d^(3/2)*log(c) + a*b*d^(3/2))*log(x^n
))*x^(3/2)), x) - 2/((b*d^(3/2)*log(c) + b*d^(3/2)*log(x^n) + a*d^(3/2))*sq
rt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \frac{1}{(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{c^{\frac{1}{2n}}\,\mathrm{Ei}\left(-\frac{\log(c)}{2n}-\frac{a}{2bn}-\frac{1}{2}\log\left(x\right)\right)e^{\left(\frac{a}{2bn}\right)}}{bd^{\frac{3}{2}}n}$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] c^(1/2/n)*Ei(-1/2*log(c)/n - 1/2*a/(b*n) - 1/2*log(x))*e^(1/2*a/(b*n))/(b*d
^(3/2)*n)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}\,(a+b\log(cx^n))}\,dx=\int \frac{1}{{(d\,x)}^{3/2}\,\left(a+b\,\ln\left(c\,x^n\right)\right)} \,d x$$

```
[In] int(1/((d*x)^(3/2)*(a + b*log(c*x^n))),x)
[Out] int(1/((d*x)^(3/2)*(a + b*log(c*x^n))), x)
```

3.106 $\int \frac{1}{(dx)^{5/2}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{1}{(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)}\,dx=\frac{e^{\frac{3a}{2bn}}\left(cx^n\right)^{\frac{3}{2}/n}\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{2bn}\right)}{bdn(dx)^{3/2}}$$

```
[Out] exp(3/2*a/b/n)*(c*x^n)^(3/2/n)*Ei(-3/2*(a+b*ln(c*x^n))/b/n)/b/d/n/(d*x)^(3/
2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2347, 2209}

$$\int \frac{1}{(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)}\,dx=\frac{e^{\frac{3a}{2bn}}\left(cx^n\right)^{\frac{3}{2}/n}\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{2bn}\right)}{bdn(dx)^{3/2}}$$

```
[In] Int[1/((d*x)^(5/2)*(a + b*Log[c*x^n])),x]
```

```
[Out] (E^((3*a)/(2*b*n))*(c*x^n)^(3/(2*n))*ExpIntegralEi[(-3*(a + b*Log[c*x^n]))/
(2*b*n)])/(b*d*n*(d*x)^(3/2))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{(cx^n)^{\frac{3}{2}/n}\,\text{Subst}\left(\int \frac{e^{-\frac{3x}{2n}}}{a+bx}\,dx, x, \log(cx^n)\right)}{dn(dx)^{3/2}}$$

$$= \frac{e^{\frac{3a}{2bn}}(cx^n)^{\frac{3}{2}/n}\,\text{Ei}\left(-\frac{3(a+b\log(cx^n))}{2bn}\right)}{bdn(dx)^{3/2}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{1}{(dx)^{5/2}\,(a+b\log(cx^n))}\,dx = \frac{e^{\frac{3a}{2bn}}x(cx^n)^{\frac{3}{2}/n}\,\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{2bn}\right)}{bn(dx)^{5/2}}$$

```
[In] Integrate[1/((d*x)^(5/2)*(a + b*Log[c*x^n])),x]
[Out] (E^((3*a)/(2*b*n))*x*(c*x^n)^(3/(2*n))*ExpIntegralEi[(-3*(a + b*Log[c*x^n])
)/(2*b*n)])/(b*n*(d*x)^(5/2))
```

Maple [F]

$$\int \frac{1}{(dx)^{\frac{5}{2}}\,(a+b\ln(c\,x^n))}dx$$

```
[In] int(1/(d*x)^(5/2)/(a+b*ln(c*x^n)),x)
[Out] int(1/(d*x)^(5/2)/(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \frac{1}{(dx)^{5/2}\,(a+b\log(cx^n))}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}}\,(b\log(cx^n)+a)}\,dx$$

```
[In] integrate(1/(d*x)^(5/2)/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(sqrt(d*x)/(b*d^3*x^3*log(c*x^n) + a*d^3*x^3), x)
```

Sympy [F]

$$\int \frac{1}{(dx)^{5/2}\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}}\,(a+b\log{(cx^n)})}\,dx$$

[In] integrate(1/(d*x)**(5/2)/(a+b*ln(c*x**n)),x)

[Out] Integral(1/((d*x)**(5/2)*(a + b*log(c*x**n))), x)

Maxima [F]

$$\int \frac{1}{(dx)^{5/2}\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}}\,(b\log{(cx^n)}+a)}\,dx$$

[In] integrate(1/(d*x)^(5/2)/(a+b*log(c*x^n)),x, algorithm="maxima")

[Out] -2*b*n*integrate(1/3/((b^2*d^(5/2)*log(c)^2 + b^2*d^(5/2)*log(x^n)^2 + 2*a*b*d^(5/2)*log(c) + a^2*d^(5/2) + 2*(b^2*d^(5/2)*log(c) + a*b*d^(5/2))*log(x^n))*x^(5/2)), x) - 2/3/((b*d^(5/2)*log(c) + b*d^(5/2)*log(x^n) + a*d^(5/2))*x^(3/2))

Giac [F]

$$\int \frac{1}{(dx)^{5/2}\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}}\,(b\log{(cx^n)}+a)}\,dx$$

[In] integrate(1/(d*x)^(5/2)/(a+b*log(c*x^n)),x, algorithm="giac")

[Out] integrate(1/((d*x)^(5/2)*(b*log(c*x^n) + a)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2}\,(a+b\log{(cx^n)})}\,dx = \int \frac{1}{{(d\,x)}^{5/2}\,\left(a+b\,\ln{(c\,x^n)}\right)}\,dx$$

[In] int(1/((d*x)^(5/2)*(a + b*log(c*x^n))),x)

[Out] int(1/((d*x)^(5/2)*(a + b*log(c*x^n))), x)

3.107 $\int \frac{(dx)^{5/2}}{(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{(dx)^{5/2}}{(a+b\log(cx^n))^2}\,dx = \frac{7e^{-\frac{7a}{2bn}}(dx)^{7/2}(cx^n)^{-\frac{7}{2}/n}\operatorname{ExpIntegralEi}\left(\frac{7(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{7/2}}{bdn(a+b\log(cx^n))}$$

```
[Out] 7/2*(d*x)^(7/2)*Ei(7/2*(a+b*ln(c*x^n))/b/n)/b^2/d/exp(7/2*a/b/n)/n^2/((c*x^
n)^(7/2/n))-(d*x)^(7/2)/b/d/n/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2343, 2347, 2209}

$$\int \frac{(dx)^{5/2}}{(a+b\log(cx^n))^2}\,dx = \frac{7(dx)^{7/2}e^{-\frac{7a}{2bn}}(cx^n)^{-\frac{7}{2}/n}\operatorname{ExpIntegralEi}\left(\frac{7(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{7/2}}{bdn(a+b\log(cx^n))}$$

```
[In] Int[(d*x)^(5/2)/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (7*(d*x)^(7/2)*ExpIntegralEi[(7*(a + b*Log[c*x^n]))/(2*b*n)])/(2*b^2*d*E^((
7*a)/(2*b*n))*n^2*(c*x^n)^(7/(2*n))) - (d*x)^(7/2)/(b*d*n*(a + b*Log[c*x^n]
))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(dx)^{7/2}}{bdn\left(a+b\log\left(cx^n\right)\right)}+\frac{7\int\frac{(dx)^{5/2}}{a+b\log(cx^n)}\,dx}{2bn}\\
&= -\frac{(dx)^{7/2}}{bdn\left(a+b\log\left(cx^n\right)\right)}+\frac{\left(7(dx)^{7/2}\left(cx^n\right)^{-\frac{7}{2}/n}\right)\operatorname{Subst}\left(\int\frac{e^{\frac{7x}{2n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{2bdn^2}\\
&= \frac{7e^{-\frac{7a}{2bn}}(dx)^{7/2}\left(cx^n\right)^{-\frac{7}{2}/n}\operatorname{Ei}\left(\frac{7(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2}-\frac{(dx)^{7/2}}{bdn\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

$$\int\frac{(dx)^{5/2}}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\frac{x(dx)^{5/2}\left(7e^{-\frac{7a}{2bn}}\left(cx^n\right)^{-\frac{7}{2}/n}\operatorname{ExpIntegralEi}\left(\frac{7(a+b\log(cx^n))}{2bn}\right)-\frac{2bn}{a+b\log(cx^n)}\right)}{2b^2n^2}$$

```
[In] Integrate[(d*x)^(5/2)/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (x*(d*x)^(5/2)*((7*ExpIntegralEi[(7*(a + b*Log[c*x^n]))/(2*b*n)])/(E^((7*a)
/(2*b*n))*(c*x^n)^(7/(2*n))) - (2*b*n)/(a + b*Log[c*x^n])))/(2*b^2*n^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.41

method	result
risch	$-\frac{2x^4d^3}{bn\sqrt{dx}\left(2a+2b\ln(c)+2b\ln\left(e^{n\ln(x)}\right)-ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}\left(ie^{n\ln(x)}\right)\mathrm{csgn}\left(ic\,e^{n\ln(x)}\right)+ib\pi\,\mathrm{csgn}(ic)\,\mathrm{csgn}\left(ic\,e^{n\ln(x)}\right)^2+ib\pi\,\mathrm{csgn}\left(ie^{n\ln(x)}\right.\right.}$

[In] int((d*x)^(5/2)/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)

[Out] -2/b/n*x^4/(d*x)^(1/2)/(2*a+2*b*ln(c)+2*b*ln(exp(n*ln(x)))-I*b*Pi*csgn(I*c)
*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))+I*b*Pi*csgn(I*c)*csgn(I*c*exp(
n*ln(x)))^2+I*b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2-I*b*Pi*csg
n(I*c*exp(n*ln(x)))^3)*d^3-7/2/d/b^2/n^2*exp(7/4*I*(b*Pi*csgn(I*c)*csgn(I*e
xp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2
-b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(
x)))^3+2*I*b*n*(ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))
+2*I*a)/b/n)*Ei(1,-7/2*ln(d*x)+7/4*I*(b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*c
sgn(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b*Pi*csgn(I*e
xp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)))^3+2*I*b*n
*(ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2*I*a)/b/n)

Fricas [F]

$$\int \frac{(dx)^{5/2}}{(a+b\log(cx^n))^2}\,dx=\int \frac{(dx)^{\frac{5}{2}}}{(b\log(cx^n)+a)^2}\,dx$$

[In] integrate((d*x)^(5/2)/(a+b*log(c*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d*x)*d^2*x^2/(b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2), x)

Sympy [F]

$$\int \frac{(dx)^{5/2}}{(a+b\log(cx^n))^2}\,dx=\int \frac{(dx)^{\frac{5}{2}}}{(a+b\log(cx^n))^2}\,dx$$

[In] integrate((d*x)**(5/2)/(a+b*ln(c*x**n))**2,x)

[Out] Integral((d*x)**(5/2)/(a + b*log(c*x**n))**2, x)

Maxima [F]

$$\int \frac{(dx)^{5/2}}{(a+b\log(cx^n))^2}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate((d*x)^(5/2)/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 4*b*d^(5/2)*n*integrate(1/7*x^(5/2)/(b^3*log(c)^3 + b^3*log(x^n)^3 + 3*a*b^
2*log(c)^2 + 3*a^2*b*log(c) + a^3 + 3*(b^3*log(c) + a*b^2)*log(x^n)^2 + 3*(
b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log(x^n)), x) + 2/7*d^(5/2)*x^(7/2)/
(b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c) + a^2 + 2*(b^2*log(c) + a*b)*
log(x^n))
```

Giac [F]

$$\int \frac{(dx)^{5/2}}{(a+b\log(cx^n))^2}\,dx = \int \frac{(dx)^{\frac{5}{2}}}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate((d*x)^(5/2)/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)/(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{5/2}}{(a+b\log(cx^n))^2}\,dx = \int \frac{(d\,x)^{5/2}}{(a+b\,\ln(c\,x^n))^2}\,dx$$

```
[In] int((d*x)^(5/2)/(a + b*log(c*x^n))^2,x)
```

```
[Out] int((d*x)^(5/2)/(a + b*log(c*x^n))^2, x)
```

3.108 $\int \frac{(dx)^{3/2}}{(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{(dx)^{3/2}}{(a+b\log(cx^n))^2}\,dx = \frac{5e^{-\frac{5a}{2bn}}(dx)^{5/2}(cx^n)^{-\frac{5}{2}/n}\,\text{ExpIntegralEi}\left(\frac{5(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{5/2}}{bdn(a+b\log(cx^n))}$$

```
[Out] 5/2*(d*x)^(5/2)*Ei(5/2*(a+b*ln(c*x^n))/b/n)/b^2/d/exp(5/2*a/b/n)/n^2/((c*x^
n)^(5/2/n))-(d*x)^(5/2)/b/d/n/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2343, 2347, 2209}

$$\int \frac{(dx)^{3/2}}{(a+b\log(cx^n))^2}\,dx = \frac{5(dx)^{5/2}e^{-\frac{5a}{2bn}}(cx^n)^{-\frac{5}{2}/n}\,\text{ExpIntegralEi}\left(\frac{5(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{5/2}}{bdn(a+b\log(cx^n))}$$

```
[In] Int[(d*x)^(3/2)/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (5*(d*x)^(5/2)*ExpIntegralEi[(5*(a + b*Log[c*x^n]))/(2*b*n)])/(2*b^2*d*E^((
5*a)/(2*b*n))*n^2*(c*x^n)^(5/(2*n))) - (d*x)^(5/2)/(b*d*n*(a + b*Log[c*x^n]
))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(dx)^{5/2}}{bdn\left(a+b\log\left(cx^n\right)\right)}+\frac{5\int\frac{(dx)^{3/2}}{a+b\log(cx^n)}\,dx}{2bn}\\
&= -\frac{(dx)^{5/2}}{bdn\left(a+b\log\left(cx^n\right)\right)}+\frac{\left(5(dx)^{5/2}\left(cx^n\right)^{-\frac{5}{2}/n}\right)\text{Subst}\left(\int\frac{e^{\frac{5x}{2n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{2bdn^2}\\
&= \frac{5e^{-\frac{5a}{2bn}}(dx)^{5/2}\left(cx^n\right)^{-\frac{5}{2}/n}\text{Ei}\left(\frac{5(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2}-\frac{(dx)^{5/2}}{bdn\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

$$\int\frac{(dx)^{3/2}}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\frac{x(dx)^{3/2}\left(5e^{-\frac{5a}{2bn}}\left(cx^n\right)^{-\frac{5}{2}/n}\text{ExpIntegralEi}\left(\frac{5(a+b\log(cx^n))}{2bn}\right)-\frac{2bn}{a+b\log(cx^n)}\right)}{2b^2n^2}$$

```
[In] Integrate[(d*x)^(3/2)/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (x*(d*x)^(3/2)*((5*ExpIntegralEi[(5*(a + b*Log[c*x^n]))/(2*b*n)])/(E^((5*a)
/(2*b*n))*(c*x^n)^(5/(2*n))) - (2*b*n)/(a + b*Log[c*x^n])))/(2*b^2*n^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.69 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.41

method	result
risch	$-\frac{2x^3d^2}{bn\sqrt{dx}\left(2a+2b\ln(c)+2b\ln\left(\mathrm{e}^{n\ln(x)}\right)-ib\pi\ \mathrm{csgn}(ic)\ \mathrm{csgn}\left(i\mathrm{e}^{n\ln(x)}\right)\ \mathrm{csgn}\left(ic\,\mathrm{e}^{n\ln(x)}\right)+ib\pi\ \mathrm{csgn}(ic)\ \mathrm{csgn}\left(ic\,\mathrm{e}^{n\ln(x)}\right)^2+ib\pi\ \mathrm{csgn}\left(i\mathrm{e}^{n\ln(x}\right.\right.}$

[In] int((d*x)^(3/2)/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)

[Out] -2/b/n*x^3/(d*x)^(1/2)/(2*a+2*b*ln(c)+2*b*ln(exp(n*ln(x)))-I*b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))+I*b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2+I*b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2-I*b*Pi*csgn(I*c*exp(n*ln(x)))^3)*d^2-5/2/d/b^2/n^2*exp(5/4*I*(b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)))^3+2*I*b*n*(ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2*I*a)/b/n)*Ei(1,-5/2*ln(d*x)+5/4*I*(b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)))^3+2*I*b*n*(ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2*I*a)/b/n)

Fricas [F]

$$\int \frac{(dx)^{3/2}}{(a+b\log(cx^n))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(b\log(cx^n)+a)^2}\,dx$$

[In] integrate((d*x)^(3/2)/(a+b*log(c*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d*x)*d*x/(b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2), x)

Sympy [F]

$$\int \frac{(dx)^{3/2}}{(a+b\log(cx^n))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(a+b\log(cx^n))^2}\,dx$$

[In] integrate((d*x)**(3/2)/(a+b*ln(c*x**n))**2,x)

[Out] Integral((d*x)**(3/2)/(a + b*log(c*x**n))**2, x)

Maxima [F]

$$\int \frac{(dx)^{3/2}}{(a+b\log(cx^n))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate((d*x)^(3/2)/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 4*b*d^(3/2)*n*integrate(1/5*x^(3/2)/(b^3*log(c)^3 + b^3*log(x^n)^3 + 3*a*b^
2*log(c)^2 + 3*a^2*b*log(c) + a^3 + 3*(b^3*log(c) + a*b^2)*log(x^n)^2 + 3*(
b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log(x^n)), x) + 2/5*d^(3/2)*x^(5/2)/
(b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c) + a^2 + 2*(b^2*log(c) + a*b)*
log(x^n))
```

Giac [F]

$$\int \frac{(dx)^{3/2}}{(a+b\log(cx^n))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate((d*x)^(3/2)/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)/(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{3/2}}{(a+b\log(cx^n))^2}\,dx = \int \frac{(d\,x)^{3/2}}{(a+b\,\ln(c\,x^n))^2}\,dx$$

```
[In] int((d*x)^(3/2)/(a + b*log(c*x^n))^2,x)
```

```
[Out] int((d*x)^(3/2)/(a + b*log(c*x^n))^2, x)
```

3.109 $\int \frac{\sqrt{dx}}{(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{\sqrt{dx}}{(a+b\log(cx^n))^2}\,dx = \frac{3e^{-\frac{3a}{2bn}}(dx)^{3/2}(cx^n)^{-\frac{3}{2}/n}\,\text{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{3/2}}{bdn(a+b\log(cx^n))}$$

```
[Out] 3/2*(d*x)^(3/2)*Ei(3/2*(a+b*ln(c*x^n))/b/n)/b^2/d/exp(3/2*a/b/n)/n^2/((c*x^
n)^(3/2/n))-(d*x)^(3/2)/b/d/n/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2343, 2347, 2209}

$$\int \frac{\sqrt{dx}}{(a+b\log(cx^n))^2}\,dx = \frac{3(dx)^{3/2}e^{-\frac{3a}{2bn}}(cx^n)^{-\frac{3}{2}/n}\,\text{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2} - \frac{(dx)^{3/2}}{bdn(a+b\log(cx^n))}$$

```
[In] Int[Sqrt[d*x]/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (3*(d*x)^(3/2)*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(2*b*n)])/(2*b^2*d*E^((
3*a)/(2*b*n))*n^2*(c*x^n)^(3/(2*n))) - (d*x)^(3/2)/(b*d*n*(a + b*Log[c*x^n]
))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(dx)^{3/2}}{bdn\left(a+b\log\left(cx^n\right)\right)}+\frac{3\int\frac{\sqrt{dx}}{a+b\log(cx^n)}\,dx}{2bn}\\
&= -\frac{(dx)^{3/2}}{bdn\left(a+b\log\left(cx^n\right)\right)}+\frac{\left(3(dx)^{3/2}\left(cx^n\right)^{-\frac{3}{2}/n}\right)\text{Subst}\left(\int\frac{e^{\frac{3x}{2n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{2bdn^2}\\
&= \frac{3e^{-\frac{3a}{2bn}}(dx)^{3/2}\left(cx^n\right)^{-\frac{3}{2}/n}\text{Ei}\left(\frac{3(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2}-\frac{(dx)^{3/2}}{bdn\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

$$\int\frac{\sqrt{dx}}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx = \frac{x\sqrt{dx}\left(3e^{-\frac{3a}{2bn}}\left(cx^n\right)^{-\frac{3}{2}/n}\text{ExpIntegralEi}\left(\frac{3(a+b\log(cx^n))}{2bn}\right)-\frac{2bn}{a+b\log(cx^n)}\right)}{2b^2n^2}$$

```
[In] Integrate[Sqrt[d*x]/(a + b*Log[c*x^n])^2,x]
```

```
[Out] (x*Sqrt[d*x]*((3*ExpIntegralEi[(3*(a + b*Log[c*x^n]))/(2*b*n)])/(E^((3*a)/(
2*b*n))*(c*x^n)^(3/(2*n))) - (2*b*n)/(a + b*Log[c*x^n])))/(2*b^2*n^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.89 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.39

method	result
risch	$-\frac{2x^2d}{bn\sqrt{dx}\left(2a+2b\ln(c)+2b\ln\left(\mathrm{e}^{n\ln(x)}\right)-ib\pi\ \mathrm{csgn}(ic)\ \mathrm{csgn}\left(i\mathrm{e}^{n\ln(x)}\right)\mathrm{csgn}\left(ic\,\mathrm{e}^{n\ln(x)}\right)+ib\pi\ \mathrm{csgn}(ic)\ \mathrm{csgn}\left(ic\,\mathrm{e}^{n\ln(x)}\right)^2+ib\pi\ \mathrm{csgn}\left(i\mathrm{e}^{n\ln(x}\right.\right.}$

```
[In] int((d*x)^(1/2)/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/n*x^2/(d*x)^(1/2)/(2*a+2*b*ln(c)+2*b*ln(exp(n*ln(x)))-I*b*Pi*csgn(I*c)
*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))+I*b*Pi*csgn(I*c)*csgn(I*c*exp(
n*ln(x)))^2+I*b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2-I*b*Pi*csg
n(I*c*exp(n*ln(x)))^3)*d-3/2/d/b^2/n^2*exp(3/4*I*(b*Pi*csgn(I*c)*csgn(I*exp
(n*ln(x)))*csgn(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b
*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)
))^3+2*I*b*n*(ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2
*I*a)/b/n)*Ei(1,-3/2*ln(d*x)+3/4*I*(b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csg
n(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b*Pi*csgn(I*exp
(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)))^3+2*I*b*n*(
ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2*I*a)/b/n)
```

Fricas [F]

$$\int \frac{\sqrt{dx}}{(a+b\log(cx^n))^2}\,dx = \int \frac{\sqrt{dx}}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/(b^2*log(c*x^n)^2 + 2*a*b*log(c*x^n) + a^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{dx}}{(a+b\log(cx^n))^2}\,dx = \int \frac{\sqrt{dx}}{(a+b\log(cx^n))^2}\,dx$$

```
[In] integrate((d*x)**(1/2)/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(sqrt(d*x)/(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{dx}}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx = \int \frac{\sqrt{dx}}{\left(b\log\left(cx^n\right)+a\right)^2}\,dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 4*b*sqrt(d)*n*integrate(1/3*sqrt(x)/(b^3*log(c)^3 + b^3*log(x^n)^3 + 3*a*b^
2*log(c)^2 + 3*a^2*b*log(c) + a^3 + 3*(b^3*log(c) + a*b^2)*log(x^n)^2 + 3*(
b^3*log(c)^2 + 2*a*b^2*log(c) + a^2*b)*log(x^n)), x) + 2/3*sqrt(d)*x^(3/2)/
(b^2*log(c)^2 + b^2*log(x^n)^2 + 2*a*b*log(c) + a^2 + 2*(b^2*log(c) + a*b)*
log(x^n))
```

Giac [F]

$$\int \frac{\sqrt{dx}}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx = \int \frac{\sqrt{dx}}{\left(b\log\left(cx^n\right)+a\right)^2}\,dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{dx}}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx = \int \frac{\sqrt{d\,x}}{\left(a+b\,\ln\left(c\,x^n\right)\right)^2}\,dx$$

```
[In] int((d*x)^(1/2)/(a + b*log(c*x^n))^2,x)
```

```
[Out] int((d*x)^(1/2)/(a + b*log(c*x^n))^2, x)
```

3.110 $\int \frac{1}{\sqrt{dx}(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))^2}\,dx = \frac{e^{-\frac{a}{2bn}}\sqrt{dx}(cx^n)^{-\frac{1}{2}/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{2bn}\right)}{2b^2dn^2} - \frac{\sqrt{dx}}{bdn\,(a+b\log(cx^n))}$$

```
[Out] 1/2*Ei(1/2*(a+b*ln(c*x^n))/b/n)*(d*x)^(1/2)/b^2/d/exp(1/2*a/b/n)/n^2/((c*x^
n)^(1/2/n))-(d*x)^(1/2)/b/d/n/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2343, 2347, 2209}

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))^2}\,dx = \frac{\sqrt{dx}e^{-\frac{a}{2bn}}(cx^n)^{-\frac{1}{2}/n}\,\text{ExpIntegralEi}\left(\frac{a+b\log(cx^n)}{2bn}\right)}{2b^2dn^2} - \frac{\sqrt{dx}}{bdn\,(a+b\log(cx^n))}$$

```
[In] Int[1/(Sqrt[d*x]*(a + b*Log[c*x^n])^2),x]
```

```
[Out] (Sqrt[d*x]*ExpIntegralEi[(a + b*Log[c*x^n])/(2*b*n)])/(2*b^2*d*E^(a/(2*b*n)
)*n^2*(c*x^n)^(1/(2*n))) - Sqrt[d*x]/(b*d*n*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\sqrt{dx}}{bdn\left(a+b\log\left(cx^n\right)\right)}+\frac{\int \frac{1}{\sqrt{dx}\left(a+b\log\left(cx^n\right)\right)}\,dx}{2bn}\\
&= -\frac{\sqrt{dx}}{bdn\left(a+b\log\left(cx^n\right)\right)}+\frac{\left(\sqrt{dx}\left(cx^n\right)^{-\frac{1}{2}/n}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{x}{2n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{2bdn^2}\\
&= \frac{e^{-\frac{a}{2bn}}\sqrt{dx}\left(cx^n\right)^{-\frac{1}{2}/n}\operatorname{Ei}\left(\frac{a+b\log\left(cx^n\right)}{2bn}\right)}{2b^2dn^2}-\frac{\sqrt{dx}}{bdn\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$
\int \frac{1}{\sqrt{dx}\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\frac{x\left(e^{-\frac{a}{2bn}}\left(cx^n\right)^{-\frac{1}{2}/n}\operatorname{ExpIntegralEi}\left(\frac{a+b\log\left(cx^n\right)}{2bn}\right)-\frac{2bn}{a+b\log\left(cx^n\right)}\right)}{2b^2n^2\sqrt{dx}}
$$

```
[In] Integrate[1/(Sqrt[d*x]*(a + b*Log[c*x^n])^2),x]
```

```
[Out] (x*(ExpIntegralEi[(a + b*Log[c*x^n])/(2*b*n)]/(E^(a/(2*b*n))*(c*x^n)^(1/(2*
n))) - (2*b*n)/(a + b*Log[c*x^n])))/(2*b^2*n^2*Sqrt[d*x])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.36

method	result
risch	$-\frac{2x}{bn\sqrt{dx}\left(2a+2b\ln(c)+2b\ln\left(e^{n\ln(x)}\right)-ib\pi\operatorname{csgn}(ic)\operatorname{csgn}\left(ie^{n\ln(x)}\right)\operatorname{csgn}\left(ic\,e^{n\ln(x)}\right)+ib\pi\operatorname{csgn}(ic)\operatorname{csgn}\left(ic\,e^{n\ln(x)}\right)^{2}+ib\pi\operatorname{csgn}\left(ie^{n\ln(x)}\right.\cdots\right.}$

```
[In] int(1/(d*x)^(1/2)/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/n*x/(d*x)^(1/2)/(2*a+2*b*ln(c)+2*b*ln(exp(n*ln(x)))-I*b*Pi*csgn(I*c)*c
sgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))+I*b*Pi*csgn(I*c)*csgn(I*c*exp(n*
ln(x)))^2+I*b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2-I*b*Pi*csgn(
I*c*exp(n*ln(x)))^3)-1/2/d/b^2/n^2*exp(1/4*I*(b*Pi*csgn(I*c)*csgn(I*exp(n*l
n(x)))*csgn(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b*Pi*
csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)))^3
+2*I*b*n*(ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2*I*a
)/b/n)*Ei(1,-1/2*ln(d*x)+1/4*I*(b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csgn(I*
c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b*Pi*csgn(I*exp(n*l
n(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)))^3+2*I*b*n*(ln(x
)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2*I*a)/b/n)
```

Fricas [F]

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))^2}\,dx = \int \frac{1}{\sqrt{dx}(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate(1/(d*x)^(1/2)/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/(b^2*d*x*log(c*x^n)^2 + 2*a*b*d*x*log(c*x^n) + a^2*d*x),
 x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))^2}\,dx = \int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))^2}\,dx$$

```
[In] integrate(1/(d*x)**(1/2)/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*log(c*x**n))**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))^2}\,dx = \int \frac{1}{\sqrt{dx}(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate(1/(d*x)^(1/2)/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 4*b*n*integrate(1/((b^3*sqrt(d)*log(c)^3 + b^3*sqrt(d)*log(x^n)^3 + 3*a*b^2
*sqrt(d)*log(c)^2 + 3*a^2*b*sqrt(d)*log(c) + a^3*sqrt(d) + 3*(b^3*sqrt(d)*l
og(c) + a*b^2*sqrt(d))*log(x^n)^2 + 3*(b^3*sqrt(d)*log(c)^2 + 2*a*b^2*sqrt(
d)*log(c) + a^2*b*sqrt(d))*log(x^n))*sqrt(x)), x) + 2*sqrt(x)/(b^2*sqrt(d)*
log(c)^2 + b^2*sqrt(d)*log(x^n)^2 + 2*a*b*sqrt(d)*log(c) + a^2*sqrt(d) + 2*
(b^2*sqrt(d)*log(c) + a*b*sqrt(d))*log(x^n))
```

Giac [F]

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))^2}\,dx = \int \frac{1}{\sqrt{dx}(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate(1/(d*x)^(1/2)/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x)*(b*log(c*x^n) + a)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{dx}\,(a+b\log(cx^n))^2}\,dx = \int \frac{1}{\sqrt{d\,x}\,(a+b\,\ln(c\,x^n))^2}\,dx$$

```
[In] int(1/((d*x)^(1/2)*(a + b*log(c*x^n))^2),x)
```

```
[Out] int(1/((d*x)^(1/2)*(a + b*log(c*x^n))^2), x)
```

3.111 $\int \frac{1}{(dx)^{3/2}(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{1}{(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx = -\frac{e^{\frac{a}{2bn}}\left(cx^n\right)^{\frac{1}{2}/n}\operatorname{ExpIntegralEi}\left(\frac{-a-b\log(cx^n)}{2bn}\right)}{2b^2dn^2\sqrt{dx}}-\frac{1}{bdn\sqrt{dx}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] -1/2*exp(1/2*a/b/n)*(c*x^n)^(1/2/n)*Ei(1/2*(-a-b*ln(c*x^n))/b/n)/b^2/d/n^2/
(d*x)^(1/2)-1/b/d/n/(a+b*ln(c*x^n))/(d*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2343, 2347, 2209}

$$\int \frac{1}{(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx = -\frac{e^{\frac{a}{2bn}}\left(cx^n\right)^{\frac{1}{2}/n}\operatorname{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{2bn}\right)}{2b^2dn^2\sqrt{dx}}-\frac{1}{bdn\sqrt{dx}\left(a+b\log\left(cx^n\right)\right)}$$

```
[In] Int[1/((d*x)^(3/2)*(a + b*Log[c*x^n])^2),x]
```

```
[Out] -1/2*(E^(a/(2*b*n))*(c*x^n)^(1/(2*n))*ExpIntegralEi[-1/2*(a + b*Log[c*x^n])
/(b*n)])/(b^2*d*n^2*Sqrt[d*x]) - 1/(b*d*n*Sqrt[d*x]*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{bdn\sqrt{dx}\,(a+b\log(cx^n))} - \frac{\int \frac{1}{(dx)^{3/2}(a+b\log(cx^n))}\,dx}{2bn} \\
&= -\frac{1}{bdn\sqrt{dx}\,(a+b\log(cx^n))} - \frac{(cx^n)^{\frac{1}{2}/n}\,\text{Subst}\left(\int \frac{e^{-\frac{x}{2n}}}{a+bx}\,dx, x, \log(cx^n)\right)}{2bdn^2\sqrt{dx}} \\
&= -\frac{e^{\frac{a}{2bn}}(cx^n)^{\frac{1}{2}/n}\,\text{Ei}\left(-\frac{a+b\log(cx^n)}{2bn}\right)}{2b^2dn^2\sqrt{dx}} - \frac{1}{bdn\sqrt{dx}\,(a+b\log(cx^n))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{1}{(dx)^{3/2}(a+b\log(cx^n))^2}\,dx = -\frac{x\left(2bn+e^{\frac{a}{2bn}}(cx^n)^{\frac{1}{2}/n}\,\text{ExpIntegralEi}\left(-\frac{a+b\log(cx^n)}{2bn}\right)(a+b\log(cx^n))\right)}{2b^2n^2(dx)^{3/2}\,(a+b\log(cx^n))}$$

```
[In] Integrate[1/((d*x)^(3/2)*(a + b*Log[c*x^n])^2),x]
[Out] -1/2*(x*(2*b*n + E^(a/(2*b*n))*(c*x^n)^(1/(2*n))*ExpIntegralEi[-1/2*(a + b*
Log[c*x^n])/(b*n)]*(a + b*Log[c*x^n])))/(b^2*n^2*(d*x)^(3/2)*(a + b*Log[c*x
^n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 429, normalized size of antiderivative = 4.25

method	result
risch	$-\frac{2}{bn\sqrt{dx}\left(2a+2b\ln(c)+2b\ln\left(\mathrm{e}^{n\ln(x)}\right)-ib\pi\ \mathrm{csgn}(ic)\ \mathrm{csgn}\left(i\mathrm{e}^{n\ln(x)}\right)\ \mathrm{csgn}\left(ic\,\mathrm{e}^{n\ln(x)}\right)+ib\pi\ \mathrm{csgn}(ic)\ \mathrm{csgn}\left(ic\,\mathrm{e}^{n\ln(x)}\right)^{2}+ib\pi\ \mathrm{csgn}\left(i\mathrm{e}^{n\ln(x)}\right.\right.}$

[In] int(1/(d*x)^(3/2)/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)

[Out] -2/b/n/(d*x)^(1/2)/(2*a+2*b*ln(c)+2*b*ln(exp(n*ln(x)))-I*b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))+I*b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2+I*b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2-I*b*Pi*csgn(I*c*exp(n*ln(x)))^3)/d+1/2/b^2/n^2*exp(-1/4*I*(b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)))^3+2*I*b*n*(ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2*I*a)/b/n)*Ei(1,1/2*ln(d*x)-1/4*I*(b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)))^3+2*I*b*n*(ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2*I*a)/b/n)/d

Fricas [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{(dx)^{\frac{3}{2}}\left(b\log\left(cx^n\right)+a\right)^2}\,dx$$

[In] integrate(1/(d*x)^(3/2)/(a+b*log(c*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d*x)/(b^2*d^2*x^2*log(c*x^n)^2 + 2*a*b*d^2*x^2*log(c*x^n) + a^2*d^2*x^2), x)

Sympy [F]

$$\int \frac{1}{(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

[In] integrate(1/(d*x)**(3/2)/(a+b*ln(c*x**n))**2,x)

[Out] Integral(1/((d*x)**(3/2)*(a + b*log(c*x**n))**2), x)

Maxima [F]

$$\int \frac{1}{(dx)^{3/2} \left(a+b\log\left(cx^n\right)\right)^2}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}} \left(b\log\left(cx^n\right)+a\right)^2}\,dx$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -4*b*n*integrate(1/((b^3*d^(3/2)*log(c)^3 + b^3*d^(3/2)*log(x^n)^3 + 3*a*b^
2*d^(3/2)*log(c)^2 + 3*a^2*b*d^(3/2)*log(c) + a^3*d^(3/2) + 3*(b^3*d^(3/2)*
log(c) + a*b^2*d^(3/2))*log(x^n)^2 + 3*(b^3*d^(3/2)*log(c)^2 + 2*a*b^2*d^(3
/2)*log(c) + a^2*b*d^(3/2))*log(x^n))*x^(3/2)), x) - 2/((b^2*d^(3/2)*log(c)
^2 + b^2*d^(3/2)*log(x^n)^2 + 2*a*b*d^(3/2)*log(c) + a^2*d^(3/2) + 2*(b^2*d
^(3/2)*log(c) + a*b*d^(3/2))*log(x^n))*sqrt(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(85) = 170$.

Time = 0.34 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.90

$$\int \frac{1}{(dx)^{3/2} \left(a+b\log\left(cx^n\right)\right)^2}\,dx = -\frac{\frac{bc^{\frac{1}{2n}} n \mathrm{Ei}\left(-\frac{\log(c)}{2n}-\frac{a}{2bn}-\frac{1}{2}\log(x)\right)e^{\left(\frac{a}{2bn}\right)}\log(x)}{b^3\sqrt{d}n^3\log(x)+b^3\sqrt{d}n^2\log(c)+ab^2\sqrt{d}n^2} + \frac{bc^{\frac{1}{2n}} \mathrm{Ei}\left(-\frac{\log(c)}{2n}-\frac{a}{2bn}-\frac{1}{2}\log(x)\right)e^{\left(\frac{a}{2bn}\right)}\log(c)}{b^3\sqrt{d}n^3\log(x)+b^3\sqrt{d}n^2\log(c)+ab^2\sqrt{d}n^2} + \frac{ac^{\frac{1}{2n}} \mathrm{Ei}\left(-\frac{\log(c)}{2n}-\frac{a}{2bn}-\frac{1}{2}\log(x)\right)e}{b^3\sqrt{d}n^3\log(x)+b^3\sqrt{d}n^2\log(c)+ab^2}}{2d}$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*c^(1/2/n)*n*Ei(-1/2*log(c)/n - 1/2*a/(b*n) - 1/2*log(x))*e^(1/2*a/(
b*n))*log(x)/(b^3*sqrt(d)*n^3*log(x) + b^3*sqrt(d)*n^2*log(c) + a*b^2*sqrt(
d)*n^2) + b*c^(1/2/n)*Ei(-1/2*log(c)/n - 1/2*a/(b*n) - 1/2*log(x))*e^(1/2*a
/(b*n))*log(c)/(b^3*sqrt(d)*n^3*log(x) + b^3*sqrt(d)*n^2*log(c) + a*b^2*sqr
t(d)*n^2) + a*c^(1/2/n)*Ei(-1/2*log(c)/n - 1/2*a/(b*n) - 1/2*log(x))*e^(1/2
*a/(b*n))/(b^3*sqrt(d)*n^3*log(x) + b^3*sqrt(d)*n^2*log(c) + a*b^2*sqrt(d)*
n^2) + 2*b*n/((b^3*sqrt(d)*n^3*log(x) + b^3*sqrt(d)*n^2*log(c) + a*b^2*sqrt
(d)*n^2)*sqrt(x)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{{\left(d\,x\right)}^{3/2}\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}\,dx$$

```
[In] int(1/((d*x)^(3/2)*(a + b*log(c*x^n))^2),x)
[Out] int(1/((d*x)^(3/2)*(a + b*log(c*x^n))^2), x)
```

3.112 $\int \frac{1}{(dx)^{5/2}(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 98

$$\int \frac{1}{(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx = -\frac{3e^{\frac{3a}{2bn}}\left(cx^n\right)^{\frac{3}{2}/n}\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2(dx)^{3/2}}-\frac{1}{bdn(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)}$$

```
[Out] -3/2*exp(3/2*a/b/n)*(c*x^n)^(3/2/n)*Ei(-3/2*(a+b*ln(c*x^n))/b/n)/b^2/d/n^2/
(d*x)^(3/2)-1/b/d/n/(d*x)^(3/2)/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2343, 2347, 2209}

$$\int \frac{1}{(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx = -\frac{3e^{\frac{3a}{2bn}}\left(cx^n\right)^{\frac{3}{2}/n}\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2(dx)^{3/2}}-\frac{1}{bdn(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)}$$

```
[In] Int[1/((d*x)^(5/2)*(a + b*Log[c*x^n])^2),x]
```

```
[Out] (-3*E^((3*a)/(2*b*n))*(c*x^n)^(3/(2*n))*ExpIntegralEi[(-3*(a + b*Log[c*x^n]
))/(2*b*n)])/(2*b^2*d*n^2*(d*x)^(3/2)) - 1/(b*d*n*(d*x)^(3/2)*(a + b*Log[c*
x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{bdn(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)}-\frac{3\int\frac{1}{(dx)^{5/2}(a+b\log(cx^n))}\,dx}{2bn}\\
&= -\frac{1}{bdn(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)}-\frac{\left(3(cx^n)^{\frac{3}{2}/n}\right)\text{Subst}\left(\int\frac{e^{-\frac{3x}{2n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{2bdn^2(dx)^{3/2}}\\
&= -\frac{3e^{\frac{3a}{2bn}}\left(cx^n\right)^{\frac{3}{2}/n}\text{Ei}\left(-\frac{3(a+b\log(cx^n))}{2bn}\right)}{2b^2dn^2(dx)^{3/2}}-\frac{1}{bdn(dx)^{3/2}\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int\frac{1}{(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx = -\frac{x\left(2bn+3e^{\frac{3a}{2bn}}\left(cx^n\right)^{\frac{3}{2}/n}\text{ExpIntegralEi}\left(-\frac{3(a+b\log(cx^n))}{2bn}\right)\left(a+b\log\left(cx^n\right)\right)\right)}{2b^2n^2(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)}$$

```
[In] Integrate[1/((d*x)^(5/2)*(a + b*Log[c*x^n])^2),x]
```

```
[Out] -1/2*(x*(2*b*n + 3*E^((3*a)/(2*b*n))*(c*x^n)^(3/(2*n))*ExpIntegralEi[(-3*(a
 + b*Log[c*x^n]))/(2*b*n)]*(a + b*Log[c*x^n])))/(b^2*n^2*(d*x)^(5/2)*(a + b
*Log[c*x^n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.87 (sec) , antiderivative size = 432, normalized size of antiderivative = 4.41

method	result
risch	$-\frac{2}{bnx\sqrt{dx}\left(2a+2b\ln(c)+2b\ln\left(\mathrm{e}^{n\ln(x)}\right)-ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}\left(i\mathrm{e}^{n\ln(x)}\right)\mathrm{csgn}\left(ic\,\mathrm{e}^{n\ln(x)}\right)+ib\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}\left(ic\,\mathrm{e}^{n\ln(x)}\right)^2+ib\pi\ \mathrm{csgn}\left(i\mathrm{e}^{n}\right.\right.}$

[In] int(1/(d*x)^(5/2)/(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)

[Out] -2/b/n/x/(d*x)^(1/2)/(2*a+2*b*ln(c)+2*b*ln(exp(n*ln(x)))-I*b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))+I*b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2+I*b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2-I*b*Pi*csgn(I*c*exp(n*ln(x)))^3)/d^2+3/2/d/b^2/n^2*exp(-3/4*I*(b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)))^3+2*I*b*n*(ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2*I*a)/b/n)*Ei(1,3/2*ln(d*x)-3/4*I*(b*Pi*csgn(I*c)*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))-b*Pi*csgn(I*c)*csgn(I*c*exp(n*ln(x)))^2-b*Pi*csgn(I*exp(n*ln(x)))*csgn(I*c*exp(n*ln(x)))^2+b*Pi*csgn(I*c*exp(n*ln(x)))^3+2*I*b*n*(ln(x)-ln(d*x))+2*I*b*ln(c)+2*I*b*(ln(exp(n*ln(x)))-n*ln(x))+2*I*a)/b/n)

Fricas [F]

$$\int \frac{1}{(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}}\left(b\log\left(cx^n\right)+a\right)^2}\,dx$$

[In] integrate(1/(d*x)^(5/2)/(a+b*log(c*x^n))^2,x, algorithm="fricas")

[Out] integral(sqrt(d*x)/(b^2*d^3*x^3*log(c*x^n)^2 + 2*a*b*d^3*x^3*log(c*x^n) + a^2*d^3*x^3), x)

Sympy [F]

$$\int \frac{1}{(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx = \int \frac{1}{(dx)^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)^2}\,dx$$

[In] integrate(1/(d*x)**(5/2)/(a+b*ln(c*x**n))**2,x)

[Out] Integral(1/((d*x)**(5/2)*(a + b*log(c*x**n))**2), x)

Maxima [F]

$$\int \frac{1}{(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{(dx)^{\frac{5}{2}}\left(b\log\left(cx^n\right)+a\right)^2}\,dx$$

[In] integrate(1/(d*x)^(5/2)/(a+b*log(c*x^n))^2,x, algorithm="maxima")

[Out] -4*b*n*integrate(1/3/((b^3*d^(5/2)*log(c)^3 + b^3*d^(5/2)*log(x^n)^3 + 3*a*b^2*d^(5/2)*log(c)^2 + 3*a^2*b*d^(5/2)*log(c) + a^3*d^(5/2) + 3*(b^3*d^(5/2)*log(c) + a*b^2*d^(5/2))*log(x^n)^2 + 3*(b^3*d^(5/2)*log(c)^2 + 2*a*b^2*d^(5/2)*log(c) + a^2*b*d^(5/2))*log(x^n))*x^(5/2)), x) - 2/3/((b^2*d^(5/2)*log(c)^2 + b^2*d^(5/2)*log(x^n)^2 + 2*a*b*d^(5/2)*log(c) + a^2*d^(5/2) + 2*(b^2*d^(5/2)*log(c) + a*b*d^(5/2))*log(x^n))*x^(3/2))

Giac [F]

$$\int \frac{1}{(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{(dx)^{\frac{5}{2}}\left(b\log\left(cx^n\right)+a\right)^2}\,dx$$

[In] integrate(1/(d*x)^(5/2)/(a+b*log(c*x^n))^2,x, algorithm="giac")

[Out] integrate(1/((d*x)^(5/2)*(b*log(c*x^n) + a)^2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(dx)^{5/2}\left(a+b\log\left(cx^n\right)\right)^2}\,dx=\int \frac{1}{{\left(d\,x\right)}^{5/2}\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2}\,dx$$

[In] int(1/((d*x)^(5/2)*(a + b*log(c*x^n))^2),x)

[Out] int(1/((d*x)^(5/2)*(a + b*log(c*x^n))^2), x)

3.113 $\int \sqrt{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \sqrt{a+b\log(cx^n)}\,dx = -\frac{1}{2}\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}x(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right) + x\sqrt{a+b\log(cx^n)}$$

```
[Out] -1/2*x*erfi((a+b*ln(c*x^n))^(1/2)/b^(1/2)/n^(1/2))*b^(1/2)*n^(1/2)*Pi^(1/2)
/exp(a/b/n)/((c*x^n)^(1/n))+x*(a+b*ln(c*x^n))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2333, 2337, 2211, 2235}

$$\int \sqrt{a+b\log(cx^n)}\,dx = x\sqrt{a+b\log(cx^n)} - \frac{1}{2}\sqrt{\pi}\sqrt{b}\sqrt{n}xe^{-\frac{a}{bn}}(cx^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{a+b\log(cx^n)}}{\sqrt{b}\sqrt{n}}\right)$$

```
[In] Int[Sqrt[a + b*Log[c*x^n]],x]
```

```
[Out] -1/2*(Sqrt[b]*Sqrt[n]*Sqrt[Pi]*x*Erfi[Sqrt[a + b*Log[c*x^n]]/(Sqrt[b]*Sqrt[
n])])/(E^(a/(b*n))*(c*x^n)^n^(-1)) + x*Sqrt[a + b*Log[c*x^n]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\sqrt{a+b\log\left(cx^n\right)}-\frac{1}{2}(bn)\int\frac{1}{\sqrt{a+b\log\left(cx^n\right)}}\,dx\\
&= x\sqrt{a+b\log\left(cx^n\right)}-\frac{1}{2}\left(bx\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{a+bx}}\,dx,x,\log\left(cx^n\right)\right)\\
&= x\sqrt{a+b\log\left(cx^n\right)}-\left(x\left(cx^n\right)^{-1/n}\right)\text{Subst}\left(\int e^{-\frac{a}{bn}+\frac{x^2}{bn}}\,dx,x,\sqrt{a+b\log\left(cx^n\right)}\right)\\
&= -\frac{1}{2}\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}x\left(cx^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log\left(cx^n\right)}}{\sqrt{b}\sqrt{n}}\right)+x\sqrt{a+b\log\left(cx^n\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int\sqrt{a+b\log\left(cx^n\right)}\,dx=-\frac{1}{2}\sqrt{b}e^{-\frac{a}{bn}}\sqrt{n}\sqrt{\pi}x\left(cx^n\right)^{-1/n}\text{erfi}\left(\frac{\sqrt{a+b\log\left(cx^n\right)}}{\sqrt{b}\sqrt{n}}\right)+x\sqrt{a+b\log\left(cx^n\right)}$$

```
[In] Integrate[Sqrt[a + b*Log[c*x^n]],x]
[Out] -1/2*(Sqrt[b]*Sqrt[n]*Sqrt[Pi]*x*Erfi[Sqrt[a + b*Log[c*x^n]]/(Sqrt[b]*Sqrt[
n]])])/(E^(a/(b*n))*(c*x^n)^n^(-1)) + x*Sqrt[a + b*Log[c*x^n]]
```

Maple [F]

$$\int \sqrt{a+b\ln\left(c\,x^{n}\right)}dx$$

```
[In] int((a+b*ln(c*x^n))^(1/2),x)
[Out] int((a+b*ln(c*x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\log\left(cx^{n}\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a+b\log\left(cx^{n}\right)}\,dx = \int \sqrt{a+b\log\left(cx^{n}\right)}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(sqrt(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \sqrt{a+b\log\left(cx^{n}\right)}\,dx = \int \sqrt{b\log\left(cx^{n}\right)+a}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\log\left(cx^n\right)}\,dx=\int \sqrt{b\log\left(cx^n\right)+a}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\log\left(cx^n\right)}\,dx=\int \sqrt{a+b\,\ln\left(c\,x^n\right)}\,dx$$

```
[In] int((a + b*log(c*x^n))^(1/2),x)
[Out] int((a + b*log(c*x^n))^(1/2), x)
```

3.114 $\int x^3\sqrt{\log(ax^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 64

$$\int x^3\sqrt{\log(ax^n)}\,dx = -\frac{1}{16}\sqrt{n}\sqrt{\pi}x^4(ax^n)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)+\frac{1}{4}x^4\sqrt{\log(ax^n)}$$

```
[Out] -1/16*x^4*erfi(2*ln(a*x^n)^(1/2)/n^(1/2))*n^(1/2)*Pi^(1/2)/((a*x^n)^(4/n))+
1/4*x^4*ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2342, 2347, 2211, 2235}

$$\int x^3\sqrt{\log(ax^n)}\,dx = \frac{1}{4}x^4\sqrt{\log(ax^n)}-\frac{1}{16}\sqrt{\pi}\sqrt{n}x^4(ax^n)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

```
[In] Int[x^3*Sqrt[Log[a*x^n]],x]
```

```
[Out] -1/16*(Sqrt[n]*Sqrt[Pi]*x^4*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^n)^(4/
n) + (x^4*Sqrt[Log[a*x^n]])/4
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\sqrt{\log\left(ax^n\right)} - \frac{1}{8}n\int \frac{x^3}{\sqrt{\log\left(ax^n\right)}}\,dx \\
&= \frac{1}{4}x^4\sqrt{\log\left(ax^n\right)} - \frac{1}{8}\left(x^4\left(ax^n\right)^{-4/n}\right)\text{Subst}\left(\int \frac{e^{\frac{4x}{n}}}{\sqrt{x}}\,dx, x, \log\left(ax^n\right)\right) \\
&= \frac{1}{4}x^4\sqrt{\log\left(ax^n\right)} - \frac{1}{4}\left(x^4\left(ax^n\right)^{-4/n}\right)\text{Subst}\left(\int e^{\frac{4x^2}{n}}\,dx, x, \sqrt{\log\left(ax^n\right)}\right) \\
&= -\frac{1}{16}\sqrt{n}\sqrt{\pi}x^4\left(ax^n\right)^{-4/n}\text{erfi}\left(\frac{2\sqrt{\log\left(ax^n\right)}}{\sqrt{n}}\right) + \frac{1}{4}x^4\sqrt{\log\left(ax^n\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int x^3\sqrt{\log\left(ax^n\right)}\,dx = \frac{1}{16}x^4\left(-\sqrt{n}\sqrt{\pi}\left(ax^n\right)^{-4/n}\text{erfi}\left(\frac{2\sqrt{\log\left(ax^n\right)}}{\sqrt{n}}\right) + 4\sqrt{\log\left(ax^n\right)}\right)$$

```
[In] Integrate[x^3*Sqrt[Log[a*x^n]],x]
[Out] (x^4*(-((Sqrt[n]*Sqrt[Pi]*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^n)^(4/n)
) + 4*Sqrt[Log[a*x^n]]))/16
```

Maple [F]

$$\int x^{3} \sqrt{\ln \left(a \,x^{n}\right)}d x$$

```
[In] int(x^3*ln(a*x^n)^(1/2),x)
[Out] int(x^3*ln(a*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \sqrt{\log (ax^n)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \sqrt{\log (ax^n)}\, dx = \int x^{3} \sqrt{\log \left(a x^{n}\right)}\, dx$$

```
[In] integrate(x**3*ln(a*x**n)**(1/2),x)
[Out] Integral(x**3*sqrt(log(a*x**n)), x)
```

Maxima [F]

$$\int x^3 \sqrt{\log (ax^n)}\, dx = \int x^{3} \sqrt{\log \left(a x^{n}\right)}\, dx$$

```
[In] integrate(x^3*log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3*sqrt(log(a*x^n)), x)
```

Giac [F]

$$\int x^3 \sqrt{\log (ax^n)}\, dx = \int x^3 \sqrt{\log (ax^n)}\, dx$$

```
[In] integrate(x^3*log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^3*sqrt(log(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt{\log (ax^n)}\, dx = \int x^3 \sqrt{\ln (a\, x^n)}\, dx$$

```
[In] int(x^3*log(a*x^n)^(1/2),x)
[Out] int(x^3*log(a*x^n)^(1/2), x)
```

3.115 $\int x^2 \sqrt{\log(ax^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int x^2\sqrt{\log(ax^n)}\,dx = -\frac{1}{6}\sqrt{n}\sqrt{\frac{\pi}{3}}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)+\frac{1}{3}x^3\sqrt{\log(ax^n)}$$

```
[Out] -1/18*x^3*erfi(3^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*n^(1/2)*3^(1/2)*Pi^(1/2)/((
a*x^n)^(3/n))+1/3*x^3*ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2342, 2347, 2211, 2235}

$$\int x^2\sqrt{\log(ax^n)}\,dx = \frac{1}{3}x^3\sqrt{\log(ax^n)}-\frac{1}{6}\sqrt{\frac{\pi}{3}}\sqrt{n}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

```
[In] Int[x^2*Sqrt[Log[a*x^n]],x]
```

```
[Out] -1/6*(Sqrt[n]*Sqrt[Pi/3]*x^3*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x
^n)^(3/n) + (x^3*Sqrt[Log[a*x^n]])/3
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\sqrt{\log(ax^n)} - \frac{1}{6}n\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx \\
&= \frac{1}{3}x^3\sqrt{\log(ax^n)} - \frac{1}{6}\left(x^3(ax^n)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right) \\
&= \frac{1}{3}x^3\sqrt{\log(ax^n)} - \frac{1}{3}\left(x^3(ax^n)^{-3/n}\right)\text{Subst}\left(\int e^{\frac{3x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right) \\
&= -\frac{1}{6}\sqrt{n}\sqrt{\frac{\pi}{3}}x^3(ax^n)^{-3/n}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{3}x^3\sqrt{\log(ax^n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int x^2\sqrt{\log(ax^n)}\,dx = \frac{1}{18}x^3\left(-\sqrt{n}\sqrt{3\pi}(ax^n)^{-3/n}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + 6\sqrt{\log(ax^n)}\right)$$

```
[In] Integrate[x^2*Sqrt[Log[a*x^n]],x]
[Out] (x^3*(-((Sqrt[n]*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^
n)^(3/n)) + 6*Sqrt[Log[a*x^n]]))/18
```

Maple [F]

$$\int x^2\sqrt{\ln(a\,x^n)}dx$$

```
[In] int(x^2*ln(a*x^n)^(1/2),x)
[Out] int(x^2*ln(a*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{\log(ax^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2\sqrt{\log(ax^n)}\,dx = \int x^2\sqrt{\log(ax^n)}\,dx$$

```
[In] integrate(x**2*ln(a*x**n)**(1/2),x)
[Out] Integral(x**2*sqrt(log(a*x**n)), x)
```

Maxima [F]

$$\int x^2\sqrt{\log(ax^n)}\,dx = \int x^2\sqrt{\log(ax^n)}\,dx$$

```
[In] integrate(x^2*log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*sqrt(log(a*x^n)), x)
```

Giac [F]

$$\int x^2\sqrt{\log\left(ax^n\right)}\,dx = \int x^2\sqrt{\log\left(ax^n\right)}\,dx$$

```
[In] integrate(x^2*log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*sqrt(log(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{\log\left(ax^n\right)}\,dx = \int x^2\,\sqrt{\ln\left(a\,x^n\right)}\,dx$$

```
[In] int(x^2*log(a*x^n)^(1/2),x)
[Out] int(x^2*log(a*x^n)^(1/2), x)
```

3.116 $\int x\sqrt{\log(ax^n)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int x\sqrt{\log(ax^n)}\,dx = -\frac{1}{4}\sqrt{n}\sqrt{\frac{\pi}{2}}x^2(ax^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)+\frac{1}{2}x^2\sqrt{\log(ax^n)}$$

```
[Out] -1/8*x^2*erfi(2^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*n^(1/2)*2^(1/2)*Pi^(1/2)/((a
*x^n)^(2/n))+1/2*x^2*ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2342, 2347, 2211, 2235}

$$\int x\sqrt{\log(ax^n)}\,dx = \frac{1}{2}x^2\sqrt{\log(ax^n)}-\frac{1}{4}\sqrt{\frac{\pi}{2}}\sqrt{n}x^2(ax^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)$$

```
[In] Int[x*Sqrt[Log[a*x^n]],x]
```

```
[Out] -1/4*(Sqrt[n]*Sqrt[Pi/2]*x^2*Erfi[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x
^n)^(2/n) + (x^2*Sqrt[Log[a*x^n]])/2
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\sqrt{\log\left(ax^n\right)} - \frac{1}{4}n\int\frac{x}{\sqrt{\log\left(ax^n\right)}}\,dx \\
&= \frac{1}{2}x^2\sqrt{\log\left(ax^n\right)} - \frac{1}{4}\left(x^2\left(ax^n\right)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{x}}\,dx, x, \log\left(ax^n\right)\right) \\
&= \frac{1}{2}x^2\sqrt{\log\left(ax^n\right)} - \frac{1}{2}\left(x^2\left(ax^n\right)^{-2/n}\right)\text{Subst}\left(\int e^{\frac{2x^2}{n}}\,dx, x, \sqrt{\log\left(ax^n\right)}\right) \\
&= -\frac{1}{4}\sqrt{n}\sqrt{\frac{\pi}{2}}x^2\left(ax^n\right)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{\log\left(ax^n\right)}}{\sqrt{n}}\right) + \frac{1}{2}x^2\sqrt{\log\left(ax^n\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int x\sqrt{\log\left(ax^n\right)}\,dx = \frac{1}{8}x^2\left(-\sqrt{n}\sqrt{2\pi}\left(ax^n\right)^{-2/n}\text{erfi}\left(\frac{\sqrt{2}\sqrt{\log\left(ax^n\right)}}{\sqrt{n}}\right) + 4\sqrt{\log\left(ax^n\right)}\right)$$

```
[In] Integrate[x*Sqrt[Log[a*x^n]],x]
[Out] (x^2*(-((Sqrt[n]*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^
n)^(2/n)) + 4*Sqrt[Log[a*x^n]]))/8
```

Maple [F]

$$\int x\sqrt{\ln\left(a\,x^{n}\right)}dx$$

```
[In] int(x*ln(a*x^n)^(1/2),x)
[Out] int(x*ln(a*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\log\left(ax^{n}\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\sqrt{\log\left(ax^{n}\right)}\,dx = \int x\sqrt{\log\left(ax^{n}\right)}\,dx$$

```
[In] integrate(x*ln(a*x**n)**(1/2),x)
[Out] Integral(x*sqrt(log(a*x**n)), x)
```

Maxima [F]

$$\int x\sqrt{\log\left(ax^{n}\right)}\,dx = \int x\sqrt{\log\left(ax^{n}\right)}\,dx$$

```
[In] integrate(x*log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(log(a*x^n)), x)
```

Giac [F]

$$\int x\sqrt{\log\left(ax^n\right)}\,dx = \int x\sqrt{\log\left(ax^n\right)}\,dx$$

```
[In] integrate(x*log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x*sqrt(log(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\log\left(ax^n\right)}\,dx = \int x\,\sqrt{\ln\left(a\,x^n\right)}\,dx$$

```
[In] int(x*log(a*x^n)^(1/2),x)
[Out] int(x*log(a*x^n)^(1/2), x)
```

3.117 $\int \sqrt{\log (a x^n)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \sqrt{\log (a x^n)}\,dx=-\frac{1}{2}\sqrt{n}\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log (ax^n)}}{\sqrt{n}}\right)+x\sqrt{\log (ax^n)}$$

```
[Out] -1/2*x*erfi(ln(a*x^n)^(1/2)/n^(1/2))*n^(1/2)*Pi^(1/2)/((a*x^n)^(1/n))+x*ln(
a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2333, 2337, 2211, 2235}

$$\int \sqrt{\log (ax^n)}\,dx=x\sqrt{\log (ax^n)}-\frac{1}{2}\sqrt{\pi}\sqrt{n}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log (ax^n)}}{\sqrt{n}}\right)$$

```
[In] Int[Sqrt[Log[a*x^n]],x]
```

```
[Out] -1/2*(Sqrt[n]*Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(a*x^n)^n^(-1) + x
*Sqrt[Log[a*x^n]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\sqrt{\log(ax^n)} - \frac{1}{2}n\int \frac{1}{\sqrt{\log(ax^n)}}\,dx \\
&= x\sqrt{\log(ax^n)} - \frac{1}{2}\left(x(ax^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right) \\
&= x\sqrt{\log(ax^n)} - \left(x(ax^n)^{-1/n}\right)\text{Subst}\left(\int e^{\frac{x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right) \\
&= -\frac{1}{2}\sqrt{n}\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + x\sqrt{\log(ax^n)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \sqrt{\log(ax^n)}\,dx = -\frac{1}{2}\sqrt{n}\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + x\sqrt{\log(ax^n)}$$

```
[In] Integrate[Sqrt[Log[a*x^n]],x]
[Out] -1/2*(Sqrt[n]*Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(a*x^n)^n^(-1) + x
*Sqrt[Log[a*x^n]]
```

Maple [F]

$$\int \sqrt{\ln\left(a\,x^{n}\right)}dx$$

```
[In] int(ln(a*x^n)^(1/2),x)
[Out] int(ln(a*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\log\left(ax^{n}\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{\log\left(ax^{n}\right)}\,dx = \int \sqrt{\log\left(ax^{n}\right)}\,dx$$

```
[In] integrate(ln(a*x**n)**(1/2),x)
[Out] Integral(sqrt(log(a*x**n)), x)
```

Maxima [F]

$$\int \sqrt{\log\left(ax^{n}\right)}\,dx = \int \sqrt{\log\left(ax^{n}\right)}\,dx$$

```
[In] integrate(log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(log(a*x^n)), x)
```

Giac [F]

$$\int \sqrt{\log \left(ax^{n}\right)}\,dx = \int \sqrt{\log \left(ax^{n}\right)}\,dx$$

```
[In] integrate(log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(log(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\log \left(ax^{n}\right)}\,dx = \int \sqrt{\ln \left(a\,x^{n}\right)}\,dx$$

```
[In] int(log(a*x^n)^(1/2),x)
[Out] int(log(a*x^n)^(1/2), x)
```

3.118 $\int \frac{\sqrt{\log(ax^n)}}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 17

$$\int \frac{\sqrt{\log(ax^n)}}{x} dx = \frac{2\log^{\frac{3}{2}}(ax^n)}{3n}$$

```
[Out] 2/3*ln(a*x^n)^(3/2)/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2339, 30}

$$\int \frac{\sqrt{\log(ax^n)}}{x} dx = \frac{2\log^{\frac{3}{2}}(ax^n)}{3n}$$

```
[In] Int[Sqrt[Log[a*x^n]]/x,x]
```

```
[Out] (2*Log[a*x^n]^(3/2))/(3*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{x}\,dx, x, \log(ax^n)\right)}{n}\\ &= \frac{2\log^{\frac{3}{2}}(ax^n)}{3n}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\log(ax^n)}}{x}\,dx = \frac{2\log^{\frac{3}{2}}(ax^n)}{3n}$$

```
[In] Integrate[Sqrt[Log[a*x^n]]/x,x]
[Out] (2*Log[a*x^n]^(3/2))/(3*n)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\ln(a\,x^n)^{\frac{3}{2}}}{3n}$	14
default	$\frac{2\ln(a\,x^n)^{\frac{3}{2}}}{3n}$	14

```
[In] int(ln(a*x^n)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/3*ln(a*x^n)^(3/2)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\log(ax^n)}}{x}\,dx = \frac{2\,(n\log(x) + \log(a))^{\frac{3}{2}}}{3\,n}$$

```
[In] integrate(log(a*x^n)^(1/2)/x,x, algorithm="fricas")
[Out] 2/3*(n*log(x) + log(a))^(3/2)/n
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{\log(ax^n)}}{x}\,dx = -\begin{cases} -\sqrt{\log(a)}\log(x) & \text{for } n = 0 \\ -\frac{2\log(ax^n)^{\frac{3}{2}}}{3n} & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(a*x**n)**(1/2)/x,x)
[Out] -Piecewise((-sqrt(log(a))*log(x), Eq(n, 0)), (-2*log(a*x**n)**(3/2)/(3*n),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{\log(ax^n)}}{x}\,dx = \frac{2\,\log(ax^n)^{\frac{3}{2}}}{3\,n}$$

```
[In] integrate(log(a*x^n)^(1/2)/x,x, algorithm="maxima")
[Out] 2/3*log(a*x^n)^(3/2)/n
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{\log(ax^n)}}{x}\,dx = \frac{2\,(n\log(x) + \log(a))^{\frac{3}{2}}}{3\,n}$$

```
[In] integrate(log(a*x^n)^(1/2)/x,x, algorithm="giac")
[Out] 2/3*(n*log(x) + log(a))^(3/2)/n
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{\log(ax^n)}}{x}\,dx = \frac{2\ln(a\,x^n)^{3/2}}{3\,n}$$

```
[In] int(log(a*x^n)^(1/2)/x,x)
[Out] (2*log(a*x^n)^(3/2))/(3*n)
```

3.119 $\int \frac{\sqrt{\log(ax^n)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \frac{\sqrt{\log(ax^n)}}{x^2}\,dx=\frac{\sqrt{n}\sqrt{\pi}(ax^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2x}-\frac{\sqrt{\log(ax^n)}}{x}$$

```
[Out] 1/2*(a*x^n)^(1/n)*erf(ln(a*x^n)^(1/2)/n^(1/2))*n^(1/2)*Pi^(1/2)/x-ln(a*x^n)
^(1/2)/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2342, 2347, 2211, 2236}

$$\int \frac{\sqrt{\log(ax^n)}}{x^2}\,dx=\frac{\sqrt{\pi}\sqrt{n}(ax^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2x}-\frac{\sqrt{\log(ax^n)}}{x}$$

```
[In] Int[Sqrt[Log[a*x^n]]/x^2,x]
```

```
[Out] (Sqrt[n]*Sqrt[Pi]*(a*x^n)^n^(-1)*Erf[Sqrt[Log[a*x^n]]/Sqrt[n]])/(2*x) - Sqr
t[Log[a*x^n]]/x
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\log(ax^n)}}{x}+\frac{1}{2}n\int\frac{1}{x^2\sqrt{\log(ax^n)}}\,dx\\
&= -\frac{\sqrt{\log(ax^n)}}{x}+\frac{(ax^n)^{\frac{1}{n}}\,\text{Subst}\left(\int\frac{e^{-\frac{x}{n}}}{\sqrt{x}}\,dx,x,\log(ax^n)\right)}{2x}\\
&= -\frac{\sqrt{\log(ax^n)}}{x}+\frac{(ax^n)^{\frac{1}{n}}\,\text{Subst}\left(\int e^{-\frac{x^2}{n}}\,dx,x,\sqrt{\log(ax^n)}\right)}{x}\\
&= \frac{\sqrt{n}\sqrt{\pi}(ax^n)^{\frac{1}{n}}\,\text{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2x}-\frac{\sqrt{\log(ax^n)}}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int\frac{\sqrt{\log(ax^n)}}{x^2}\,dx=-\frac{2\log(ax^n)+n(ax^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{2},\frac{\log(ax^n)}{n}\right)\sqrt{\frac{\log(ax^n)}{n}}}{2x\sqrt{\log(ax^n)}}$$

```
[In] Integrate[Sqrt[Log[a*x^n]]/x^2,x]
```

```
[Out] -1/2*(2*Log[a*x^n] + n*(a*x^n)^n^(-1)*Gamma[1/2, Log[a*x^n]/n]*Sqrt[Log[a*x
^n]/n])/(x*Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{\sqrt{\ln \left(a\,x^{n}\right)}}{x^{2}}dx$$

```
[In] int(ln(a*x^n)^(1/2)/x^2,x)
[Out] int(ln(a*x^n)^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\log \left(ax^{n}\right)}}{x^{2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(a*x^n)^(1/2)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{\log \left(ax^{n}\right)}}{x^{2}}\,dx = \int \frac{\sqrt{\log \left(ax^{n}\right)}}{x^{2}}\,dx$$

```
[In] integrate(ln(a*x**n)**(1/2)/x**2,x)
[Out] Integral(sqrt(log(a*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{\log \left(ax^{n}\right)}}{x^{2}}\,dx = \int \frac{\sqrt{\log \left(ax^{n}\right)}}{x^{2}}\,dx$$

```
[In] integrate(log(a*x^n)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(log(a*x^n))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{\log(ax^n)}}{x^2}\,dx = \int \frac{\sqrt{\log(ax^n)}}{x^2}\,dx$$

[In] integrate(log(a*x^n)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(log(a*x^n))/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\log(ax^n)}}{x^2}\,dx = \int \frac{\sqrt{\ln(a\,x^n)}}{x^2}\,dx$$

[In] int(log(a*x^n)^(1/2)/x^2,x)

[Out] int(log(a*x^n)^(1/2)/x^2, x)

3.120 $\int \frac{\sqrt{\log(ax^n)}}{x^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int \frac{\sqrt{\log(ax^n)}}{x^3} \, dx = \frac{\sqrt{n}\sqrt{\frac{\pi}{2}}(ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x^2} - \frac{\sqrt{\log(ax^n)}}{2x^2}$$

```
[Out] 1/8*(a*x^n)^(2/n)*erf(2^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*n^(1/2)*2^(1/2)*Pi^(
1/2)/x^2-1/2*ln(a*x^n)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2342, 2347, 2211, 2236}

$$\int \frac{\sqrt{\log(ax^n)}}{x^3} \, dx = \frac{\sqrt{\frac{\pi}{2}}\sqrt{n}(ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x^2} - \frac{\sqrt{\log(ax^n)}}{2x^2}$$

```
[In] Int[Sqrt[Log[a*x^n]]/x^3,x]
```

```
[Out] (Sqrt[n]*Sqrt[Pi/2]*(a*x^n)^(2/n)*Erf[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/
(4*x^2) - Sqrt[Log[a*x^n]]/(2*x^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{\log(ax^n)}}{2x^2}+\frac{1}{4}n\int\frac{1}{x^3\sqrt{\log(ax^n)}}\,dx\\
&= -\frac{\sqrt{\log(ax^n)}}{2x^2}+\frac{(ax^n)^{2/n}\,\text{Subst}\left(\int\frac{e^{-\frac{2x}{n}}}{\sqrt{x}}\,dx,x,\log(ax^n)\right)}{4x^2}\\
&= -\frac{\sqrt{\log(ax^n)}}{2x^2}+\frac{(ax^n)^{2/n}\,\text{Subst}\left(\int e^{-\frac{2x^2}{n}}\,dx,x,\sqrt{\log(ax^n)}\right)}{2x^2}\\
&= \frac{\sqrt{n}\sqrt{\frac{\pi}{2}}(ax^n)^{2/n}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x^2}-\frac{\sqrt{\log(ax^n)}}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int\frac{\sqrt{\log(ax^n)}}{x^3}\,dx=-\frac{4\log(ax^n)+\sqrt{2}n(ax^n)^{2/n}\,\Gamma\left(\frac{1}{2},\frac{2\log(ax^n)}{n}\right)\sqrt{\frac{\log(ax^n)}{n}}}{8x^2\sqrt{\log(ax^n)}}$$

```
[In] Integrate[Sqrt[Log[a*x^n]]/x^3,x]
```

```
[Out] -1/8*(4*Log[a*x^n] + Sqrt[2]*n*(a*x^n)^(2/n)*Gamma[1/2, (2*Log[a*x^n])/n]*S
qrt[Log[a*x^n]/n])/(x^2*Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{\sqrt{\ln \left(a\,x^{n}\right)}}{x^{3}}dx$$

```
[In] int(ln(a*x^n)^(1/2)/x^3,x)
```

```
[Out] int(ln(a*x^n)^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\log \left(ax^{n}\right)}}{x^{3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(a*x^n)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\sqrt{\log \left(ax^{n}\right)}}{x^{3}}\,dx = \int \frac{\sqrt{\log \left(ax^{n}\right)}}{x^{3}}\,dx$$

```
[In] integrate(ln(a*x**n)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(log(a*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{\log \left(ax^{n}\right)}}{x^{3}}\,dx = \int \frac{\sqrt{\log \left(ax^{n}\right)}}{x^{3}}\,dx$$

```
[In] integrate(log(a*x^n)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(log(a*x^n))/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{\log(ax^n)}}{x^3}\,dx = \int \frac{\sqrt{\log(ax^n)}}{x^3}\,dx$$

```
[In] integrate(log(a*x^n)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(log(a*x^n))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\log(ax^n)}}{x^3}\,dx = \int \frac{\sqrt{\ln(a\,x^n)}}{x^3}\,dx$$

```
[In] int(log(a*x^n)^(1/2)/x^3,x)
[Out] int(log(a*x^n)^(1/2)/x^3, x)
```

3.121 $\int x^3 \log^{\frac{3}{2}}(ax^n)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 82

$$\int x^3 \log^{\frac{3}{2}}(ax^n)\, dx = \frac{3}{128} n^{3/2} \sqrt{\pi} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{3}{32} n x^4 \sqrt{\log(ax^n)} + \frac{1}{4} x^4 \log^{\frac{3}{2}}(ax^n)$$

```
[Out] 1/4*x^4*ln(a*x^n)^(3/2)+3/128*n^(3/2)*x^4*erfi(2*ln(a*x^n)^(1/2)/n^(1/2))*P
i^(1/2)/((a*x^n)^(4/n))-3/32*n*x^4*ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2342, 2347, 2211, 2235}

$$\int x^3 \log^{\frac{3}{2}}(ax^n)\, dx = \frac{3}{128} \sqrt{\pi} n^{3/2} x^4 (ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{4} x^4 \log^{\frac{3}{2}}(ax^n) - \frac{3}{32} n x^4 \sqrt{\log(ax^n)}$$

```
[In] Int[x^3*Log[a*x^n]^(3/2),x]
```

```
[Out] (3*n^(3/2)*Sqrt[Pi]*x^4*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(128*(a*x^n)^(4
/n)) - (3*n*x^4*Sqrt[Log[a*x^n]])/32 + (x^4*Log[a*x^n]^(3/2))/4
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\log^{\frac{3}{2}}(ax^n) - \frac{1}{8}(3n)\int x^3\sqrt{\log(ax^n)}\,dx \\
&= -\frac{3}{32}nx^4\sqrt{\log(ax^n)} + \frac{1}{4}x^4\log^{\frac{3}{2}}(ax^n) + \frac{1}{64}\left(3n^2\right)\int \frac{x^3}{\sqrt{\log(ax^n)}}\,dx \\
&= -\frac{3}{32}nx^4\sqrt{\log(ax^n)} + \frac{1}{4}x^4\log^{\frac{3}{2}}(ax^n) + \frac{1}{64}\left(3nx^4(ax^n)^{-4/n}\right)\text{Subst}\left(\int \frac{e^{\frac{4x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right) \\
&= -\frac{3}{32}nx^4\sqrt{\log(ax^n)} + \frac{1}{4}x^4\log^{\frac{3}{2}}(ax^n) \\
&\quad + \frac{1}{32}\left(3nx^4(ax^n)^{-4/n}\right)\text{Subst}\left(\int e^{\frac{4x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right) \\
&= \frac{3}{128}n^{3/2}\sqrt{\pi}x^4(ax^n)^{-4/n}\,\text{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{3}{32}nx^4\sqrt{\log(ax^n)} + \frac{1}{4}x^4\log^{\frac{3}{2}}(ax^n)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$\int x^3 \log^{\frac{3}{2}}(ax^n)\,dx$$
$$= \frac{1}{128}x^4\left(3n^{3/2}\sqrt{\pi}(ax^n)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)+4\sqrt{\log(ax^n)}(-3n+8\log(ax^n))\right)$$

```
[In] Integrate[x^3*Log[a*x^n]^(3/2),x]
[Out] (x^4*((3*n^(3/2)*Sqrt[Pi]*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^n)^(4/n)
 + 4*Sqrt[Log[a*x^n]]*(-3*n + 8*Log[a*x^n])))/128
```

Maple [F]

$$\int x^3 \ln(a\,x^n)^{\frac{3}{2}}\,dx$$

```
[In] int(x^3*ln(a*x^n)^(3/2),x)
[Out] int(x^3*ln(a*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \log^{\frac{3}{2}}(ax^n)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*log(a*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \log^{\frac{3}{2}}(ax^n)\,dx = \int x^3 \log(ax^n)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**3*ln(a*x**n)**(3/2),x)
[Out] Integral(x**3*log(a*x**n)**(3/2), x)
```

Maxima [F]

$$\int x^3 \log^{\frac{3}{2}}(ax^n)\, dx = \int x^3 \log(ax^n)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x^3*log(a*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3*log(a*x^n)^(3/2), x)
```

Giac [F]

$$\int x^3 \log^{\frac{3}{2}}(ax^n)\, dx = \int x^3 \log(ax^n)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x^3*log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(x^3*log(a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \log^{\frac{3}{2}}(ax^n)\, dx = \int x^3 \ln\left(a\,x^n\right)^{3/2} dx$$

```
[In] int(x^3*log(a*x^n)^(3/2),x)
[Out] int(x^3*log(a*x^n)^(3/2), x)
```

3.122 $\int x^2 \log^{\frac{3}{2}}(ax^n)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int x^2 \log^{\frac{3}{2}}(ax^n)\, dx = \frac{1}{12} n^{3/2} \sqrt{\frac{\pi}{3}} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{1}{6} n x^3 \sqrt{\log(ax^n)} + \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n)$$

```
[Out] 1/3*x^3*ln(a*x^n)^(3/2)+1/36*n^(3/2)*x^3*erfi(3^(1/2)*ln(a*x^n)^(1/2)/n^(1/
2))*3^(1/2)*Pi^(1/2)/((a*x^n)^(3/n))-1/6*n*x^3*ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2342, 2347, 2211, 2235}

$$\int x^2 \log^{\frac{3}{2}}(ax^n)\, dx = \frac{1}{12} \sqrt{\frac{\pi}{3}} n^{3/2} x^3 (ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{3} x^3 \log^{\frac{3}{2}}(ax^n) - \frac{1}{6} n x^3 \sqrt{\log(ax^n)}$$

```
[In] Int[x^2*Log[a*x^n]^(3/2),x]
```

```
[Out] (n^(3/2)*Sqrt[Pi/3]*x^3*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(12*(a*x^
n)^(3/n)) - (n*x^3*Sqrt[Log[a*x^n]])/6 + (x^3*Log[a*x^n]^(3/2))/3
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\log^{\frac{3}{2}}(ax^n) - \frac{1}{2}n\int x^2\sqrt{\log(ax^n)}\,dx \\
&= -\frac{1}{6}nx^3\sqrt{\log(ax^n)} + \frac{1}{3}x^3\log^{\frac{3}{2}}(ax^n) + \frac{1}{12}n^2\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx \\
&= -\frac{1}{6}nx^3\sqrt{\log(ax^n)} + \frac{1}{3}x^3\log^{\frac{3}{2}}(ax^n) + \frac{1}{12}\left(nx^3(ax^n)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right) \\
&= -\frac{1}{6}nx^3\sqrt{\log(ax^n)} + \frac{1}{3}x^3\log^{\frac{3}{2}}(ax^n) + \frac{1}{6}\left(nx^3(ax^n)^{-3/n}\right)\text{Subst}\left(\int e^{\frac{3x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right) \\
&= \frac{1}{12}n^{3/2}\sqrt{\frac{\pi}{3}}x^3(ax^n)^{-3/n}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{1}{6}nx^3\sqrt{\log(ax^n)} + \frac{1}{3}x^3\log^{\frac{3}{2}}(ax^n)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int x^2 \log^{\frac{3}{2}}(ax^n)\, dx = \frac{1}{36}x^3 \left(n^{3/2}\sqrt{3\pi}(ax^n)^{-3/n} \operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - 6(n - 2\log(ax^n))\sqrt{\log(ax^n)}\right)$$

```
[In] Integrate[x^2*Log[a*x^n]^(3/2),x]
[Out] (x^3*((n^(3/2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^n)
^(3/n) - 6*(n - 2*Log[a*x^n])*Sqrt[Log[a*x^n]]))/36
```

Maple [F]

$$\int x^2 \ln(a\,x^n)^{\frac{3}{2}}\, dx$$

```
[In] int(x^2*ln(a*x^n)^(3/2),x)
[Out] int(x^2*ln(a*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \log^{\frac{3}{2}}(ax^n)\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*log(a*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \log^{\frac{3}{2}}(ax^n)\, dx = \int x^2 \log(ax^n)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x**2*ln(a*x**n)**(3/2),x)
[Out] Integral(x**2*log(a*x**n)**(3/2), x)
```

Maxima [F]

$$\int x^2 \log^{\frac{3}{2}} (ax^n) \, dx = \int x^2 \log (ax^n)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x^2*log(a*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2*log(a*x^n)^(3/2), x)
```

Giac [F]

$$\int x^2 \log^{\frac{3}{2}} (ax^n) \, dx = \int x^2 \log (ax^n)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x^2*log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(x^2*log(a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \log^{\frac{3}{2}} (ax^n) \, dx = \int x^2 \ln \left(a\,x^n\right)^{3/2} \,dx$$

```
[In] int(x^2*log(a*x^n)^(3/2),x)
[Out] int(x^2*log(a*x^n)^(3/2), x)
```

3.123 $\int x \log^{\frac{3}{2}}(ax^n)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int x \log^{\frac{3}{2}}(ax^n)\,dx = \frac{3}{16}n^{3/2}\sqrt{\frac{\pi}{2}}x^2(ax^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{3}{8}nx^2\sqrt{\log(ax^n)} + \frac{1}{2}x^2\log^{\frac{3}{2}}(ax^n)$$

```
[Out] 1/2*x^2*ln(a*x^n)^(3/2)+3/32*n^(3/2)*x^2*erfi(2^(1/2)*ln(a*x^n)^(1/2)/n^(1/
2))*2^(1/2)*Pi^(1/2)/((a*x^n)^(2/n))-3/8*n*x^2*ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2342, 2347, 2211, 2235}

$$\int x \log^{\frac{3}{2}}(ax^n)\,dx = \frac{3}{16}\sqrt{\frac{\pi}{2}}n^{3/2}x^2(ax^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + \frac{1}{2}x^2\log^{\frac{3}{2}}(ax^n) - \frac{3}{8}nx^2\sqrt{\log(ax^n)}$$

```
[In] Int[x*Log[a*x^n]^(3/2),x]
```

```
[Out] (3*n^(3/2)*Sqrt[Pi/2]*x^2*Erfi[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(16*(a*
x^n)^(2/n)) - (3*n*x^2*Sqrt[Log[a*x^n]])/8 + (x^2*Log[a*x^n]^(3/2))/2
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\log^{\frac{3}{2}}(ax^n) - \frac{1}{4}(3n)\int x\sqrt{\log(ax^n)}\,dx \\
&= -\frac{3}{8}nx^2\sqrt{\log(ax^n)} + \frac{1}{2}x^2\log^{\frac{3}{2}}(ax^n) + \frac{1}{16}\left(3n^2\right)\int \frac{x}{\sqrt{\log(ax^n)}}\,dx \\
&= -\frac{3}{8}nx^2\sqrt{\log(ax^n)} + \frac{1}{2}x^2\log^{\frac{3}{2}}(ax^n) + \frac{1}{16}\left(3nx^2(ax^n)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right) \\
&= -\frac{3}{8}nx^2\sqrt{\log(ax^n)} + \frac{1}{2}x^2\log^{\frac{3}{2}}(ax^n) + \frac{1}{8}\left(3nx^2(ax^n)^{-2/n}\right)\text{Subst}\left(\int e^{\frac{2x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right) \\
&= \frac{3}{16}n^{3/2}\sqrt{\frac{\pi}{2}}x^2(ax^n)^{-2/n}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{3}{8}nx^2\sqrt{\log(ax^n)} + \frac{1}{2}x^2\log^{\frac{3}{2}}(ax^n)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

$$\int x\log^{\frac{3}{2}}(ax^n)\,dx = \frac{1}{32}x^2\left(3n^{3/2}\sqrt{2\pi}(ax^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)+4\sqrt{\log(ax^n)}(-3n+4\log(ax^n))\right)$$

```
[In] Integrate[x*Log[a*x^n]^(3/2),x]
[Out] (x^2*((3*n^(3/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(a*x^
n)^(2/n) + 4*Sqrt[Log[a*x^n]]*(-3*n + 4*Log[a*x^n])))/32
```

Maple [F]

$$\int x\ln(a\,x^n)^{\frac{3}{2}}\,dx$$

```
[In] int(x*ln(a*x^n)^(3/2),x)
[Out] int(x*ln(a*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\log^{\frac{3}{2}}(ax^n)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*log(a*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\log^{\frac{3}{2}}(ax^n)\,dx = \int x\log(ax^n)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*ln(a*x**n)**(3/2),x)
[Out] Integral(x*log(a*x**n)**(3/2), x)
```

Maxima [F]

$$\int x \log^{\frac{3}{2}}(ax^n)\,dx = \int x \log(ax^n)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*log(a*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(x*log(a*x^n)^(3/2), x)
```

Giac [F]

$$\int x \log^{\frac{3}{2}}(ax^n)\,dx = \int x \log(ax^n)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(x*log(a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \log^{\frac{3}{2}}(ax^n)\,dx = \int x \ln(a\,x^n)^{3/2}\,dx$$

```
[In] int(x*log(a*x^n)^(3/2),x)
[Out] int(x*log(a*x^n)^(3/2), x)
```

3.124 $\int \log^{\frac{3}{2}}(ax^n)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 72

$$\int \log^{\frac{3}{2}}(ax^n)\,dx = \frac{3}{4}n^{3/2}\sqrt{\pi}x(ax^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{3}{2}nx\sqrt{\log(ax^n)} + x\log^{\frac{3}{2}}(ax^n)$$

```
[Out] x*ln(a*x^n)^(3/2)+3/4*n^(3/2)*x*erfi(ln(a*x^n)^(1/2)/n^(1/2))*Pi^(1/2)/((a*
x^n)^(1/n))-3/2*n*x*ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2333, 2337, 2211, 2235}

$$\int \log^{\frac{3}{2}}(ax^n)\,dx = \frac{3}{4}\sqrt{\pi}n^{3/2}x(ax^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right) + x\log^{\frac{3}{2}}(ax^n) - \frac{3}{2}nx\sqrt{\log(ax^n)}$$

```
[In] Int[Log[a*x^n]^(3/2),x]
```

```
[Out] (3*n^(3/2)*Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(4*(a*x^n)^n^(-1)) -
(3*n*x*Sqrt[Log[a*x^n]])/2 + x*Log[a*x^n]^(3/2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\log^{\frac{3}{2}}(ax^n) - \frac{1}{2}(3n)\int\sqrt{\log(ax^n)}\,dx \\
&= -\frac{3}{2}nx\sqrt{\log(ax^n)} + x\log^{\frac{3}{2}}(ax^n) + \frac{1}{4}(3n^2)\int\frac{1}{\sqrt{\log(ax^n)}}\,dx \\
&= -\frac{3}{2}nx\sqrt{\log(ax^n)} + x\log^{\frac{3}{2}}(ax^n) + \frac{1}{4}\left(3nx(ax^n)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right) \\
&= -\frac{3}{2}nx\sqrt{\log(ax^n)} + x\log^{\frac{3}{2}}(ax^n) + \frac{1}{2}\left(3nx(ax^n)^{-1/n}\right)\text{Subst}\left(\int e^{\frac{x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right) \\
&= \frac{3}{4}n^{3/2}\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{3}{2}nx\sqrt{\log(ax^n)} + x\log^{\frac{3}{2}}(ax^n)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int\log^{\frac{3}{2}}(ax^n)\,dx = \frac{3}{4}n^{3/2}\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right) - \frac{3}{2}nx\sqrt{\log(ax^n)} + x\log^{\frac{3}{2}}(ax^n)$$

```
[In] Integrate[Log[a*x^n]^(3/2),x]
[Out] (3*n^(3/2)*Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(4*(a*x^n)^n^(-1)) -
(3*n*x*Sqrt[Log[a*x^n]])/2 + x*Log[a*x^n]^(3/2)
```

Maple [F]

$$\int \ln \left(a\,x^{n}\right)^{\frac{3}{2}}dx$$

[In] int(ln(a*x^n)^(3/2),x)

[Out] int(ln(a*x^n)^(3/2),x)

Fricas [F(-2)]

Exception generated.

$$\int \log^{\frac{3}{2}}\left(ax^{n}\right)\,dx=\text{Exception raised: TypeError}$$

[In] integrate(log(a*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [F]

$$\int \log^{\frac{3}{2}}\left(ax^{n}\right)\,dx=\int \log \left(ax^{n}\right)^{\frac{3}{2}}\,dx$$

[In] integrate(ln(a*x**n)**(3/2),x)

[Out] Integral(log(a*x**n)**(3/2), x)

Maxima [F]

$$\int \log^{\frac{3}{2}}\left(ax^{n}\right)\,dx=\int \log \left(ax^{n}\right)^{\frac{3}{2}}\,dx$$

[In] integrate(log(a*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(log(a*x^n)^(3/2), x)

Giac [F]

$$\int \log^{\frac{3}{2}}(ax^n)\,dx = \int \log(ax^n)^{\frac{3}{2}}\,dx$$

```
[In] integrate(log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(log(a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \log^{\frac{3}{2}}(ax^n)\,dx = \int \ln\left(a\,x^n\right)^{3/2}\,dx$$

```
[In] int(log(a*x^n)^(3/2),x)
[Out] int(log(a*x^n)^(3/2), x)
```

3.125 $\int \frac{\log^{\frac{3}{2}}(ax^n)}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 17

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x} dx = \frac{2\log^{\frac{5}{2}}(ax^n)}{5n}$$

```
[Out] 2/5*ln(a*x^n)^(5/2)/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2339, 30}

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x} dx = \frac{2\log^{\frac{5}{2}}(ax^n)}{5n}$$

```
[In] Int[Log[a*x^n]^(3/2)/x,x]
```

```
[Out] (2*Log[a*x^n]^(5/2))/(5*n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int x^{3/2}\,dx, x, \log\left(ax^n\right)\right)}{n}$$
$$= \frac{2\log^{\frac{5}{2}}\left(ax^n\right)}{5n}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\log^{\frac{3}{2}}\left(ax^n\right)}{x}\,dx = \frac{2\log^{\frac{5}{2}}\left(ax^n\right)}{5n}$$

```
[In] Integrate[Log[a*x^n]^(3/2)/x,x]
[Out] (2*Log[a*x^n]^(5/2))/(5*n)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\ln\left(a\,x^{n}\right)^{\frac{5}{2}}}{5n}$	14
default	$\frac{2\ln\left(a\,x^{n}\right)^{\frac{5}{2}}}{5n}$	14

```
[In] int(ln(a*x^n)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/5*ln(a*x^n)^(5/2)/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.00

$$\int \frac{\log^{\frac{3}{2}}\left(ax^n\right)}{x}\,dx = \frac{2\left(n^2\log\left(x\right)^2 + 2\,n\log\left(a\right)\log\left(x\right) + \log\left(a\right)^2\right)\sqrt{n\log\left(x\right) + \log\left(a\right)}}{5\,n}$$

```
[In] integrate(log(a*x^n)^(3/2)/x,x, algorithm="fricas")
[Out] 2/5*(n^2*log(x)^2 + 2*n*log(a)*log(x) + log(a)^2)*sqrt(n*log(x) + log(a))/n
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x}\,dx = \begin{cases} \frac{2\log(ax^n)^{\frac{5}{2}}}{5n} & \text{for } n \neq 0 \\ \log(a)^{\frac{3}{2}}\log(x) & \text{otherwise} \end{cases}$$

```
[In] integrate(ln(a*x**n)**(3/2)/x,x)
```

```
[Out] Piecewise((2*log(a*x**n)**(5/2)/(5*n), Ne(n, 0)), (log(a)**(3/2)*log(x), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x}\,dx = \frac{2\log(ax^n)^{\frac{5}{2}}}{5n}$$

```
[In] integrate(log(a*x^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2/5*log(a*x^n)^(5/2)/n
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(13) = 26.

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.24

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x}\,dx = \frac{2\left(3(n\log(x)+\log(a))^{\frac{5}{2}} - 10(n\log(x)+\log(a))^{\frac{3}{2}}\log(a) + 30\sqrt{n\log(x)+\log(a)}\log(a)^2 + 10\left((n\log\right.\right.}{15n}$$

```
[In] integrate(log(a*x^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2/15*(3*(n*log(x) + log(a))^(5/2) - 10*(n*log(x) + log(a))^(3/2)*log(a) + 3
0*sqrt(n*log(x) + log(a))*log(a)^2 + 10*((n*log(x) + log(a))^(3/2) - 3*sqrt
(n*log(x) + log(a))*log(a))*log(a))/n
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x} \, dx = \frac{2 \ln (a x^n)^{5/2}}{5 n}$$

```
[In] int(log(a*x^n)^(3/2)/x,x)
[Out] (2*log(a*x^n)^(5/2))/(5*n)
```

3.126 $\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^2} \, dx = \frac{3n^{3/2}\sqrt{\pi}(ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x} - \frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x}$$

```
[Out] -ln(a*x^n)^(3/2)/x+3/4*n^(3/2)*(a*x^n)^(1/n)*erf(ln(a*x^n)^(1/2)/n^(1/2))*P
i^(1/2)/x-3/2*n*ln(a*x^n)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2342, 2347, 2211, 2236}

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^2} \, dx = \frac{3\sqrt{\pi}n^{3/2}(ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} - \frac{3n\sqrt{\log(ax^n)}}{2x}$$

```
[In] Int[Log[a*x^n]^(3/2)/x^2,x]
```

```
[Out] (3*n^(3/2)*Sqrt[Pi]*(a*x^n)^n^(-1)*Erf[Sqrt[Log[a*x^n]]/Sqrt[n]])/(4*x) - (
3*n*Sqrt[Log[a*x^n]])/(2*x) - Log[a*x^n]^(3/2)/x
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{1}{2}(3n)\int \frac{\sqrt{\log(ax^n)}}{x^2}\,dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{1}{4}(3n^2)\int \frac{1}{x^2\sqrt{\log(ax^n)}}\,dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{\left(3n(ax^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{4x} \\
&= -\frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x} + \frac{\left(3n(ax^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int e^{-\frac{x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{2x} \\
&= \frac{3n^{3/2}\sqrt{\pi}(ax^n)^{\frac{1}{n}}\,\text{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4x} - \frac{3n\sqrt{\log(ax^n)}}{2x} - \frac{\log^{\frac{3}{2}}(ax^n)}{x}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^2}\,dx = -\frac{6n\log(ax^n) + 4\log^2(ax^n) + 3n^2(ax^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right)\sqrt{\frac{\log(ax^n)}{n}}}{4x\sqrt{\log(ax^n)}}$$

```
[In] Integrate[Log[a*x^n]^(3/2)/x^2,x]
[Out] -1/4*(6*n*Log[a*x^n] + 4*Log[a*x^n]^2 + 3*n^2*(a*x^n)^n^(-1)*Gamma[1/2, Log
[a*x^n]/n]*Sqrt[Log[a*x^n]/n])/(x*Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{\ln\left(a\,x^{n}\right)^{\frac{3}{2}}}{x^{2}}dx$$

```
[In] int(ln(a*x^n)^(3/2)/x^2,x)
[Out] int(ln(a*x^n)^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\log^{\frac{3}{2}}\left(ax^{n}\right)}{x^{2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(a*x^n)^(3/2)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\log^{\frac{3}{2}}\left(ax^{n}\right)}{x^{2}}\,dx = \int \frac{\log\left(ax^{n}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

```
[In] integrate(ln(a*x**n)**(3/2)/x**2,x)
[Out] Integral(log(a*x**n)**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{\log^{\frac{3}{2}}\left(ax^{n}\right)}{x^{2}}\,dx = \int \frac{\log\left(ax^{n}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

```
[In] integrate(log(a*x^n)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate(log(a*x^n)^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^2}\,dx = \int \frac{\log(ax^n)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate(log(a*x^n)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate(log(a*x^n)^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^2}\,dx = \int \frac{\ln(a\,x^n)^{3/2}}{x^2}\,dx$$

```
[In] int(log(a*x^n)^(3/2)/x^2,x)
[Out] int(log(a*x^n)^(3/2)/x^2, x)
```

3.127 $\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 90

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^3} dx = \frac{3n^{3/2}\sqrt{\frac{\pi}{2}}(ax^n)^{2/n}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{16x^2} - \frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2}$$

```
[Out] -1/2*ln(a*x^n)^(3/2)/x^2+3/32*n^(3/2)*(a*x^n)^(2/n)*erf(2^(1/2)*ln(a*x^n)^(
1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/x^2-3/8*n*ln(a*x^n)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2342, 2347, 2211, 2236}

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^3} dx = \frac{3\sqrt{\frac{\pi}{2}}n^{3/2}(ax^n)^{2/n}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{16x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} - \frac{3n\sqrt{\log(ax^n)}}{8x^2}$$

```
[In] Int[Log[a*x^n]^(3/2)/x^3,x]
```

```
[Out] (3*n^(3/2)*Sqrt[Pi/2]*(a*x^n)^(2/n)*Erf[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]]
)/(16*x^2) - (3*n*Sqrt[Log[a*x^n]])/(8*x^2) - Log[a*x^n]^(3/2)/(2*x^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{1}{4}(3n)\int \frac{\sqrt{\log(ax^n)}}{x^3}\,dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{1}{16}(3n^2)\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx \\
&= -\frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{\left(3n(ax^n)^{2/n}\right)\text{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{16x^2} \\
&= -\frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2} + \frac{\left(3n(ax^n)^{2/n}\right)\text{Subst}\left(\int e^{-\frac{2x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{8x^2} \\
&= \frac{3n^{3/2}\sqrt{\frac{\pi}{2}}(ax^n)^{2/n}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{16x^2} - \frac{3n\sqrt{\log(ax^n)}}{8x^2} - \frac{\log^{\frac{3}{2}}(ax^n)}{2x^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^3}\,dx = -\frac{3\sqrt{2}n^2(ax^n)^{2/n}\,\Gamma\left(\frac{1}{2}, \frac{2\log(ax^n)}{n}\right)\sqrt{\frac{\log(ax^n)}{n}} + 4\log(ax^n)(3n + 4\log(ax^n))}{32x^2\sqrt{\log(ax^n)}}$$

```
[In] Integrate[Log[a*x^n]^(3/2)/x^3,x]
[Out] -1/32*(3*Sqrt[2]*n^2*(a*x^n)^(2/n)*Gamma[1/2, (2*Log[a*x^n])/n]*Sqrt[Log[a*
x^n]/n] + 4*Log[a*x^n]*(3*n + 4*Log[a*x^n]))/(x^2*Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{\ln \left(a\,x^{n}\right)^{\frac{3}{2}}}{x^{3}}dx$$

```
[In] int(ln(a*x^n)^(3/2)/x^3,x)
```

```
[Out] int(ln(a*x^n)^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\log^{\frac{3}{2}}\left(ax^{n}\right)}{x^{3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(log(a*x^n)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{\log^{\frac{3}{2}}\left(ax^{n}\right)}{x^{3}}\,dx = \int \frac{\log\left(ax^{n}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

```
[In] integrate(ln(a*x**n)**(3/2)/x**3,x)
```

```
[Out] Integral(log(a*x**n)**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\log^{\frac{3}{2}}\left(ax^{n}\right)}{x^{3}}\,dx = \int \frac{\log\left(ax^{n}\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

```
[In] integrate(log(a*x^n)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(log(a*x^n)^(3/2)/x^3, x)
```

Giac [F]

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^3} \, dx = \int \frac{\log(ax^n)^{\frac{3}{2}}}{x^3} \, dx$$

```
[In] integrate(log(a*x^n)^(3/2)/x^3,x, algorithm="giac")
[Out] integrate(log(a*x^n)^(3/2)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\log^{\frac{3}{2}}(ax^n)}{x^3} \, dx = \int \frac{\ln(a\,x^n)^{3/2}}{x^3} \, dx$$

```
[In] int(log(a*x^n)^(3/2)/x^3,x)
[Out] int(log(a*x^n)^(3/2)/x^3, x)
```

3.128 $\int \frac{x^3}{\sqrt{\log(ax^n)}} dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{x^3}{\sqrt{\log(ax^n)}} dx = \frac{\sqrt{\pi}x^4(ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}$$

```
[Out] 1/2*x^4*erfi(2*ln(a*x^n)^(1/2)/n^(1/2))*Pi^(1/2)/((a*x^n)^(4/n))/n^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2347, 2211, 2235}

$$\int \frac{x^3}{\sqrt{\log(ax^n)}} dx = \frac{\sqrt{\pi}x^4(ax^n)^{-4/n} \operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}$$

```
[In] Int[x^3/Sqrt[Log[a*x^n]],x]
[Out] (Sqrt[Pi]*x^4*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(2*Sqrt[n]*(a*x^n)^(4/n))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x^4(ax^n)^{-4/n}\right)\text{Subst}\left(\int\frac{e^{\frac{4x}{n}}}{\sqrt{x}}\,dx, x, \log\left(ax^n\right)\right)}{n}\\ &= \frac{\left(2x^4(ax^n)^{-4/n}\right)\text{Subst}\left(\int e^{\frac{4x^2}{n}}\,dx, x, \sqrt{\log\left(ax^n\right)}\right)}{n}\\ &= \frac{\sqrt{\pi}x^4(ax^n)^{-4/n}\,\text{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int\frac{x^3}{\sqrt{\log\left(ax^n\right)}}\,dx = \frac{\sqrt{\pi}x^4(ax^n)^{-4/n}\,\text{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2\sqrt{n}}$$

```
[In] Integrate[x^3/Sqrt[Log[a*x^n]],x]
[Out] (Sqrt[Pi]*x^4*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(2*Sqrt[n]*(a*x^n)^(4/n))
```

Maple [F]

$$\int\frac{x^3}{\sqrt{\ln\left(a\,x^n\right)}}dx$$

```
[In] int(x^3/ln(a*x^n)^(1/2),x)
[Out] int(x^3/ln(a*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\log(ax^n)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{\log(ax^n)}}\,dx = \int \frac{x^3}{\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(x**3/ln(a*x**n)**(1/2),x)
[Out] Integral(x**3/sqrt(log(a*x**n)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{\log(ax^n)}}\,dx = \int \frac{x^3}{\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(x^3/log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(log(a*x^n)), x)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{\log(ax^n)}}\,dx = \int \frac{x^3}{\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(x^3/log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(log(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\log(ax^n)}}\,dx = \int \frac{x^3}{\sqrt{\ln(a\,x^n)}}\,dx$$

```
[In] int(x^3/log(a*x^n)^(1/2),x)
[Out] int(x^3/log(a*x^n)^(1/2), x)
```

3.129 $\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx = \frac{\sqrt{\frac{\pi}{3}}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[Out] 1/3*x^3*erfi(3^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)/((a*x^n)^(3/
n))/n^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2347, 2211, 2235}

$$\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx = \frac{\sqrt{\frac{\pi}{3}}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[In] Int[x^2/Sqrt[Log[a*x^n]],x]
```

```
[Out] (Sqrt[Pi/3]*x^3*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(Sqrt[n]*(a*x^n)^
(3/n))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^3(ax^n)^{-3/n}\right)\operatorname{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{\sqrt{x}}\,dx, x, \log\left(ax^n\right)\right)}{n} \\
&= \frac{\left(2x^3(ax^n)^{-3/n}\right)\operatorname{Subst}\left(\int e^{\frac{3x^2}{n}}\,dx, x, \sqrt{\log\left(ax^n\right)}\right)}{n} \\
&= \frac{\sqrt{\frac{\pi}{3}}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$
\int \frac{x^2}{\sqrt{\log\left(ax^n\right)}}\,dx = \frac{\sqrt{\frac{\pi}{3}}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}
$$

```
[In] Integrate[x^2/Sqrt[Log[a*x^n]],x]
[Out] (Sqrt[Pi/3]*x^3*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(Sqrt[n]*(a*x^n)^
(3/n))
```

Maple [F]

$$
\int \frac{x^2}{\sqrt{\ln\left(a\,x^n\right)}}dx
$$

```
[In] int(x^2/ln(a*x^n)^(1/2),x)
[Out] int(x^2/ln(a*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx = \int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(x**2/ln(a*x**n)**(1/2),x)
[Out] Integral(x**2/sqrt(log(a*x**n)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx = \int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(x^2/log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(log(a*x^n)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx = \int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(x^2/log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(log(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\log (ax^n)}}\,dx = \int \frac{x^2}{\sqrt{\ln (a\,x^n)}}\,dx$$

```
[In] int(x^2/log(a*x^n)^(1/2),x)
[Out] int(x^2/log(a*x^n)^(1/2), x)
```

3.130 $\int \frac{x}{\sqrt{\log(ax^n)}} dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{x}{\sqrt{\log(ax^n)}} dx = \frac{\sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[Out] 1/2*x^2*erfi(2^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/((a*x^n)^(2/
n))/n^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2347, 2211, 2235}

$$\int \frac{x}{\sqrt{\log(ax^n)}} dx = \frac{\sqrt{\frac{\pi}{2}} x^2 (ax^n)^{-2/n} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[In] Int[x/Sqrt[Log[a*x^n]],x]
```

```
[Out] (Sqrt[Pi/2]*x^2*Erfi[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(Sqrt[n]*(a*x^n)^
(2/n))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x^2(ax^n)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{n}\\ &= \frac{\left(2x^2(ax^n)^{-2/n}\right)\text{Subst}\left(\int e^{\frac{2x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{n}\\ &= \frac{\sqrt{\frac{\pi}{2}}x^2(ax^n)^{-2/n}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\log(ax^n)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}x^2(ax^n)^{-2/n}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[In] Integrate[x/Sqrt[Log[a*x^n]],x]
[Out] (Sqrt[Pi/2]*x^2*Erfi[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(Sqrt[n]*(a*x^n)^
(2/n))
```

Maple [F]

$$\int \frac{x}{\sqrt{\ln(a\,x^n)}}dx$$

```
[In] int(x/ln(a*x^n)^(1/2),x)
[Out] int(x/ln(a*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\log\left(ax^{n}\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{\log\left(ax^{n}\right)}}\,dx = \int \frac{x}{\sqrt{\log\left(ax^{n}\right)}}\,dx$$

```
[In] integrate(x/ln(a*x**n)**(1/2),x)
[Out] Integral(x/sqrt(log(a*x**n)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{\log\left(ax^{n}\right)}}\,dx = \int \frac{x}{\sqrt{\log\left(ax^{n}\right)}}\,dx$$

```
[In] integrate(x/log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(log(a*x^n)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{\log\left(ax^{n}\right)}}\,dx = \int \frac{x}{\sqrt{\log\left(ax^{n}\right)}}\,dx$$

```
[In] integrate(x/log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(log(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\log (ax^n)}} \, dx = \int \frac{x}{\sqrt{\ln (a\,x^n)}} \, dx$$

```
[In] int(x/log(a*x^n)^(1/2),x)
[Out] int(x/log(a*x^n)^(1/2), x)
```

3.131 $\int \frac{1}{\sqrt{\log(ax^n)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \frac{1}{\sqrt{\log(ax^n)}}\,dx=\frac{\sqrt{\pi}x(ax^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[Out] x*erfi(ln(a*x^n)^(1/2)/n^(1/2))*Pi^(1/2)/((a*x^n)^(1/n))/n^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2337, 2211, 2235}

$$\int \frac{1}{\sqrt{\log(ax^n)}}\,dx=\frac{\sqrt{\pi}x(ax^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[In] Int[1/Sqrt[Log[a*x^n]],x]
```

```
[Out] (Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*(a*x^n)^n^(-1))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x(ax^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{n}\\ &= \frac{\left(2x(ax^n)^{-1/n}\right)\text{Subst}\left(\int e^{\frac{x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{n}\\ &= \frac{\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\log(ax^n)}}\,dx = \frac{\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}}$$

```
[In] Integrate[1/Sqrt[Log[a*x^n]],x]
[Out] (Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*(a*x^n)^n^(-1))
```

Maple [F]

$$\int \frac{1}{\sqrt{\ln(a\,x^n)}}dx$$

```
[In] int(1/ln(a*x^n)^(1/2),x)
[Out] int(1/ln(a*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\log(ax^n)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\log(ax^n)}}\,dx = \int \frac{1}{\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(1/ln(a*x**n)**(1/2),x)
[Out] Integral(1/sqrt(log(a*x**n)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\log(ax^n)}}\,dx = \int \frac{1}{\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(1/log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(log(a*x^n)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\log(ax^n)}}\,dx = \int \frac{1}{\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(1/log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(log(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\log (ax^n)}}\,dx = \int \frac{1}{\sqrt{\ln (a\,x^n)}}\,dx$$

```
[In] int(1/log(a*x^n)^(1/2),x)
[Out] int(1/log(a*x^n)^(1/2), x)
```

3.132 $\int \frac{1}{x\sqrt{\log(ax^n)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{1}{x\sqrt{\log(ax^n)}}\,dx = \frac{2\sqrt{\log(ax^n)}}{n}$$

```
[Out] 2*ln(a*x^n)^(1/2)/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2339, 30}

$$\int \frac{1}{x\sqrt{\log(ax^n)}}\,dx = \frac{2\sqrt{\log(ax^n)}}{n}$$

```
[In] Int[1/(x*Sqrt[Log[a*x^n]]),x]
```

```
[Out] (2*Sqrt[Log[a*x^n]])/n
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \log\left(ax^n\right)\right)}{n}$$
$$= \frac{2\sqrt{\log\left(ax^n\right)}}{n}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\log\left(ax^n\right)}}\,dx = \frac{2\sqrt{\log\left(ax^n\right)}}{n}$$

```
[In] Integrate[1/(x*Sqrt[Log[a*x^n]]),x]
[Out] (2*Sqrt[Log[a*x^n]])/n
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2\sqrt{\ln\left(a\,x^{n}\right)}}{n}$	14
default	$\frac{2\sqrt{\ln\left(a\,x^{n}\right)}}{n}$	14

```
[In] int(1/x/ln(a*x^n)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*ln(a*x^n)^(1/2)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{\log\left(ax^n\right)}}\,dx = \frac{2\sqrt{n\log\left(x\right)+\log\left(a\right)}}{n}$$

```
[In] integrate(1/x/log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(n*log(x) + log(a))/n
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{1}{x\sqrt{\log\left(ax^{n}\right)}}\,dx = \begin{cases} \frac{2\sqrt{\log\left(ax^{n}\right)}}{n} & \text{for}\ n \neq 0 \\ \frac{\log\left(x\right)}{\sqrt{\log\left(a\right)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/ln(a*x**n)**(1/2),x)
[Out] Piecewise((2*sqrt(log(a*x**n))/n, Ne(n, 0)), (log(x)/sqrt(log(a)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{\log\left(ax^{n}\right)}}\,dx = \frac{2\sqrt{\log\left(ax^{n}\right)}}{n}$$

```
[In] integrate(1/x/log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(log(a*x^n))/n
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\sqrt{\log\left(ax^{n}\right)}}\,dx = \frac{2\sqrt{n\log\left(x\right) + \log\left(a\right)}}{n}$$

```
[In] integrate(1/x/log(a*x^n)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(n*log(x) + log(a))/n
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{\log(ax^n)}}\,dx = \frac{2\sqrt{\ln(a\,x^n)}}{n}$$

```
[In] int(1/(x*log(a*x^n)^(1/2)),x)
[Out] (2*log(a*x^n)^(1/2))/n
```

3.133 $\int \frac{1}{x^2\sqrt{\log(ax^n)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \frac{1}{x^2\sqrt{\log(ax^n)}}\,dx=\frac{\sqrt{\pi}(ax^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}x}$$

```
[Out] (a*x^n)^(1/n)*erf(ln(a*x^n)^(1/2)/n^(1/2))*Pi^(1/2)/x/n^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2347, 2211, 2236}

$$\int \frac{1}{x^2\sqrt{\log(ax^n)}}\,dx=\frac{\sqrt{\pi}(ax^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}x}$$

```
[In] Int[1/(x^2*Sqrt[Log[a*x^n]]),x]
```

```
[Out] (Sqrt[Pi]*(a*x^n)^n^(-1)*Erf[Sqrt[Log[a*x^n]]/Sqrt[n]])/(Sqrt[n]*x)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(ax^n)^{\frac{1}{n}}\,\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{nx}\\ &= \frac{\left(2(ax^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int e^{-\frac{x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{nx}\\ &= \frac{\sqrt{\pi}(ax^n)^{\frac{1}{n}}\,\text{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}x}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^2\sqrt{\log(ax^n)}}\,dx = -\frac{(ax^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right)\sqrt{\frac{\log(ax^n)}{n}}}{x\sqrt{\log(ax^n)}}$$

```
[In] Integrate[1/(x^2*Sqrt[Log[a*x^n]]),x]
[Out] -(((a*x^n)^n^(-1)*Gamma[1/2, Log[a*x^n]/n]*Sqrt[Log[a*x^n]/n])/(x*Sqrt[Log[
a*x^n]]))
```

Maple [F]

$$\int \frac{1}{x^2\sqrt{\ln(a\,x^n)}}dx$$

```
[In] int(1/x^2/ln(a*x^n)^(1/2),x)
[Out] int(1/x^2/ln(a*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{\log\left(ax^n\right)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{\log\left(ax^n\right)}}\,dx = \int \frac{1}{x^2\sqrt{\log\left(ax^n\right)}}\,dx$$

```
[In] integrate(1/x**2/ln(a*x**n)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(log(a*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{\log\left(ax^n\right)}}\,dx = \int \frac{1}{x^2\sqrt{\log\left(ax^n\right)}}\,dx$$

```
[In] integrate(1/x^2/log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x^2*sqrt(log(a*x^n))), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{\log\left(ax^n\right)}}\,dx = \int \frac{1}{x^2\sqrt{\log\left(ax^n\right)}}\,dx$$

```
[In] integrate(1/x^2/log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(x^2*sqrt(log(a*x^n))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{\log{(ax^n)}}}\,dx = \int \frac{1}{x^2\,\sqrt{\ln{(a\,x^n)}}}\,dx$$

```
[In] int(1/(x^2*log(a*x^n)^(1/2)),x)
[Out] int(1/(x^2*log(a*x^n)^(1/2)), x)
```

3.134 $\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}(ax^n)^{2/n}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}x^2}$$

```
[Out] 1/2*(a*x^n)^(2/n)*erf(2^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/x^2
/n^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2347, 2211, 2236}

$$\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}(ax^n)^{2/n}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}x^2}$$

```
[In] Int[1/(x^3*Sqrt[Log[a*x^n]]),x]
```

```
[Out] (Sqrt[Pi/2]*(a*x^n)^(2/n)*Erf[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(Sqrt[n]
*x^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(ax^n)^{2/n}\,\text{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{nx^2}\\ &= \frac{\left(2(ax^n)^{2/n}\right)\text{Subst}\left(\int e^{-\frac{2x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{nx^2}\\ &= \frac{\sqrt{\frac{\pi}{2}}(ax^n)^{2/n}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{n}x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx = -\frac{(ax^n)^{2/n}\,\Gamma\left(\frac{1}{2}, \frac{2\log(ax^n)}{n}\right)\sqrt{\frac{\log(ax^n)}{n}}}{\sqrt{2}x^2\sqrt{\log(ax^n)}}$$

```
[In] Integrate[1/(x^3*Sqrt[Log[a*x^n]]),x]
[Out] -(((a*x^n)^(2/n)*Gamma[1/2, (2*Log[a*x^n])/n]*Sqrt[Log[a*x^n]/n])/(Sqrt[2]*
x^2*Sqrt[Log[a*x^n]]))
```

Maple [F]

$$\int \frac{1}{x^3\sqrt{\ln(a\,x^n)}}dx$$

```
[In] int(1/x^3/ln(a*x^n)^(1/2),x)
[Out] int(1/x^3/ln(a*x^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^3/log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx = \int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(1/x**3/ln(a*x**n)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(log(a*x**n))), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx = \int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(1/x^3/log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x^3*sqrt(log(a*x^n))), x)
```

Giac [F]

$$\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx = \int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(1/x^3/log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(x^3*sqrt(log(a*x^n))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx = \int \frac{1}{x^3\,\sqrt{\ln(a\,x^n)}}\,dx$$

```
[In] int(1/(x^3*log(a*x^n)^(1/2)),x)
[Out] int(1/(x^3*log(a*x^n)^(1/2)), x)
```

3.135 $\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)}\,dx=\frac{4\sqrt{\pi}x^4(ax^n)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}}-\frac{2x^4}{n\sqrt{\log(ax^n)}}$$

```
[Out] 4*x^4*erfi(2*ln(a*x^n)^(1/2)/n^(1/2))*Pi^(1/2)/n^(3/2)/((a*x^n)^(4/n))-2*x^
4/n/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2343, 2347, 2211, 2235}

$$\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)}\,dx=\frac{4\sqrt{\pi}x^4(ax^n)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}}-\frac{2x^4}{n\sqrt{\log(ax^n)}}$$

```
[In] Int[x^3/Log[a*x^n]^(3/2),x]
```

```
[Out] (4*Sqrt[Pi]*x^4*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(n^(3/2)*(a*x^n)^(4/n))
 - (2*x^4)/(n*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^4}{n\sqrt{\log(ax^n)}}+\frac{8\int\frac{x^3}{\sqrt{\log(ax^n)}}\,dx}{n}\\
&= -\frac{2x^4}{n\sqrt{\log(ax^n)}}+\frac{\left(8x^4(ax^n)^{-4/n}\right)\text{Subst}\left(\int\frac{e^{\frac{4x}{n}}}{\sqrt{x}}\,dx,x,\log(ax^n)\right)}{n^2}\\
&= -\frac{2x^4}{n\sqrt{\log(ax^n)}}+\frac{\left(16x^4(ax^n)^{-4/n}\right)\text{Subst}\left(\int e^{\frac{4x^2}{n}}\,dx,x,\sqrt{\log(ax^n)}\right)}{n^2}\\
&= \frac{4\sqrt{\pi}x^4(ax^n)^{-4/n}\,\text{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}}-\frac{2x^4}{n\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

$$\int\frac{x^3}{\log^{\frac{3}{2}}(ax^n)}\,dx=-\frac{2x^4(ax^n)^{-4/n}\left((ax^n)^{4/n}-2\Gamma\left(\frac{1}{2},-\frac{4\log(ax^n)}{n}\right)\sqrt{-\frac{\log(ax^n)}{n}}\right)}{n\sqrt{\log(ax^n)}}$$

```
[In] Integrate[x^3/Log[a*x^n]^(3/2),x]
[Out] (-2*x^4*((a*x^n)^(4/n) - 2*Gamma[1/2, (-4*Log[a*x^n])/n]*Sqrt[-(Log[a*x^n]/
n)]))/(n*(a*x^n)^(4/n)*Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{x^3}{\ln(a\,x^n)^{\frac{3}{2}}}dx$$

[In] int(x^3/ln(a*x^n)^(3/2),x)

[Out] int(x^3/ln(a*x^n)^(3/2),x)

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^3/log(a*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^3}{\log(ax^n)^{\frac{3}{2}}}\,dx$$

[In] integrate(x**3/ln(a*x**n)**(3/2),x)

[Out] Integral(x**3/log(a*x**n)**(3/2), x)

Maxima [F]

$$\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^3}{\log(ax^n)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^3/log(a*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(x^3/log(a*x^n)^(3/2), x)

Giac [F]

$$\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^3}{\log(ax^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/log(a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^3}{\ln(a\,x^n)^{3/2}}\,dx$$

```
[In] int(x^3/log(a*x^n)^(3/2),x)
[Out] int(x^3/log(a*x^n)^(3/2), x)
```

3.136 $\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)}\,dx = \frac{2\sqrt{3\pi}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^3}{n\sqrt{\log(ax^n)}}$$

```
[Out] 2*x^3*erfi(3^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*3^(1/2)*Pi^(1/2)/n^(3/2)/((a*x^
n)^(3/n))-2*x^3/n/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2343, 2347, 2211, 2235}

$$\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)}\,dx = \frac{2\sqrt{3\pi}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^3}{n\sqrt{\log(ax^n)}}$$

```
[In] Int[x^2/Log[a*x^n]^(3/2),x]
```

```
[Out] (2*Sqrt[3*Pi]*x^3*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(n^(3/2)*(a*x^n
)^(3/n)) - (2*x^3)/(n*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^3}{n\sqrt{\log(ax^n)}} + \frac{6\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx}{n}\\
&= -\frac{2x^3}{n\sqrt{\log(ax^n)}} + \frac{\left(6x^3(ax^n)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{n^2}\\
&= -\frac{2x^3}{n\sqrt{\log(ax^n)}} + \frac{\left(12x^3(ax^n)^{-3/n}\right)\text{Subst}\left(\int e^{\frac{3x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{n^2}\\
&= \frac{2\sqrt{3\pi}x^3(ax^n)^{-3/n}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^3}{n\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)}\,dx = -\frac{2x^3(ax^n)^{-3/n}\left((ax^n)^{3/n} - \sqrt{3}\Gamma\left(\frac{1}{2}, -\frac{3\log(ax^n)}{n}\right)\sqrt{-\frac{\log(ax^n)}{n}}\right)}{n\sqrt{\log(ax^n)}}$$

```
[In] Integrate[x^2/Log[a*x^n]^(3/2),x]
[Out] (-2*x^3*((a*x^n)^(3/n) - Sqrt[3]*Gamma[1/2, (-3*Log[a*x^n])/n]*Sqrt[-(Log[a
*x^n]/n)]))/(n*(a*x^n)^(3/n)*Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{x^2}{\ln (a\,x^n)^{\frac{3}{2}}}dx$$

```
[In] int(x^2/ln(a*x^n)^(3/2),x)
[Out] int(x^2/ln(a*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/log(a*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^2}{\log (ax^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/ln(a*x**n)**(3/2),x)
[Out] Integral(x**2/log(a*x**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^2}{\log (ax^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/log(a*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/log(a*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^2}{\log(ax^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/log(a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^2}{\ln(a\,x^n)^{3/2}}\,dx$$

```
[In] int(x^2/log(a*x^n)^(3/2),x)
[Out] int(x^2/log(a*x^n)^(3/2), x)
```

3.137 $\int \frac{x}{\log^{\frac{3}{2}}(ax^n)} dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \frac{x}{\log^{\frac{3}{2}}(ax^n)} dx = \frac{2\sqrt{2\pi}x^2(ax^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^2}{n\sqrt{\log(ax^n)}}$$

```
[Out] 2*x^2*erfi(2^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/n^(3/2)/((a*x^
n)^(2/n))-2*x^2/n/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2343, 2347, 2211, 2235}

$$\int \frac{x}{\log^{\frac{3}{2}}(ax^n)} dx = \frac{2\sqrt{2\pi}x^2(ax^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^2}{n\sqrt{\log(ax^n)}}$$

```
[In] Int[x/Log[a*x^n]^(3/2),x]
```

```
[Out] (2*Sqrt[2*Pi]*x^2*Erfi[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(n^(3/2)*(a*x^n
)^(2/n)) - (2*x^2)/(n*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x^2}{n\sqrt{\log(ax^n)}} + \frac{4\int \frac{x}{\sqrt{\log(ax^n)}}\,dx}{n} \\
&= -\frac{2x^2}{n\sqrt{\log(ax^n)}} + \frac{\left(4x^2(ax^n)^{-2/n}\right)\text{Subst}\left(\int \frac{e^{\frac{2x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{n^2} \\
&= -\frac{2x^2}{n\sqrt{\log(ax^n)}} + \frac{\left(8x^2(ax^n)^{-2/n}\right)\text{Subst}\left(\int e^{\frac{2x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{n^2} \\
&= \frac{2\sqrt{2\pi}x^2(ax^n)^{-2/n}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^2}{n\sqrt{\log(ax^n)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$
\int \frac{x}{\log^{\frac{3}{2}}(ax^n)}\,dx = -\frac{2x^2(ax^n)^{-2/n}\left((ax^n)^{2/n} - \sqrt{2}\Gamma\left(\frac{1}{2}, -\frac{2\log(ax^n)}{n}\right)\sqrt{-\frac{\log(ax^n)}{n}}\right)}{n\sqrt{\log(ax^n)}}
$$

```
[In] Integrate[x/Log[a*x^n]^(3/2),x]
```

```
[Out] (-2*x^2*((a*x^n)^(2/n) - Sqrt[2]*Gamma[1/2, (-2*Log[a*x^n])/n]*Sqrt[-(Log[a
*x^n]/n)]))/(n*(a*x^n)^(2/n)*Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{x}{\ln\left(a\,x^{n}\right)^{\frac{3}{2}}}dx$$

```
[In] int(x/ln(a*x^n)^(3/2),x)
[Out] int(x/ln(a*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\log^{\frac{3}{2}}(ax^{n})}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/log(a*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\log^{\frac{3}{2}}(ax^{n})}\,dx = \int \frac{x}{\log\left(ax^{n}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/ln(a*x**n)**(3/2),x)
[Out] Integral(x/log(a*x**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\log^{\frac{3}{2}}(ax^{n})}\,dx = \int \frac{x}{\log\left(ax^{n}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/log(a*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(x/log(a*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x}{\log(ax^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(x/log(a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x}{\ln(a\,x^n)^{3/2}}\,dx$$

```
[In] int(x/log(a*x^n)^(3/2),x)
[Out] int(x/log(a*x^n)^(3/2), x)
```

3.138 $\int \frac{1}{\log^{\frac{3}{2}}(ax^n)} dx$

Optimal result

Integrand size = 10, antiderivative size = 58

$$\int \frac{1}{\log^{\frac{3}{2}}(ax^n)} dx = \frac{2\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x}{n\sqrt{\log(ax^n)}}$$

```
[Out] 2*x*erfi(ln(a*x^n)^(1/2)/n^(1/2))*Pi^(1/2)/n^(3/2)/((a*x^n)^(1/n))-2*x/n/ln
(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2334, 2337, 2211, 2235}

$$\int \frac{1}{\log^{\frac{3}{2}}(ax^n)} dx = \frac{2\sqrt{\pi}x(ax^n)^{-1/n} \operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x}{n\sqrt{\log(ax^n)}}$$

```
[In] Int[Log[a*x^n]^(-3/2),x]
```

```
[Out] (2*Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(n^(3/2)*(a*x^n)^n^(-1)) - (2
*x)/(n*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x}{n\sqrt{\log(ax^n)}}+\frac{2\int\frac{1}{\sqrt{\log(ax^n)}}\,dx}{n}\\
&= -\frac{2x}{n\sqrt{\log(ax^n)}}+\frac{\left(2x(ax^n)^{-1/n}\right)\text{Subst}\left(\int\frac{e^{\frac{x}{n}}}{\sqrt{x}}\,dx,x,\log(ax^n)\right)}{n^2}\\
&= -\frac{2x}{n\sqrt{\log(ax^n)}}+\frac{\left(4x(ax^n)^{-1/n}\right)\text{Subst}\left(\int e^{\frac{x^2}{n}}\,dx,x,\sqrt{\log(ax^n)}\right)}{n^2}\\
&= \frac{2\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}}-\frac{2x}{n\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int\frac{1}{\log^{\frac{3}{2}}(ax^n)}\,dx=-\frac{2x(ax^n)^{-1/n}\left((ax^n)^{\frac{1}{n}}-\Gamma\left(\frac{1}{2},-\frac{\log(ax^n)}{n}\right)\sqrt{-\frac{\log(ax^n)}{n}}\right)}{n\sqrt{\log(ax^n)}}$$

```
[In] Integrate[Log[a*x^n]^(-3/2),x]
[Out] (-2*x*((a*x^n)^n^(-1) - Gamma[1/2, -(Log[a*x^n]/n)]*Sqrt[-(Log[a*x^n]/n)]))
/(n*(a*x^n)^n^(-1)*Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{1}{\ln\left(a\,x^{n}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/ln(a*x^n)^(3/2),x)
[Out] int(1/ln(a*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\log^{\frac{3}{2}}(ax^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/log(a*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{1}{\log(ax^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/ln(a*x**n)**(3/2),x)
[Out] Integral(log(a*x**n)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{1}{\log(ax^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/log(a*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(log(a*x^n)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{1}{\log(ax^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(log(a*x^n)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{1}{\ln(a\,x^n)^{3/2}}\,dx$$

```
[In] int(1/log(a*x^n)^(3/2),x)
[Out] int(1/log(a*x^n)^(3/2), x)
```

3.139 $\int \frac{1}{x \log^{\frac{3}{2}}(ax^n)} dx$

Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{1}{x \log^{\frac{3}{2}}(ax^n)} dx = -\frac{2}{n\sqrt{\log(ax^n)}}$$

```
[Out] -2/n/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2339, 30}

$$\int \frac{1}{x \log^{\frac{3}{2}}(ax^n)} dx = -\frac{2}{n\sqrt{\log(ax^n)}}$$

```
[In] Int[1/(x*Log[a*x^n]^(3/2)),x]
```

```
[Out] -2/(n*Sqrt[Log[a*x^n]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \log\left(ax^n\right)\right)}{n}$$
$$= -\frac{2}{n\sqrt{\log\left(ax^n\right)}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log^{\frac{3}{2}}\left(ax^n\right)}\,dx = -\frac{2}{n\sqrt{\log\left(ax^n\right)}}$$

```
[In] Integrate[1/(x*Log[a*x^n]^(3/2)),x]
[Out] -2/(n*Sqrt[Log[a*x^n]])
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{2}{n\sqrt{\ln(a\,x^n)}}$	14
default	$-\frac{2}{n\sqrt{\ln(a\,x^n)}}$	14

```
[In] int(1/x/ln(a*x^n)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/n/ln(a*x^n)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{1}{x\log^{\frac{3}{2}}\left(ax^n\right)}\,dx = -\frac{2\sqrt{n\log\left(x\right)+\log\left(a\right)}}{n^2\log\left(x\right)+n\log\left(a\right)}$$

```
[In] integrate(1/x/log(a*x^n)^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(n*log(x) + log(a))/(n^2*log(x) + n*log(a))
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{1}{x\log^{\frac{3}{2}}(ax^n)}\,dx = \begin{cases} -\frac{2}{n\sqrt{\log(ax^n)}} & \text{for } n \neq 0 \\ \frac{\log(x)}{\log(a)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/ln(a*x**n)**(3/2),x)
[Out] Piecewise((-2/(n*sqrt(log(a*x**n))), Ne(n, 0)), (log(x)/log(a)**(3/2), True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\log^{\frac{3}{2}}(ax^n)}\,dx = -\frac{2}{n\sqrt{\log(ax^n)}}$$

```
[In] integrate(1/x/log(a*x^n)^(3/2),x, algorithm="maxima")
[Out] -2/(n*sqrt(log(a*x^n)))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\log^{\frac{3}{2}}(ax^n)}\,dx = -\frac{2}{\sqrt{n\log(x)+\log(a)}n}$$

```
[In] integrate(1/x/log(a*x^n)^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(n*log(x) + log(a))*n)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x \log^{\frac{3}{2}}(ax^n)}\,dx = -\frac{2}{n\,\sqrt{\ln\left(a\,x^n\right)}}$$

```
[In] int(1/(x*log(a*x^n)^(3/2)),x)
[Out] -2/(n*log(a*x^n)^(1/2))
```

3.140 $\int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)}\,dx = -\frac{2\sqrt{\pi}(ax^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x} - \frac{2}{nx\sqrt{\log(ax^n)}}$$

```
[Out] -2*(a*x^n)^(1/n)*erf(ln(a*x^n)^(1/2)/n^(1/2))*Pi^(1/2)/n^(3/2)/x-2/n/x/ln(a
*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2343, 2347, 2211, 2236}

$$\int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)}\,dx = -\frac{2\sqrt{\pi}(ax^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x} - \frac{2}{nx\sqrt{\log(ax^n)}}$$

```
[In] Int[1/(x^2*Log[a*x^n]^(3/2)),x]
```

```
[Out] (-2*Sqrt[Pi]*(a*x^n)^n^(-1)*Erf[Sqrt[Log[a*x^n]]/Sqrt[n]])/(n^(3/2)*x) - 2/
(n*x*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{nx\sqrt{\log(ax^n)}} - \frac{2\int \frac{1}{x^2\sqrt{\log(ax^n)}}\,dx}{n} \\
&= -\frac{2}{nx\sqrt{\log(ax^n)}} - \frac{\left(2(ax^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{n^2x} \\
&= -\frac{2}{nx\sqrt{\log(ax^n)}} - \frac{\left(4(ax^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int e^{-\frac{x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{n^2x} \\
&= -\frac{2\sqrt{\pi}(ax^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x} - \frac{2}{nx\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2\log^{\frac{3}{2}}(ax^n)}\,dx = \frac{2\left(-1+(ax^n)^{\frac{1}{n}}\,\Gamma\left(\frac{1}{2},\frac{\log(ax^n)}{n}\right)\sqrt{\frac{\log(ax^n)}{n}}\right)}{nx\sqrt{\log(ax^n)}}$$

```
[In] Integrate[1/(x^2*Log[a*x^n]^(3/2)),x]
[Out] (2*(-1 + (a*x^n)^n^(-1)*Gamma[1/2, Log[a*x^n]/n]*Sqrt[Log[a*x^n]/n]))/(n*x*
Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{1}{x^2 \ln \left(a\, x^n\right)^{\frac{3}{2}}}dx$$

[In] int(1/x^2/ln(a*x^n)^(3/2),x)

[Out] int(1/x^2/ln(a*x^n)^(3/2),x)

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)}\, dx = \text{Exception raised: TypeError}$$

[In] integrate(1/x^2/log(a*x^n)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)}\, dx = \int \frac{1}{x^2 \log{(ax^n)}^{\frac{3}{2}}}\, dx$$

[In] integrate(1/x**2/ln(a*x**n)**(3/2),x)

[Out] Integral(1/(x**2*log(a*x**n)**(3/2)), x)

Maxima [F]

$$\int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)}\, dx = \int \frac{1}{x^2 \log\left(ax^n\right)^{\frac{3}{2}}}\, dx$$

[In] integrate(1/x^2/log(a*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate(1/(x^2*log(a*x^n)^(3/2)), x)

Giac [F]

$$\int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{1}{x^2 \log(ax^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^2/log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/(x^2*log(a*x^n)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{1}{x^2 \ln(a\,x^n)^{3/2}}\,dx$$

```
[In] int(1/(x^2*log(a*x^n)^(3/2)),x)
[Out] int(1/(x^2*log(a*x^n)^(3/2)), x)
```

3.141 $\int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)} dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)} dx = -\frac{2\sqrt{2\pi}(ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x^2} - \frac{2}{nx^2\sqrt{\log(ax^n)}}$$

```
[Out] -2*(a*x^n)^(2/n)*erf(2^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*2^(1/2)*Pi^(1/2)/n^(3
/2)/x^2-2/n/x^2/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2343, 2347, 2211, 2236}

$$\int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)} dx = -\frac{2\sqrt{2\pi}(ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x^2} - \frac{2}{nx^2\sqrt{\log(ax^n)}}$$

```
[In] Int[1/(x^3*Log[a*x^n]^(3/2)),x]
```

```
[Out] (-2*Sqrt[2*Pi]*(a*x^n)^(2/n)*Erf[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(n^(3
/2)*x^2) - 2/(n*x^2*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{nx^2\sqrt{\log(ax^n)}} - \frac{4\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx}{n} \\
&= -\frac{2}{nx^2\sqrt{\log(ax^n)}} - \frac{\left(4(ax^n)^{2/n}\right)\text{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{n^2x^2} \\
&= -\frac{2}{nx^2\sqrt{\log(ax^n)}} - \frac{\left(8(ax^n)^{2/n}\right)\text{Subst}\left(\int e^{-\frac{2x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{n^2x^2} \\
&= -\frac{2\sqrt{2\pi}(ax^n)^{2/n}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}x^2} - \frac{2}{nx^2\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3\log^{\frac{3}{2}}(ax^n)}\,dx = \frac{2\left(-1+\sqrt{2}(ax^n)^{2/n}\,\Gamma\left(\frac{1}{2},\frac{2\log(ax^n)}{n}\right)\sqrt{\frac{\log(ax^n)}{n}}\right)}{nx^2\sqrt{\log(ax^n)}}$$

```
[In] Integrate[1/(x^3*Log[a*x^n]^(3/2)),x]
[Out] (2*(-1 + Sqrt[2]*(a*x^n)^(2/n)*Gamma[1/2, (2*Log[a*x^n])/n]*Sqrt[Log[a*x^n]
/n]))/(n*x^2*Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{1}{x^{3} \ln \left(a\,x^{n}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/x^3/ln(a*x^n)^(3/2),x)
```

```
[Out] int(1/x^3/ln(a*x^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^3/log(a*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{1}{x^{3} \log{\left(a x^{n} \right)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**3/ln(a*x**n)**(3/2),x)
```

```
[Out] Integral(1/(x**3*log(a*x**n)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)}\,dx = \int { \frac{1}{x^{3} \log\left(a x^{n}\right)^{\frac{3}{2}}} \,d x }$$

```
[In] integrate(1/x^3/log(a*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^3*log(a*x^n)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)}\, dx = \int \frac{1}{x^3 \log{(ax^n)}^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/x^3/log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/(x^3*log(a*x^n)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \log^{\frac{3}{2}}(ax^n)}\, dx = \int \frac{1}{x^3 \ln{(a\,x^n)}^{3/2}}\, dx$$

```
[In] int(1/(x^3*log(a*x^n)^(3/2)),x)
[Out] int(1/(x^3*log(a*x^n)^(3/2)), x)
```

3.142 $\int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)} dx$

Optimal result

Integrand size = 14, antiderivative size = 87

$$\int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)} dx = \frac{32\sqrt{\pi}x^4(ax^n)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x^4}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{16x^4}{3n^2\sqrt{\log(ax^n)}}$$

```
[Out] -2/3*x^4/n/ln(a*x^n)^(3/2)+32/3*x^4*erfi(2*ln(a*x^n)^(1/2)/n^(1/2))*Pi^(1/2
)/n^(5/2)/((a*x^n)^(4/n))-16/3*x^4/n^2/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2343, 2347, 2211, 2235}

$$\int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)} dx = \frac{32\sqrt{\pi}x^4(ax^n)^{-4/n}\operatorname{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{16x^4}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^4}{3n\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Int[x^3/Log[a*x^n]^(5/2),x]
```

```
[Out] (32*Sqrt[Pi]*x^4*Erfi[(2*Sqrt[Log[a*x^n]])/Sqrt[n]])/(3*n^(5/2)*(a*x^n)^(4/
n)) - (2*x^4)/(3*n*Log[a*x^n]^(3/2)) - (16*x^4)/(3*n^2*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^4}{3n\log^{\frac{3}{2}}(ax^n)}+\frac{8\int \frac{x^3}{\log^{\frac{3}{2}}(ax^n)}\,dx}{3n}\\
&= -\frac{2x^4}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{16x^4}{3n^2\sqrt{\log(ax^n)}}+\frac{64\int \frac{x^3}{\sqrt{\log(ax^n)}}\,dx}{3n^2}\\
&= -\frac{2x^4}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{16x^4}{3n^2\sqrt{\log(ax^n)}}+\frac{\left(64x^4(ax^n)^{-4/n}\right)\text{Subst}\left(\int \frac{e^{\frac{4x}{n}}}{\sqrt{x}}\,dx,x,\log(ax^n)\right)}{3n^3}\\
&= -\frac{2x^4}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{16x^4}{3n^2\sqrt{\log(ax^n)}}+\frac{\left(128x^4(ax^n)^{-4/n}\right)\text{Subst}\left(\int e^{\frac{4x^2}{n}}\,dx,x,\sqrt{\log(ax^n)}\right)}{3n^3}\\
&= \frac{32\sqrt{\pi}x^4(ax^n)^{-4/n}\,\text{erfi}\left(\frac{2\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}}-\frac{2x^4}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{16x^4}{3n^2\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)}\,dx = -\frac{2x^4(ax^n)^{-4/n}\left(16n\Gamma\left(\frac{1}{2},-\frac{4\log(ax^n)}{n}\right)\left(-\frac{\log(ax^n)}{n}\right)^{3/2}+(ax^n)^{4/n}(n+8\log(ax^n))\right)}{3n^2\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Integrate[x^3/Log[a*x^n]^(5/2),x]
```

```
[Out] (-2*x^4*(16*n*Gamma[1/2, (-4*Log[a*x^n])/n]*(-(Log[a*x^n]/n))^(3/2) + (a*x^
n)^(4/n)*(n + 8*Log[a*x^n])))/(3*n^2*(a*x^n)^(4/n)*Log[a*x^n]^(3/2))
```

Maple [F]

$$\int \frac{x^3}{\ln(a\,x^n)^{\frac{5}{2}}}dx$$

```
[In] int(x^3/ln(a*x^n)^(5/2),x)
```

```
[Out] int(x^3/ln(a*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/log(a*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^3}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**3/ln(a*x**n)**(5/2),x)
[Out] Integral(x**3/log(a*x**n)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^3}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3/log(a*x^n)^(5/2),x, algorithm="maxima")
[Out] integrate(x^3/log(a*x^n)^(5/2), x)
```

Giac [F]

$$\int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^3}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3/log(a*x^n)^(5/2),x, algorithm="giac")
[Out] integrate(x^3/log(a*x^n)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^3}{\ln(a\,x^n)^{5/2}}\,dx$$

```
[In] int(x^3/log(a*x^n)^(5/2),x)
[Out] int(x^3/log(a*x^n)^(5/2), x)
```

3.143 $\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)} dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)} dx = \frac{4\sqrt{3\pi}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{5/2}} - \frac{2x^3}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2\sqrt{\log(ax^n)}}$$

```
[Out] -2/3*x^3/n/ln(a*x^n)^(3/2)+4*x^3*erfi(3^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*3^(1
/2)*Pi^(1/2)/n^(5/2)/((a*x^n)^(3/n))-4*x^3/n^2/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2343, 2347, 2211, 2235}

$$\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)} dx = \frac{4\sqrt{3\pi}x^3(ax^n)^{-3/n}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{5/2}} - \frac{4x^3}{n^2\sqrt{\log(ax^n)}} - \frac{2x^3}{3n\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Int[x^2/Log[a*x^n]^(5/2),x]
```

```
[Out] (4*Sqrt[3*Pi]*x^3*Erfi[(Sqrt[3]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(n^(5/2)*(a*x^n
)^(3/n)) - (2*x^3)/(3*n*Log[a*x^n]^(3/2)) - (4*x^3)/(n^2*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^3}{3n\log^{\frac{3}{2}}(ax^n)} + \frac{2\int \frac{x^2}{\log^{\frac{3}{2}}(ax^n)}\,dx}{n} \\
&= -\frac{2x^3}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2\sqrt{\log(ax^n)}} + \frac{12\int \frac{x^2}{\sqrt{\log(ax^n)}}\,dx}{n^2} \\
&= -\frac{2x^3}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2\sqrt{\log(ax^n)}} + \frac{\left(12x^3(ax^n)^{-3/n}\right)\text{Subst}\left(\int \frac{e^{\frac{3x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{n^3} \\
&= -\frac{2x^3}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2\sqrt{\log(ax^n)}} + \frac{\left(24x^3(ax^n)^{-3/n}\right)\text{Subst}\left(\int e^{\frac{3x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{n^3} \\
&= \frac{4\sqrt{3\pi}x^3(ax^n)^{-3/n}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{5/2}} - \frac{2x^3}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4x^3}{n^2\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)}\,dx = -\frac{2x^3(ax^n)^{-3/n}\left(6\sqrt{3}n\Gamma\left(\frac{1}{2},-\frac{3\log(ax^n)}{n}\right)\left(-\frac{\log(ax^n)}{n}\right)^{3/2}+(ax^n)^{3/n}(n+6\log(ax^n))\right)}{3n^2\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Integrate[x^2/Log[a*x^n]^(5/2),x]
```

```
[Out] (-2*x^3*(6*Sqrt[3]*n*Gamma[1/2, (-3*Log[a*x^n])/n]*(-(Log[a*x^n]/n))^(3/2)
+ (a*x^n)^(3/n)*(n + 6*Log[a*x^n])))/(3*n^2*(a*x^n)^(3/n)*Log[a*x^n]^(3/2))
```

Maple [F]

$$\int \frac{x^2}{\ln(a\,x^n)^{\frac{5}{2}}}dx$$

```
[In] int(x^2/ln(a*x^n)^(5/2),x)
```

```
[Out] int(x^2/ln(a*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/log(a*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^2}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**2/ln(a*x**n)**(5/2),x)
[Out] Integral(x**2/log(a*x**n)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^2}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/log(a*x^n)^(5/2),x, algorithm="maxima")
[Out] integrate(x^2/log(a*x^n)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^2}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/log(a*x^n)^(5/2),x, algorithm="giac")
[Out] integrate(x^2/log(a*x^n)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^2}{\ln(a\,x^n)^{5/2}}\,dx$$

```
[In] int(x^2/log(a*x^n)^(5/2),x)
[Out] int(x^2/log(a*x^n)^(5/2), x)
```

3.144 $\int \frac{x}{\log^{\frac{5}{2}}(ax^n)} dx$

Optimal result

Integrand size = 12, antiderivative size = 93

$$\int \frac{x}{\log^{\frac{5}{2}}(ax^n)} dx = \frac{8\sqrt{2\pi}x^2(ax^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x^2}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{8x^2}{3n^2\sqrt{\log(ax^n)}}$$

```
[Out] -2/3*x^2/n/ln(a*x^n)^(3/2)+8/3*x^2*erfi(2^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*2^
(1/2)*Pi^(1/2)/n^(5/2)/((a*x^n)^(2/n))-8/3*x^2/n^2/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2343, 2347, 2211, 2235}

$$\int \frac{x}{\log^{\frac{5}{2}}(ax^n)} dx = \frac{8\sqrt{2\pi}x^2(ax^n)^{-2/n}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{8x^2}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^2}{3n\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Int[x/Log[a*x^n]^(5/2),x]
```

```
[Out] (8*Sqrt[2*Pi]*x^2*Erfi[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(3*n^(5/2)*(a*x
^n)^(2/n)) - (2*x^2)/(3*n*Log[a*x^n]^(3/2)) - (8*x^2)/(3*n^2*Sqrt[Log[a*x^n
]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^2}{3n\log^{\frac{3}{2}}(ax^n)}+\frac{4\int\frac{x}{\log^{\frac{3}{2}}(ax^n)}\,dx}{3n}\\
&= -\frac{2x^2}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{8x^2}{3n^2\sqrt{\log(ax^n)}}+\frac{16\int\frac{x}{\sqrt{\log(ax^n)}}\,dx}{3n^2}\\
&= -\frac{2x^2}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{8x^2}{3n^2\sqrt{\log(ax^n)}}+\frac{\left(16x^2(ax^n)^{-2/n}\right)\text{Subst}\left(\int\frac{e^{\frac{2x}{n}}}{\sqrt{x}}\,dx,x,\log(ax^n)\right)}{3n^3}\\
&= -\frac{2x^2}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{8x^2}{3n^2\sqrt{\log(ax^n)}}+\frac{\left(32x^2(ax^n)^{-2/n}\right)\text{Subst}\left(\int e^{\frac{2x^2}{n}}\,dx,x,\sqrt{\log(ax^n)}\right)}{3n^3}\\
&= \frac{8\sqrt{2\pi}x^2(ax^n)^{-2/n}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}}-\frac{2x^2}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{8x^2}{3n^2\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \frac{x}{\log^{\frac{5}{2}}(ax^n)}\,dx = -\frac{2x^2(ax^n)^{-2/n}\left(4\sqrt{2}n\Gamma\left(\frac{1}{2},-\frac{2\log(ax^n)}{n}\right)\left(-\frac{\log(ax^n)}{n}\right)^{3/2}+(ax^n)^{2/n}(n+4\log(ax^n))\right)}{3n^2\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Integrate[x/Log[a*x^n]^(5/2),x]
[Out] (-2*x^2*(4*Sqrt[2]*n*Gamma[1/2, (-2*Log[a*x^n])/n]*(-(Log[a*x^n]/n))^(3/2)
+ (a*x^n)^(2/n)*(n + 4*Log[a*x^n])))/(3*n^2*(a*x^n)^(2/n)*Log[a*x^n]^(3/2))
```

Maple [F]

$$\int \frac{x}{\ln(a\,x^n)^{\frac{5}{2}}}dx$$

```
[In] int(x/ln(a*x^n)^(5/2),x)
[Out] int(x/ln(a*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\log^{\frac{5}{2}}(ax^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/log(a*x^n)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/ln(a*x**n)**(5/2),x)
[Out] Integral(x/log(a*x**n)**(5/2), x)
```

Maxima [F]

$$\int \frac{x}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/log(a*x^n)^(5/2),x, algorithm="maxima")
[Out] integrate(x/log(a*x^n)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/log(a*x^n)^(5/2),x, algorithm="giac")
[Out] integrate(x/log(a*x^n)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x}{\ln(a\,x^n)^{5/2}}\,dx$$

```
[In] int(x/log(a*x^n)^(5/2),x)
[Out] int(x/log(a*x^n)^(5/2), x)
```

3.145 $\int \frac{1}{\log^{\frac{5}{2}}(ax^n)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \frac{1}{\log^{\frac{5}{2}}(ax^n)}\,dx = \frac{4\sqrt{\pi}x(ax^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2\sqrt{\log(ax^n)}}$$

```
[Out] -2/3*x/n/ln(a*x^n)^(3/2)+4/3*x*erfi(ln(a*x^n)^(1/2)/n^(1/2))*Pi^(1/2)/n^(5/
2)/((a*x^n)^(1/n))-4/3*x/n^2/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2334, 2337, 2211, 2235}

$$\int \frac{1}{\log^{\frac{5}{2}}(ax^n)}\,dx = \frac{4\sqrt{\pi}x(ax^n)^{-1/n}\operatorname{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{4x}{3n^2\sqrt{\log(ax^n)}} - \frac{2x}{3n\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Int[Log[a*x^n]^(-5/2),x]
```

```
[Out] (4*Sqrt[Pi]*x*Erfi[Sqrt[Log[a*x^n]]/Sqrt[n]])/(3*n^(5/2)*(a*x^n)^n^(-1)) -
(2*x)/(3*n*Log[a*x^n]^(3/2)) - (4*x)/(3*n^2*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2334

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x*((a + b
*Log[c*x^n])^(p + 1)/(b*n*(p + 1))), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x}{3n\log^{\frac{3}{2}}(ax^n)} + \frac{2\int \frac{1}{\log^{\frac{3}{2}}(ax^n)}\,dx}{3n} \\
&= -\frac{2x}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2\sqrt{\log(ax^n)}} + \frac{4\int \frac{1}{\sqrt{\log(ax^n)}}\,dx}{3n^2} \\
&= -\frac{2x}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2\sqrt{\log(ax^n)}} + \frac{\left(4x(ax^n)^{-1/n}\right)\text{Subst}\left(\int \frac{e^{\frac{x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{3n^3} \\
&= -\frac{2x}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2\sqrt{\log(ax^n)}} + \frac{\left(8x(ax^n)^{-1/n}\right)\text{Subst}\left(\int e^{\frac{x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{3n^3} \\
&= \frac{4\sqrt{\pi}x(ax^n)^{-1/n}\,\text{erfi}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4x}{3n^2\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{1}{\log^{\frac{5}{2}}(ax^n)}\,dx = -\frac{2x(ax^n)^{-1/n}\left(2n\Gamma\left(\frac{1}{2},-\frac{\log(ax^n)}{n}\right)\left(-\frac{\log(ax^n)}{n}\right)^{3/2}+(ax^n)^{\frac{1}{n}}(n+2\log(ax^n))\right)}{3n^2\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Integrate[Log[a*x^n]^(-5/2),x]
[Out] (-2*x*(2*n*Gamma[1/2, -(Log[a*x^n]/n)]*(-(Log[a*x^n]/n))^(3/2) + (a*x^n)^n^
(-1)*(n + 2*Log[a*x^n])))/(3*n^2*(a*x^n)^n^(-1)*Log[a*x^n]^(3/2))
```

Maple [F]

$$\int \frac{1}{\ln(a\,x^n)^{\frac{5}{2}}}dx$$

```
[In] int(1/ln(a*x^n)^(5/2),x)
[Out] int(1/ln(a*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\log^{\frac{5}{2}}(ax^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/log(a*x^n)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/ln(a*x**n)**(5/2),x)
[Out] Integral(log(a*x**n)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/log(a*x^n)^(5/2),x, algorithm="maxima")
[Out] integrate(log(a*x^n)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/log(a*x^n)^(5/2),x, algorithm="giac")
[Out] integrate(log(a*x^n)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{\ln(a\,x^n)^{5/2}}\,dx$$

```
[In] int(1/log(a*x^n)^(5/2),x)
[Out] int(1/log(a*x^n)^(5/2), x)
```

3.146 $\int \frac{1}{x \log^{\frac{5}{2}}(ax^n)} dx$

Optimal result

Integrand size = 14, antiderivative size = 17

$$\int \frac{1}{x \log^{\frac{5}{2}}(ax^n)} dx = -\frac{2}{3n \log^{\frac{3}{2}}(ax^n)}$$

```
[Out] -2/3/n/ln(a*x^n)^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2339, 30}

$$\int \frac{1}{x \log^{\frac{5}{2}}(ax^n)} dx = -\frac{2}{3n \log^{\frac{3}{2}}(ax^n)}$$

```
[In] Int[1/(x*Log[a*x^n]^(5/2)),x]
```

```
[Out] -2/(3*n*Log[a*x^n]^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, \log\left(ax^n\right)\right)}{n}$$
$$= -\frac{2}{3n\log^{\frac{3}{2}}\left(ax^n\right)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\log^{\frac{5}{2}}\left(ax^n\right)}\,dx = -\frac{2}{3n\log^{\frac{3}{2}}\left(ax^n\right)}$$

```
[In] Integrate[1/(x*Log[a*x^n]^(5/2)),x]
[Out] -2/(3*n*Log[a*x^n]^(3/2))
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2}{3n\ln\left(a\,x^{n}\right)^{\frac{3}{2}}}$	14
default	$-\frac{2}{3n\ln\left(a\,x^{n}\right)^{\frac{3}{2}}}$	14

```
[In] int(1/x/ln(a*x^n)^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3/n/ln(a*x^n)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{x\log^{\frac{5}{2}}\left(ax^n\right)}\,dx = -\frac{2\sqrt{n\log\left(x\right)+\log\left(a\right)}}{3\left(n^3\log\left(x\right)^2+2\,n^2\log\left(a\right)\log\left(x\right)+n\log\left(a\right)^2\right)}$$

```
[In] integrate(1/x/log(a*x^n)^(5/2),x, algorithm="fricas")
[Out] -2/3*sqrt(n*log(x) + log(a))/(n^3*log(x)^2 + 2*n^2*log(a)*log(x) + n*log(a)
^2)
```

Sympy [A] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1}{x\log^{\frac{5}{2}}(ax^n)}\,dx = \begin{cases} -\frac{2}{3n\log(ax^n)^{\frac{3}{2}}} & \text{for } n \neq 0 \\ \frac{\log(x)}{\log(a)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/x/ln(a*x**n)**(5/2),x)
[Out] Piecewise((-2/(3*n*log(a*x**n)**(3/2)), Ne(n, 0)), (log(x)/log(a)**(5/2), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\log^{\frac{5}{2}}(ax^n)}\,dx = -\frac{2}{3\,n\log(ax^n)^{\frac{3}{2}}}$$

```
[In] integrate(1/x/log(a*x^n)^(5/2),x, algorithm="maxima")
[Out] -2/3/(n*log(a*x^n)^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\log^{\frac{5}{2}}(ax^n)}\,dx = -\frac{2}{3\,(n\log(x)+\log(a))^{\frac{3}{2}}n}$$

```
[In] integrate(1/x/log(a*x^n)^(5/2),x, algorithm="giac")
[Out] -2/3/((n*log(x) + log(a))^(3/2)*n)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{x \log^{\frac{5}{2}}(ax^n)} \, dx = -\frac{2}{3\, n \ln\left(a\, x^n\right)^{3/2}}$$

```
[In] int(1/(x*log(a*x^n)^(5/2)),x)
[Out] -2/(3*n*log(a*x^n)^(3/2))
```

3.147 $\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)} dx$

Optimal result

Integrand size = 14, antiderivative size = 84

$$\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)} dx = \frac{4\sqrt{\pi}(ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x} - \frac{2}{3nx \log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2x\sqrt{\log(ax^n)}}$$

```
[Out] -2/3/n/x/ln(a*x^n)^(3/2)+4/3*(a*x^n)^(1/n)*erf(ln(a*x^n)^(1/2)/n^(1/2))*Pi^
(1/2)/n^(5/2)/x+4/3/n^2/x/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2343, 2347, 2211, 2236}

$$\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)} dx = \frac{4\sqrt{\pi}(ax^n)^{\frac{1}{n}} \operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x} + \frac{4}{3n^2x\sqrt{\log(ax^n)}} - \frac{2}{3nx \log^{\frac{3}{2}}(ax^n)}$$

```
[In] Int[1/(x^2*Log[a*x^n]^(5/2)),x]
```

```
[Out] (4*Sqrt[Pi]*(a*x^n)^n^(-1)*Erf[Sqrt[Log[a*x^n]]/Sqrt[n]])/(3*n^(5/2)*x) - 2
/(3*n*x*Log[a*x^n]^(3/2)) + 4/(3*n^2*x*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = -\frac{2}{3nx\log^{\frac{3}{2}}(ax^n)} - \frac{2\int \frac{1}{x^2\log^{\frac{3}{2}}(ax^n)}\,dx}{3n}$$

$$= -\frac{2}{3nx\log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2x\sqrt{\log(ax^n)}} + \frac{4\int \frac{1}{x^2\sqrt{\log(ax^n)}}\,dx}{3n^2}$$

$$= -\frac{2}{3nx\log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2x\sqrt{\log(ax^n)}} + \frac{\left(4(ax^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int \frac{e^{-\frac{x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{3n^3x}$$

$$= -\frac{2}{3nx\log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2x\sqrt{\log(ax^n)}} + \frac{\left(8(ax^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int e^{-\frac{x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{3n^3x}$$

$$= \frac{4\sqrt{\pi}(ax^n)^{\frac{1}{n}}\operatorname{erf}\left(\frac{\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x} - \frac{2}{3nx\log^{\frac{3}{2}}(ax^n)} + \frac{4}{3n^2x\sqrt{\log(ax^n)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)} \, dx = -\frac{2\left(n - 2\log(ax^n) + 2n(ax^n)^{\frac{1}{n}}\Gamma\left(\frac{1}{2}, \frac{\log(ax^n)}{n}\right)\left(\frac{\log(ax^n)}{n}\right)^{3/2}\right)}{3n^2 x \log^{\frac{3}{2}}(ax^n)}$$

```
[In] Integrate[1/(x^2*Log[a*x^n]^(5/2)),x]
[Out] (-2*(n - 2*Log[a*x^n] + 2*n*(a*x^n)^n^(-1)*Gamma[1/2, Log[a*x^n]/n]*(Log[a*
x^n]/n)^(3/2)))/(3*n^2*x*Log[a*x^n]^(3/2))
```

Maple [F]

$$\int \frac{1}{x^2 \ln(a\, x^n)^{\frac{5}{2}}} dx$$

```
[In] int(1/x^2/ln(a*x^n)^(5/2),x)
[Out] int(1/x^2/ln(a*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/log(a*x^n)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)} \, dx = \int \frac{1}{x^2 \log(ax^n)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x**2/ln(a*x**n)**(5/2),x)
[Out] Integral(1/(x**2*log(a*x**n)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{x^2 \log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^2/log(a*x^n)^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x^2*log(a*x^n)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{x^2 \log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^2/log(a*x^n)^(5/2),x, algorithm="giac")
[Out] integrate(1/(x^2*log(a*x^n)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{x^2 \ln(a\,x^n)^{5/2}}\,dx$$

```
[In] int(1/(x^2*log(a*x^n)^(5/2)),x)
[Out] int(1/(x^2*log(a*x^n)^(5/2)), x)
```

3.148 $\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)} dx$

Optimal result

Integrand size = 14, antiderivative size = 93

$$\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)} dx = \frac{8\sqrt{2\pi}(ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x^2} - \frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2x^2\sqrt{\log(ax^n)}}$$

```
[Out] -2/3/n/x^2/ln(a*x^n)^(3/2)+8/3*(a*x^n)^(2/n)*erf(2^(1/2)*ln(a*x^n)^(1/2)/n^
(1/2))*2^(1/2)*Pi^(1/2)/n^(5/2)/x^2+8/3/n^2/x^2/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2343, 2347, 2211, 2236}

$$\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)} dx = \frac{8\sqrt{2\pi}(ax^n)^{2/n} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x^2} + \frac{8}{3n^2x^2\sqrt{\log(ax^n)}} - \frac{2}{3nx^2 \log^{\frac{3}{2}}(ax^n)}$$

```
[In] Int[1/(x^3*Log[a*x^n]^(5/2)),x]
```

```
[Out] (8*Sqrt[2*Pi]*(a*x^n)^(2/n)*Erf[(Sqrt[2]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(3*n^(
5/2)*x^2) - 2/(3*n*x^2*Log[a*x^n]^(3/2)) + 8/(3*n^2*x^2*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{3nx^2\log^{\frac{3}{2}}(ax^n)} - \frac{4\int \frac{1}{x^3\log^{\frac{3}{2}}(ax^n)}\,dx}{3n} \\
&= -\frac{2}{3nx^2\log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2x^2\sqrt{\log(ax^n)}} + \frac{16\int \frac{1}{x^3\sqrt{\log(ax^n)}}\,dx}{3n^2} \\
&= -\frac{2}{3nx^2\log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2x^2\sqrt{\log(ax^n)}} + \frac{\left(16(ax^n)^{2/n}\right)\operatorname{Subst}\left(\int \frac{e^{-\frac{2x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{3n^3x^2} \\
&= -\frac{2}{3nx^2\log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2x^2\sqrt{\log(ax^n)}} + \frac{\left(32(ax^n)^{2/n}\right)\operatorname{Subst}\left(\int e^{-\frac{2x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{3n^3x^2} \\
&= \frac{8\sqrt{2\pi}(ax^n)^{2/n}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}x^2} - \frac{2}{3nx^2\log^{\frac{3}{2}}(ax^n)} + \frac{8}{3n^2x^2\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)}\,dx = -\frac{2\left(n-4\log(ax^n)+4\sqrt{2}n(ax^n)^{2/n}\Gamma\left(\frac{1}{2},\frac{2\log(ax^n)}{n}\right)\left(\frac{\log(ax^n)}{n}\right)^{3/2}\right)}{3n^2x^2\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Integrate[1/(x^3*Log[a*x^n]^(5/2)),x]
[Out] (-2*(n - 4*Log[a*x^n] + 4*Sqrt[2]*n*(a*x^n)^(2/n)*Gamma[1/2, (2*Log[a*x^n])
/n]*(Log[a*x^n]/n)^(3/2)))/(3*n^2*x^2*Log[a*x^n]^(3/2))
```

Maple [F]

$$\int \frac{1}{x^3 \ln(a\,x^n)^{\frac{5}{2}}}dx$$

```
[In] int(1/x^3/ln(a*x^n)^(5/2),x)
[Out] int(1/x^3/ln(a*x^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^3/log(a*x^n)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{x^3 \log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x**3/ln(a*x**n)**(5/2),x)
[Out] Integral(1/(x**3*log(a*x**n)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{x^3 \log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^3/log(a*x^n)^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x^3*log(a*x^n)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{x^3 \log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^3/log(a*x^n)^(5/2),x, algorithm="giac")
[Out] integrate(1/(x^3*log(a*x^n)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{1}{x^3 \ln(a\,x^n)^{5/2}}\,dx$$

```
[In] int(1/(x^3*log(a*x^n)^(5/2)),x)
[Out] int(1/(x^3*log(a*x^n)^(5/2)), x)
```

3.149 $\int (dx)^m \left(a+\frac{a(1+m)\log(cx^n)}{n}\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 21

$$\int (dx)^m \left(a+\frac{a(1+m)\log\left(cx^n\right)}{n}\right)\,dx=\frac{a(dx)^{1+m}\log\left(cx^n\right)}{dn}$$

```
[Out] a*(d*x)^(1+m)*ln(c*x^n)/d/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2340}

$$\int (dx)^m \left(a+\frac{a(1+m)\log\left(cx^n\right)}{n}\right)\,dx=\frac{a(dx)^{m+1}\log\left(cx^n\right)}{dn}$$

```
[In] Int[(d*x)^m*(a + (a*(1 + m)*Log[c*x^n])/n),x]
```

```
[Out] (a*(d*x)^(1 + m)*Log[c*x^n])/(d*n)
```

Rule 2340

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[b*(d*x)^(m + 1)*(Log[c*x^n]/(d*(m + 1))), x] /; FreeQ[{a, b, c, d, m,
n}, x] && NeQ[m, -1] && EqQ[a*(m + 1) - b*n, 0]
```

Rubi steps

$$\text{integral}=\frac{a(dx)^{1+m}\log\left(cx^n\right)}{dn}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int (dx)^m \left(a+\frac{a(1+m)\log(cx^n)}{n}\right)\,dx=\frac{ax(dx)^m\log(cx^n)}{n}$$

```
[In] Integrate[(d*x)^m*(a + (a*(1 + m)*Log[c*x^n])/n),x]
[Out] (a*x*(d*x)^m*Log[c*x^n])/n
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{x(dx)^m\ln(c\,x^n)a}{n}$
risch	$\frac{a x\, x^m d^m \mathrm{e}^{\frac{i\,\mathrm{csgn}(idx)\pi m(\mathrm{csgn}(idx)-\mathrm{csgn}(ix))(-\mathrm{csgn}(idx)+\mathrm{csgn}(id))}{2}}\ln(x^n)}{n}+\frac{a\left(-i\pi\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)+i\pi\ \mathrm{csgn}(ic)\,\mathrm{cs}\right.}{}$

```
[In] int((d*x)^m*(a+a*(1+m)*ln(c*x^n)/n),x,method=_RETURNVERBOSE)
[Out] x*(d*x)^m*ln(c*x^n)*a/n
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int (dx)^m \left(a+\frac{a(1+m)\log(cx^n)}{n}\right)\,dx=\frac{(anx\log(x)+ax\log(c))e^{(m\log(d)+m\log(x))}}{n}$$

```
[In] integrate((d*x)^m*(a+a*(1+m)*log(c*x^n)/n),x, algorithm="fricas")
[Out] (a*n*x*log(x) + a*x*log(c))*e^(m*log(d) + m*log(x))/n
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int (dx)^m \left(a+\frac{a(1+m)\log(cx^n)}{n}\right) dx = \frac{ax(dx)^m \log(cx^n)}{n}$$

```
[In] integrate((d*x)**m*(a+a*(1+m)*ln(c*x**n)/n),x)
[Out] a*x*(d*x)**m*log(c*x**n)/n
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(21) = 42.

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.86

$$\int (dx)^m \left(a+\frac{a(1+m)\log(cx^n)}{n}\right) dx = -\frac{ad^m mxx^m}{(m+1)^2} - \frac{ad^m xx^m}{(m+1)^2} + \frac{(dx)^{m+1} am\log(cx^n)}{d(m+1)n} + \frac{(dx)^{m+1} a}{d(m+1)} + \frac{(dx)^{m+1} a\log(cx^n)}{d(m+1)n}$$

```
[In] integrate((d*x)^m*(a+a*(1+m)*log(c*x^n)/n),x, algorithm="maxima")
[Out] -a*d^m*m*x*x^m/(m + 1)^2 - a*d^m*x*x^m/(m + 1)^2 + (d*x)^(m + 1)*a*m*log(c*
x^n)/(d*(m + 1)*n) + (d*x)^(m + 1)*a/(d*(m + 1)) + (d*x)^(m + 1)*a*log(c*x^
n)/(d*(m + 1)*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(21) = 42.

Time = 0.41 (sec) , antiderivative size = 175, normalized size of antiderivative = 8.33

$$\int (dx)^m \left(a+\frac{a(1+m)\log(cx^n)}{n}\right) dx = \frac{ad^m m^2 xx^m \log(x)}{m^2+2m+1} + \frac{2ad^m mxx^m \log(x)}{m^2+2m+1} - \frac{ad^m mxx^m}{m^2+2m+1} + \frac{ad^{m+1} mxx^m \log(c)}{(dm+d)n} + \frac{ad^m xx^m \log(x)}{m^2+2m+1} + \frac{ad^{m+1} xx^m}{dm+d} - \frac{ad^m xx^m}{m^2+2m+1} + \frac{ad^{m+1} xx^m \log(c)}{(dm+d)n}$$

```
[In] integrate((d*x)^m*(a+a*(1+m)*log(c*x^n)/n),x, algorithm="giac")
[Out] a*d^m*m^2*x*x^m*log(x)/(m^2 + 2*m + 1) + 2*a*d^m*m*x*x^m*log(x)/(m^2 + 2*m
+ 1) - a*d^m*m*x*x^m/(m^2 + 2*m + 1) + a*d^(m + 1)*m*x*x^m*log(c)/((d*m + d
)*n) + a*d^m*x*x^m*log(x)/(m^2 + 2*m + 1) + a*d^(m + 1)*x*x^m/(d*m + d) - a
*d^m*x*x^m/(m^2 + 2*m + 1) + a*d^(m + 1)*x*x^m*log(c)/((d*m + d)*n)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+\frac{a(1+m)\log(cx^n)}{n}\right)\,dx=\int (d\,x)^m \left(a+\frac{a\,\ln\left(c\,x^n\right)\,(m+1)}{n}\right)\,dx$$

```
[In] int((d*x)^m*(a + (a*log(c*x^n)*(m + 1))/n),x)
[Out] int((d*x)^m*(a + (a*log(c*x^n)*(m + 1))/n), x)
```

3.150 $\int (dx)^m (a+b\log(cx^n))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 116

$$\int (dx)^m (a+b\log(cx^n))^3\,dx = -\frac{6b^3n^3(dx)^{1+m}}{d(1+m)^4}+\frac{6b^2n^2(dx)^{1+m}(a+b\log(cx^n))}{d(1+m)^3}-\frac{3bn(dx)^{1+m}(a+b\log(cx^n))^2}{d(1+m)^2}+\frac{(dx)^{1+m}(a+b\log(cx^n))^3}{d(1+m)}$$

```
[Out] -6*b^3*n^3*(d*x)^(1+m)/d/(1+m)^4+6*b^2*n^2*(d*x)^(1+m)*(a+b*ln(c*x^n))/d/(1
+m)^3-3*b*n*(d*x)^(1+m)*(a+b*ln(c*x^n))^2/d/(1+m)^2+(d*x)^(1+m)*(a+b*ln(c*x
^n))^3/d/(1+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2342, 2341}

$$\int (dx)^m (a+b\log(cx^n))^3\,dx = \frac{6b^2n^2(dx)^{m+1}(a+b\log(cx^n))}{d(m+1)^3}+\frac{(dx)^{m+1}(a+b\log(cx^n))^3}{d(m+1)}-\frac{3bn(dx)^{m+1}(a+b\log(cx^n))^2}{d(m+1)^2}-\frac{6b^3n^3(dx)^{m+1}}{d(m+1)^4}$$

```
[In] Int[(d*x)^m*(a + b*Log[c*x^n])^3,x]
```

```
[Out] (-6*b^3*n^3*(d*x)^(1 + m))/(d*(1 + m)^4) + (6*b^2*n^2*(d*x)^(1 + m)*(a + b*
Log[c*x^n]))/(d*(1 + m)^3) - (3*b*n*(d*x)^(1 + m)*(a + b*Log[c*x^n])^2)/(d*
(1 + m)^2) + ((d*x)^(1 + m)*(a + b*Log[c*x^n])^3)/(d*(1 + m))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(dx)^{1+m}(a+b\log(cx^n))^3}{d(1+m)} - \frac{(3bn)\int (dx)^m(a+b\log(cx^n))^2\,dx}{1+m} \\
&= -\frac{3bn(dx)^{1+m}(a+b\log(cx^n))^2}{d(1+m)^2} + \frac{(dx)^{1+m}(a+b\log(cx^n))^3}{d(1+m)} \\
&\quad + \frac{(6b^2n^2)\int (dx)^m(a+b\log(cx^n))\,dx}{(1+m)^2} \\
&= -\frac{6b^3n^3(dx)^{1+m}}{d(1+m)^4} + \frac{6b^2n^2(dx)^{1+m}(a+b\log(cx^n))}{d(1+m)^3} \\
&\quad - \frac{3bn(dx)^{1+m}(a+b\log(cx^n))^2}{d(1+m)^2} + \frac{(dx)^{1+m}(a+b\log(cx^n))^3}{d(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int (dx)^m(a+b\log(cx^n))^3\,dx = \frac{x(dx)^m\left((a+b\log(cx^n))^3 - \frac{3bn\left((1+m)^2(a+b\log(cx^n))^2+2bn(bn-(1+m)(a+b\log(cx^n)))\right)}{(1+m)^3}\right)}{1+m}$$

```
[In] Integrate[(d*x)^m*(a + b*Log[c*x^n])^3,x]
[Out] (x*(d*x)^m*((a + b*Log[c*x^n])^3 - (3*b*n*((1 + m)^2*(a + b*Log[c*x^n])^2 +
 2*b*n*(b*n - (1 + m)*(a + b*Log[c*x^n]))))/(1 + m)^3))/(1 + m)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(116) = 232$.

Time = 0.78 (sec) , antiderivative size = 578, normalized size of antiderivative = 4.98

method	result
parallelrisch	$-\frac{-x(dx)^m a^3+6x(dx)^m \ln(c\,x^n)a\,b^2m^2n+12x(dx)^m \ln(c\,x^n)a\,b^2mn-x(dx)^m \ln(c\,x^n)^3b^3m^3-3x(dx)^m \ln(c\,x^n)^3b^3m^2-3x(d}{}$
risch	Expression too large to display

```
[In] int((d*x)^m*(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(-x*(d*x)^m*a^3+6*x*(d*x)^m*ln(c*x^n)*a*b^2*m^2*n+12*x*(d*x)^m*ln(c*x^n)*a
*b^2*m*n-x*(d*x)^m*ln(c*x^n)^3*b^3*m^3-3*x*(d*x)^m*ln(c*x^n)^3*b^3*m^2-3*x*
(d*x)^m*ln(c*x^n)^3*b^3*m+3*x*(d*x)^m*ln(c*x^n)^2*b^3*n-6*x*(d*x)^m*ln(c*x^
n)*b^3*n^2-3*x*(d*x)^m*ln(c*x^n)^2*a*b^2-6*x*(d*x)^m*a*b^2*n^2-3*x*(d*x)^m*
ln(c*x^n)*a^2*b+3*x*(d*x)^m*a^2*b*n-x*(d*x)^m*ln(c*x^n)^3*b^3-x*(d*x)^m*a^3
*m^3+6*x*(d*x)^m*b^3*n^3-3*x*(d*x)^m*a^3*m^2-3*x*(d*x)^m*a^3*m-9*x*(d*x)^m*
ln(c*x^n)^2*a*b^2*m-9*x*(d*x)^m*ln(c*x^n)*a^2*b*m^2+3*x*(d*x)^m*a^2*b*m^2*n
-6*x*(d*x)^m*a*b^2*m*n^2-9*x*(d*x)^m*ln(c*x^n)*a^2*b*m+6*x*(d*x)^m*ln(c*x^n
)*a*b^2*n+6*x*(d*x)^m*a^2*b*m*n-3*x*(d*x)^m*ln(c*x^n)^2*a*b^2*m^3+3*x*(d*x)
^m*ln(c*x^n)^2*b^3*m^2*n-9*x*(d*x)^m*ln(c*x^n)^2*a*b^2*m^2+6*x*(d*x)^m*ln(c
*x^n)^2*b^3*m*n-3*x*(d*x)^m*ln(c*x^n)*a^2*b*m^3-6*x*(d*x)^m*ln(c*x^n)*b^3*m
*n^2)/(m^3+3*m^2+3*m+1)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(116) = 232$.

Time = 0.33 (sec) , antiderivative size = 574, normalized size of antiderivative = 4.95

$$\int (dx)^m (a + b\log(cx^n))^3\, dx$$
$$= \frac{\left((b^3m^3 + 3\,b^3m^2 + 3\,b^3m + b^3)n^3x\log(x)^3 + (b^3m^3 + 3\,b^3m^2 + 3\,b^3m + b^3)x\log(c)^3 + 3\,(ab^2m^3 + 3\,ab^2m\right.}{}$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3)*n^3*x*log(x)^3 + (b^3*m^3 + 3*b^3*m^
2 + 3*b^3*m + b^3)*x*log(c)^3 + 3*(a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*
b^2 - (b^3*m^2 + 2*b^3*m + b^3)*n)*x*log(c)^2 + 3*(a^2*b*m^3 + 3*a^2*b*m^2
+ 3*a^2*b*m + a^2*b + 2*(b^3*m + b^3)*n^2 - 2*(a*b^2*m^2 + 2*a*b^2*m + a*b^
2)*n)*x*log(c) + 3*((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3)*n^2*x*log(c) - ((
b^3*m^2 + 2*b^3*m + b^3)*n^3 - (a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*b^2
)*n^2)*x)*log(x)^2 + (a^3*m^3 - 6*b^3*n^3 + 3*a^3*m^2 + 3*a^3*m + a^3 + 6*(
a*b^2*m + a*b^2)*n^2 - 3*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n)*x + 3*((b^3*m^3
```

```
 + 3*b^3*m^2 + 3*b^3*m + b^3)*n*x*log(c)^2 - 2*((b^3*m^2 + 2*b^3*m + b^3)*n
^2 - (a*b^2*m^3 + 3*a*b^2*m^2 + 3*a*b^2*m + a*b^2)*n)*x*log(c) + (2*(b^3*m
+ b^3)*n^3 - 2*(a*b^2*m^2 + 2*a*b^2*m + a*b^2)*n^2 + (a^2*b*m^3 + 3*a^2*b*m
^2 + 3*a^2*b*m + a^2*b)*n)*x)*log(x))*e^(m*log(d) + m*log(x))/(m^4 + 4*m^3
+ 6*m^2 + 4*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1273 vs. $2(107) = 214$.

Time = 8.99 (sec) , antiderivative size = 1273, normalized size of antiderivative = 10.97

$$\int (dx)^m (a + b\log(cx^n))^3 \, dx = \text{Too large to display}$$

```
[In] integrate((d*x)**m*(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((a**3*m**3*x*(d*x)**m/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 3*a**3
*m**2*x*(d*x)**m/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 3*a**3*m*x*(d*x)**m/(
m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + a**3*x*(d*x)**m/(m**4 + 4*m**3 + 6*m**2
 + 4*m + 1) + 3*a**2*b*m**3*x*(d*x)**m*log(c*x**n)/(m**4 + 4*m**3 + 6*m**2
+ 4*m + 1) - 3*a**2*b*m**2*n*x*(d*x)**m/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1)
+ 9*a**2*b*m**2*x*(d*x)**m*log(c*x**n)/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) -
 6*a**2*b*m*n*x*(d*x)**m/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 9*a**2*b*m*x*
(d*x)**m*log(c*x**n)/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 3*a**2*b*n*x*(d*x
)**m/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 3*a**2*b*x*(d*x)**m*log(c*x**n)/(
m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 3*a*b**2*m**3*x*(d*x)**m*log(c*x**n)**2
/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 6*a*b**2*m**2*n*x*(d*x)**m*log(c*x**n
)/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 9*a*b**2*m**2*x*(d*x)**m*log(c*x**n)
**2/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 6*a*b**2*m*n**2*x*(d*x)**m/(m**4 +
 4*m**3 + 6*m**2 + 4*m + 1) - 12*a*b**2*m*n*x*(d*x)**m*log(c*x**n)/(m**4 +
4*m**3 + 6*m**2 + 4*m + 1) + 9*a*b**2*m*x*(d*x)**m*log(c*x**n)**2/(m**4 + 4
*m**3 + 6*m**2 + 4*m + 1) + 6*a*b**2*n**2*x*(d*x)**m/(m**4 + 4*m**3 + 6*m**
2 + 4*m + 1) - 6*a*b**2*n*x*(d*x)**m*log(c*x**n)/(m**4 + 4*m**3 + 6*m**2 +
4*m + 1) + 3*a*b**2*x*(d*x)**m*log(c*x**n)**2/(m**4 + 4*m**3 + 6*m**2 + 4*m
 + 1) + b**3*m**3*x*(d*x)**m*log(c*x**n)**3/(m**4 + 4*m**3 + 6*m**2 + 4*m +
 1) - 3*b**3*m**2*n*x*(d*x)**m*log(c*x**n)**2/(m**4 + 4*m**3 + 6*m**2 + 4*m
 + 1) + 3*b**3*m**2*x*(d*x)**m*log(c*x**n)**3/(m**4 + 4*m**3 + 6*m**2 + 4*m
 + 1) + 6*b**3*m*n**2*x*(d*x)**m*log(c*x**n)/(m**4 + 4*m**3 + 6*m**2 + 4*m
+ 1) - 6*b**3*m*n*x*(d*x)**m*log(c*x**n)**2/(m**4 + 4*m**3 + 6*m**2 + 4*m +
 1) + 3*b**3*m*x*(d*x)**m*log(c*x**n)**3/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1)
 - 6*b**3*n**3*x*(d*x)**m/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 6*b**3*n**2*
x*(d*x)**m*log(c*x**n)/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 3*b**3*n*x*(d*x
)**m*log(c*x**n)**2/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + b**3*x*(d*x)**m*lo
g(c*x**n)**3/(m**4 + 4*m**3 + 6*m**2 + 4*m + 1), Ne(m, -1)), (Piecewise(((a
**3*log(c*x**n) + 3*a**2*b*log(c*x**n)**2/2 + a*b**2*log(c*x**n)**3 + b**3*
```

```
log(c*x**n)**4/4)/n, Ne(n, 0)), ((a**3 + 3*a**2*b*log(c) + 3*a*b**2*log(c)*
*2 + b**3*log(c)**3)*log(x), True))/d, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(116) = 232$.

Time = 0.21 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.13

$$\begin{aligned}\int (dx)^m\left(a+b\log\left(cx^n\right)\right)^3\,dx &= -\frac{3\,a^2bd^mnxx^m}{(m+1)^2}+\frac{(dx)^{m+1}\,b^3\log\left(cx^n\right)^3}{d(m+1)}-6\left(\frac{d^mnxx^m\log\left(cx^n\right)}{(m+1)^2}-\frac{d^mn^2xx^m}{(m+1)^3}\right)ab^2\\ &\quad-3\left(\frac{d^mnxx^m\log\left(cx^n\right)^2}{(m+1)^2}-\frac{2\left(\frac{d^{m+1}nxx^m\log(cx^n)}{(m+1)^2}-\frac{d^{m+1}n^2xx^m}{(m+1)^3}\right)n}{d(m+1)}\right)b^3\\ &\quad+\frac{3\,(dx)^{m+1}\,ab^2\log\left(cx^n\right)^2}{d(m+1)}+\frac{3\,(dx)^{m+1}\,a^2b\log\left(cx^n\right)}{d(m+1)}+\frac{(dx)^{m+1}\,a^3}{d(m+1)}\end{aligned}$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] -3*a^2*b*d^m*n*x*x^m/(m + 1)^2 + (d*x)^(m + 1)*b^3*log(c*x^n)^3/(d*(m + 1))
 - 6*(d^m*n*x*x^m*log(c*x^n)/(m + 1)^2 - d^m*n^2*x*x^m/(m + 1)^3)*a*b^2 - 3
*(d^m*n*x*x^m*log(c*x^n)^2/(m + 1)^2 - 2*(d^(m + 1)*n*x*x^m*log(c*x^n)/(m +
 1)^2 - d^(m + 1)*n^2*x*x^m/(m + 1)^3)*n/(d*(m + 1)))*b^3 + 3*(d*x)^(m + 1)
*a*b^2*log(c*x^n)^2/(d*(m + 1)) + 3*(d*x)^(m + 1)*a^2*b*log(c*x^n)/(d*(m +
1)) + (d*x)^(m + 1)*a^3/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(116) = 232$.

Time = 0.41 (sec) , antiderivative size = 1133, normalized size of antiderivative = 9.77

$$\int (dx)^m\left(a+b\log\left(cx^n\right)\right)^3\,dx=\text{Too large to display}$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] b^3*d^m*m^3*n^3*x*x^m*log(x)^3/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 3*b^3*d^m*
m^2*n^3*x*x^m*log(x)^3/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) - 3*b^3*d^m*m^2*n^3*
x*x^m*log(x)^2/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 3*b^3*d^m*m^2*n^2*x*x^m*lo
g(c)*log(x)^2/(m^3 + 3*m^2 + 3*m + 1) + 3*b^3*d^m*m*n^3*x*x^m*log(x)^3/(m^4
 + 4*m^3 + 6*m^2 + 4*m + 1) + 3*a*b^2*d^m*m^2*n^2*x*x^m*log(x)^2/(m^3 + 3*m
^2 + 3*m + 1) - 6*b^3*d^m*m*n^3*x*x^m*log(x)^2/(m^4 + 4*m^3 + 6*m^2 + 4*m +
```

```
 1) + 6*b^3*d^m*m*n^2*x*x^m*log(c)*log(x)^2/(m^3 + 3*m^2 + 3*m + 1) + b^3*d
^m*n^3*x*x^m*log(x)^3/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 6*b^3*d^m*m*n^3*x*x
^m*log(x)/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) - 6*b^3*d^m*m*n^2*x*x^m*log(c)*lo
g(x)/(m^3 + 3*m^2 + 3*m + 1) + 3*b^3*d^m*m*n*x*x^m*log(c)^2*log(x)/(m^2 + 2
*m + 1) + 6*a*b^2*d^m*m*n^2*x*x^m*log(x)^2/(m^3 + 3*m^2 + 3*m + 1) - 3*b^3*
d^m*n^3*x*x^m*log(x)^2/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 3*b^3*d^m*n^2*x*x^
m*log(c)*log(x)^2/(m^3 + 3*m^2 + 3*m + 1) - 6*a*b^2*d^m*m*n^2*x*x^m*log(x)/
(m^3 + 3*m^2 + 3*m + 1) + 6*b^3*d^m*n^3*x*x^m*log(x)/(m^4 + 4*m^3 + 6*m^2 +
 4*m + 1) + 6*a*b^2*d^m*m*n*x*x^m*log(c)*log(x)/(m^2 + 2*m + 1) - 6*b^3*d^m
*n^2*x*x^m*log(c)*log(x)/(m^3 + 3*m^2 + 3*m + 1) + 3*b^3*d^m*n*x*x^m*log(c)
^2*log(x)/(m^2 + 2*m + 1) + 3*a*b^2*d^m*n^2*x*x^m*log(x)^2/(m^3 + 3*m^2 + 3
*m + 1) - 6*b^3*d^m*n^3*x*x^m/(m^4 + 4*m^3 + 6*m^2 + 4*m + 1) + 6*b^3*d^m*n
^2*x*x^m*log(c)/(m^3 + 3*m^2 + 3*m + 1) - 3*b^3*d^m*n*x*x^m*log(c)^2/(m^2 +
 2*m + 1) + 3*a^2*b*d^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) - 6*a*b^2*d^m*n^2*
x*x^m*log(x)/(m^3 + 3*m^2 + 3*m + 1) + 6*a*b^2*d^m*n*x*x^m*log(c)*log(x)/(m
^2 + 2*m + 1) + 6*a*b^2*d^m*n^2*x*x^m/(m^3 + 3*m^2 + 3*m + 1) - 6*a*b^2*d^m
*n*x*x^m*log(c)/(m^2 + 2*m + 1) + (d*x)^m*b^3*x*log(c)^3/(m + 1) + 3*a^2*b*
d^m*n*x*x^m*log(x)/(m^2 + 2*m + 1) - 3*a^2*b*d^m*n*x*x^m/(m^2 + 2*m + 1) +
3*(d*x)^m*a*b^2*x*log(c)^2/(m + 1) + 3*(d*x)^m*a^2*b*x*log(c)/(m + 1) + (d*
x)^m*a^3*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m (a+b\log(cx^n))^3 \, dx = \int (d\,x)^m \left(a+b\,\ln\left(c\,x^n\right)\right)^3 dx$$

```
[In] int((d*x)^m*(a + b*log(c*x^n))^3,x)
[Out] int((d*x)^m*(a + b*log(c*x^n))^3, x)
```

3.151 $\int (dx)^m (a+b\log(cx^n))^2 \, dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int (dx)^m (a+b\log(cx^n))^2 \, dx = \frac{2b^2n^2(dx)^{1+m}}{d(1+m)^3} - \frac{2bn(dx)^{1+m}(a+b\log(cx^n))}{d(1+m)^2} + \frac{(dx)^{1+m}(a+b\log(cx^n))^2}{d(1+m)}$$

```
[Out] 2*b^2*n^2*(d*x)^(1+m)/d/(1+m)^3-2*b*n*(d*x)^(1+m)*(a+b*ln(c*x^n))/d/(1+m)^2
+(d*x)^(1+m)*(a+b*ln(c*x^n))^2/d/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2342, 2341}

$$\int (dx)^m (a+b\log(cx^n))^2 \, dx = \frac{(dx)^{m+1}(a+b\log(cx^n))^2}{d(m+1)} - \frac{2bn(dx)^{m+1}(a+b\log(cx^n))}{d(m+1)^2} + \frac{2b^2n^2(dx)^{m+1}}{d(m+1)^3}$$

```
[In] Int[(d*x)^m*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (2*b^2*n^2*(d*x)^(1 + m))/(d*(1 + m)^3) - (2*b*n*(d*x)^(1 + m)*(a + b*Log[c
*x^n]))/(d*(1 + m)^2) + ((d*x)^(1 + m)*(a + b*Log[c*x^n])^2)/(d*(1 + m))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
```

```
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{(dx)^{1+m}(a+b\log(cx^n))^2}{d(1+m)} - \frac{(2bn)\int(dx)^m(a+b\log(cx^n))\,dx}{1+m}$$
$$= \frac{2b^2n^2(dx)^{1+m}}{d(1+m)^3} - \frac{2bn(dx)^{1+m}(a+b\log(cx^n))}{d(1+m)^2} + \frac{(dx)^{1+m}(a+b\log(cx^n))^2}{d(1+m)}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int(dx)^m(a+b\log(cx^n))^2\,dx$$
$$= \frac{x(dx)^m\left(a^2(1+m)^2-2ab(1+m)n+2b^2n^2+2b(1+m)(a+am-bn)\log(cx^n)+b^2(1+m)^2\log^2(cx^n)\right)}{(1+m)^3}$$

```
[In] Integrate[(d*x)^m*(a + b*Log[c*x^n])^2,x]
```

```
[Out] (x*(d*x)^m*(a^2*(1 + m)^2 - 2*a*b*(1 + m)*n + 2*b^2*n^2 + 2*b*(1 + m)*(a +
a*m - b*n)*Log[c*x^n] + b^2*(1 + m)^2*Log[c*x^n]^2))/(1 + m)^3
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(81) = 162.

Time = 0.22 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.00

method	result
parallelrisch	$-\frac{-x(dx)^m a^2m^2-2x(dx)^m b^2n^2-2x(dx)^m a^2m-x(dx)^m \ln(cx^n)^2b^2-2x(dx)^m \ln(cx^n)ab\,m^2+2x(dx)^m \ln(cx^n)b^2mn-4x(dx)}{}$
risch	Expression too large to display

```
[In] int((d*x)^m*(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(-x*(d*x)^m*a^2*m^2-2*x*(d*x)^m*b^2*n^2-2*x*(d*x)^m*a^2*m-x*(d*x)^m*ln(c*x
^n)^2*b^2-2*x*(d*x)^m*ln(c*x^n)*a*b*m^2+2*x*(d*x)^m*ln(c*x^n)*b^2*m*n-4*x*(
d*x)^m*ln(c*x^n)*a*b*m+2*x*(d*x)^m*a*b*m*n-x*(d*x)^m*a^2-x*(d*x)^m*ln(c*x^n
)^2*b^2*m^2-2*x*(d*x)^m*ln(c*x^n)^2*b^2*m+2*x*(d*x)^m*ln(c*x^n)*b^2*n-2*x*(
d*x)^m*ln(c*x^n)*a*b+2*x*(d*x)^m*a*b*n)/(m^3+3*m^2+3*m+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(81) = 162$.

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.57

$$\int (dx)^m (a + b\log(cx^n))^2 \, dx = \frac{((b^2m^2 + 2b^2m + b^2)n^2x\log(x)^2 + (b^2m^2 + 2b^2m + b^2)x\log(c)^2 + 2(abm^2 + 2abm + ab - (b^2m + b^2)n}{}$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 2*b^2*m + b^2)*n^2*x*log(x)^2 + (b^2*m^2 + 2*b^2*m + b^2)*x*log
(c)^2 + 2*(a*b*m^2 + 2*a*b*m + a*b - (b^2*m + b^2)*n)*x*log(c) + (a^2*m^2 +
 2*b^2*n^2 + 2*a^2*m + a^2 - 2*(a*b*m + a*b)*n)*x + 2*((b^2*m^2 + 2*b^2*m +
 b^2)*n*x*log(c) - ((b^2*m + b^2)*n^2 - (a*b*m^2 + 2*a*b*m + a*b)*n)*x)*log
(x))*e^(m*log(d) + m*log(x))/(m^3 + 3*m^2 + 3*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(73) = 146$.

Time = 6.38 (sec) , antiderivative size = 502, normalized size of antiderivative = 6.20

$$\int (dx)^m (a + b\log(cx^n))^2 \, dx = \begin{cases} \frac{a^2m^2x(dx)^m}{m^3+3m^2+3m+1} + \frac{2a^2mx(dx)^m}{m^3+3m^2+3m+1} + \frac{a^2x(dx)^m}{m^3+3m^2+3m+1} + \frac{2abm^2x(dx)^m\log(cx^n)}{m^3+3m^2+3m+1} - \frac{2abmnx(dx)^m}{m^3+3m^2+3m+1} + \frac{4abmx(dx)^m\log(cx^n)}{m^3+3m^2+3m+1} \\ \frac{\begin{cases} \frac{a^2\log(cx^n) + ab\log(cx^n)^2 + \frac{b^2\log(cx^n)^3}{3}}{n} & \text{for } n \neq 0 \\ (a^2 + 2ab\log(c) + b^2\log(c)^2)\log(x) & \text{otherwise} \end{cases}}{d} \end{cases}$$

```
[In] integrate((d*x)**m*(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((a**2*m**2*x*(d*x)**m/(m**3 + 3*m**2 + 3*m + 1) + 2*a**2*m*x*(d*x
)**m/(m**3 + 3*m**2 + 3*m + 1) + a**2*x*(d*x)**m/(m**3 + 3*m**2 + 3*m + 1)
+ 2*a*b*m**2*x*(d*x)**m*log(c*x**n)/(m**3 + 3*m**2 + 3*m + 1) - 2*a*b*m*n*x
*(d*x)**m/(m**3 + 3*m**2 + 3*m + 1) + 4*a*b*m*x*(d*x)**m*log(c*x**n)/(m**3
```

```
+ 3*m**2 + 3*m + 1) - 2*a*b*n*x*(d*x)**m/(m**3 + 3*m**2 + 3*m + 1) + 2*a*b*
x*(d*x)**m*log(c*x**n)/(m**3 + 3*m**2 + 3*m + 1) + b**2*m**2*x*(d*x)**m*log
(c*x**n)**2/(m**3 + 3*m**2 + 3*m + 1) - 2*b**2*m*n*x*(d*x)**m*log(c*x**n)/(
m**3 + 3*m**2 + 3*m + 1) + 2*b**2*m*x*(d*x)**m*log(c*x**n)**2/(m**3 + 3*m**
2 + 3*m + 1) + 2*b**2*n**2*x*(d*x)**m/(m**3 + 3*m**2 + 3*m + 1) - 2*b**2*n*
x*(d*x)**m*log(c*x**n)/(m**3 + 3*m**2 + 3*m + 1) + b**2*x*(d*x)**m*log(c*x*
*n)**2/(m**3 + 3*m**2 + 3*m + 1), Ne(m, -1)), (Piecewise(((a**2*log(c*x**n)
 + a*b*log(c*x**n)**2 + b**2*log(c*x**n)**3/3)/n, Ne(n, 0)), ((a**2 + 2*a*b
*log(c) + b**2*log(c)**2)*log(x), True))/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.63

$$\int (dx)^m \left(a+b\log\left(cx^n\right)\right)^2\, dx = -\frac{2\,abd^m nxx^m}{(m+1)^2} - 2\left(\frac{d^m nxx^m\log\left(cx^n\right)}{(m+1)^2} - \frac{d^m n^2xx^m}{(m+1)^3}\right)b^2 + \frac{(dx)^{m+1}\,b^2\log\left(cx^n\right)^2}{d(m+1)} + \frac{2\,(dx)^{m+1}\,ab\log\left(cx^n\right)}{d(m+1)} + \frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*d^m*n*x*x^m/(m + 1)^2 - 2*(d^m*n*x*x^m*log(c*x^n)/(m + 1)^2 - d^m*n^
2*x*x^m/(m + 1)^3)*b^2 + (d*x)^(m + 1)*b^2*log(c*x^n)^2/(d*(m + 1)) + 2*(d*
x)^(m + 1)*a*b*log(c*x^n)/(d*(m + 1)) + (d*x)^(m + 1)*a^2/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(81) = 162.

Time = 0.36 (sec) , antiderivative size = 402, normalized size of antiderivative = 4.96

$$\int (dx)^m \left(a+b\log\left(cx^n\right)\right)^2\,dx = \frac{b^2d^m m^2n^2xx^m\log\left(x\right)^2}{m^3+3\,m^2+3\,m+1}+\frac{2\,b^2d^m mn^2xx^m\log\left(x\right)^2}{m^3+3\,m^2+3\,m+1} - \frac{2\,b^2d^m mn^2xx^m\log\left(x\right)}{m^3+3\,m^2+3\,m+1}+\frac{2\,b^2d^m mnxx^m\log\left(c\right)\log\left(x\right)}{m^2+2\,m+1} + \frac{b^2d^m n^2xx^m\log\left(x\right)^2}{m^3+3\,m^2+3\,m+1}+\frac{2\,abd^m mnxx^m\log\left(x\right)}{m^2+2\,m+1} - \frac{2\,b^2d^m n^2xx^m\log\left(x\right)}{m^3+3\,m^2+3\,m+1}+\frac{2\,b^2d^m nxx^m\log\left(c\right)\log\left(x\right)}{m^2+2\,m+1} + \frac{2\,b^2d^m n^2xx^m}{m^3+3\,m^2+3\,m+1}-\frac{2\,b^2d^m nxx^m\log\left(c\right)}{m^2+2\,m+1} + \frac{2\,abd^m nxx^m\log\left(x\right)}{m^2+2\,m+1}-\frac{2\,abd^m nxx^m}{m^2+2\,m+1} + \frac{(dx)^m\,b^2x\log\left(c\right)^2}{m+1}+\frac{2\,(dx)^m\,abx\log\left(c\right)}{m+1}+\frac{(dx)^m\,a^2x}{m+1}$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] b^2*d^m*m^2*n^2*x*x^m*log(x)^2/(m^3 + 3*m^2 + 3*m + 1) + 2*b^2*d^m*m*n^2*x*
x^m*log(x)^2/(m^3 + 3*m^2 + 3*m + 1) - 2*b^2*d^m*m*n^2*x*x^m*log(x)/(m^3 +
3*m^2 + 3*m + 1) + 2*b^2*d^m*m*n*x*x^m*log(c)*log(x)/(m^2 + 2*m + 1) + b^2*
d^m*n^2*x*x^m*log(x)^2/(m^3 + 3*m^2 + 3*m + 1) + 2*a*b*d^m*m*n*x*x^m*log(x)
/(m^2 + 2*m + 1) - 2*b^2*d^m*n^2*x*x^m*log(x)/(m^3 + 3*m^2 + 3*m + 1) + 2*b
^2*d^m*n*x*x^m*log(c)*log(x)/(m^2 + 2*m + 1) + 2*b^2*d^m*n^2*x*x^m/(m^3 + 3
*m^2 + 3*m + 1) - 2*b^2*d^m*n*x*x^m*log(c)/(m^2 + 2*m + 1) + 2*a*b*d^m*n*x*
x^m*log(x)/(m^2 + 2*m + 1) - 2*a*b*d^m*n*x*x^m/(m^2 + 2*m + 1) + (d*x)^m*b^
2*x*log(c)^2/(m + 1) + 2*(d*x)^m*a*b*x*log(c)/(m + 1) + (d*x)^m*a^2*x/(m +
1)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\log\left(cx^n\right)\right)^2\,dx = \int {\left(d\,x\right)}^m\,{\left(a+b\,\ln\left(c\,x^n\right)\right)}^2 \,d x$$

```
[In] int((d*x)^m*(a + b*log(c*x^n))^2,x)
```

```
[Out] int((d*x)^m*(a + b*log(c*x^n))^2, x)
```

3.152 $\int (dx)^m (a + b\log(cx^n))\, dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int (dx)^m (a + b\log(cx^n))\, dx = -\frac{bn(dx)^{1+m}}{d(1+m)^2} + \frac{(dx)^{1+m}(a + b\log(cx^n))}{d(1+m)}$$

```
[Out] -b*n*(d*x)^(1+m)/d/(1+m)^2+(d*x)^(1+m)*(a+b*ln(c*x^n))/d/(1+m)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2341}

$$\int (dx)^m (a + b\log(cx^n))\, dx = \frac{(dx)^{m+1}(a + b\log(cx^n))}{d(m+1)} - \frac{bn(dx)^{m+1}}{d(m+1)^2}$$

```
[In] Int[(d*x)^m*(a + b*Log[c*x^n]),x]
```

```
[Out] -((b*n*(d*x)^(1 + m))/(d*(1 + m)^2)) + ((d*x)^(1 + m)*(a + b*Log[c*x^n]))/(
d*(1 + m))
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{bn(dx)^{1+m}}{d(1+m)^2} + \frac{(dx)^{1+m}(a + b\log(cx^n))}{d(1+m)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int (dx)^m (a+b\log(cx^n))\ dx = \frac{x(dx)^m (a+am-bn+b(1+m)\log(cx^n))}{(1+m)^2}$$

```
[In] Integrate[(d*x)^m*(a + b*Log[c*x^n]),x]
[Out] (x*(d*x)^m*(a + a*m - b*n + b*(1 + m)*Log[c*x^n]))/(1 + m)^2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result
parallelrisch	$-\frac{-x(dx)^m \ln(c\,x^n)bm-x(dx)^m \ln(c\,x^n)b-x(dx)^m am+x(dx)^m bn-x(dx)^m a}{(1+m)^2}$
risch	$\frac{bx\,x^m d^m \mathrm{e}^{\frac{i\,\mathrm{csgn}(idx)\pi m(\mathrm{csgn}(idx)-\mathrm{csgn}(ix))(-\,\mathrm{csgn}(idx)+\mathrm{csgn}(id))}{2}} \ln(x^n)}{1+m} - \frac{\left(i\pi b\ \mathrm{csgn}(ic)\,\mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)m-i\pi b\ \mathrm{csgn}(ic\right.}{}$

```
[In] int((d*x)^m*(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] -(-x*(d*x)^m*ln(c*x^n)*b*m-x*(d*x)^m*ln(c*x^n)*b-x*(d*x)^m*a*m+x*(d*x)^m*b*
n-x*(d*x)^m*a)/(1+m)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int (dx)^m (a+b\log(cx^n))\ dx = \frac{((bm+b)nx\log(x)+(bm+b)x\log(c)+(am-bn+a)x)e^{(m\log(d)+m\log(x))}}{m^2+2\,m+1}$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] ((b*m + b)*n*x*log(x) + (b*m + b)*x*log(c) + (a*m - b*n + a)*x)*e^(m*log(d)
 + m*log(x))/(m^2 + 2*m + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(37) = 74$.

Time = 1.99 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.07

$$\int (dx)^m (a + b\log(cx^n))\,dx = \begin{cases} \frac{amx(dx)^m}{m^2+2m+1} + \frac{ax(dx)^m}{m^2+2m+1} + \frac{bmx(dx)^m\log(cx^n)}{m^2+2m+1} - \frac{bnx(dx)^m}{m^2+2m+1} + \frac{bx(dx)^m\log(cx^n)}{m^2+2m+1} & \text{for } m \neq -1 \\ \frac{\begin{cases} a\log(x) & \text{for } b = 0 \\ -(-a - b\log(c))\log(x) & \text{for } n = 0 \\ \frac{(-a-b\log(cx^n))^2}{2bn} & \text{otherwise} \end{cases}}{d} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x)**m*(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((a*m*x*(d*x)**m/(m**2 + 2*m + 1) + a*x*(d*x)**m/(m**2 + 2*m + 1)
+ b*m*x*(d*x)**m*log(c*x**n)/(m**2 + 2*m + 1) - b*n*x*(d*x)**m/(m**2 + 2*m
+ 1) + b*x*(d*x)**m*log(c*x**n)/(m**2 + 2*m + 1), Ne(m, -1)), (Piecewise((a
*log(x), Eq(b, 0)), (-(-a - b*log(c))*log(x), Eq(n, 0)), ((-a - b*log(c*x**
n))**2/(2*b*n), True))/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int (dx)^m (a + b\log(cx^n))\,dx = -\frac{bd^m nxx^m}{(m+1)^2} + \frac{(dx)^{m+1}b\log(cx^n)}{d(m+1)} + \frac{(dx)^{m+1}a}{d(m+1)}$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] -b*d^m*n*x*x^m/(m + 1)^2 + (d*x)^(m + 1)*b*log(c*x^n)/(d*(m + 1)) + (d*x)^(
m + 1)*a/(d*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int (dx)^m (a + b\log(cx^n))\,dx = \frac{bd^m mnxx^m\log(x)}{m^2+2m+1} + \frac{bd^m nxx^m\log(x)}{m^2+2m+1} - \frac{bd^m nxx^m}{m^2+2m+1} + \frac{(dx)^m bx\log(c)}{m+1} + \frac{(dx)^m ax}{m+1}$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] b*d^m*m*n*x*x^m*log(x)/(m^2 + 2*m + 1) + b*d^m*n*x*x^m*log(x)/(m^2 + 2*m +
1) - b*d^m*n*x*x^m/(m^2 + 2*m + 1) + (d*x)^m*b*x*log(c)/(m + 1) + (d*x)^m*a
*x/(m + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m (a + b\log(cx^n))\,dx = \int (d\,x)^m\,\left(a + b\,\ln\left(c\,x^n\right)\right)\,dx$$

```
[In] int((d*x)^m*(a + b*log(c*x^n)),x)
```

```
[Out] int((d*x)^m*(a + b*log(c*x^n)), x)
```

3.153 $\int \frac{(dx)^m}{a+b\log(cx^n)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{(dx)^m}{a+b\log(cx^n)}\,dx = \frac{e^{-\frac{a(1+m)}{bn}}(dx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\,\text{ExpIntegralEi}\left(\frac{(1+m)(a+b\log(cx^n))}{bn}\right)}{bdn}$$

```
[Out] (d*x)^(1+m)*Ei((1+m)*(a+b*ln(c*x^n))/b/n)/b/d/exp(a*(1+m)/b/n)/n/((c*x^n)^(
(1+m)/n))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2347, 2209}

$$\int \frac{(dx)^m}{a+b\log(cx^n)}\,dx = \frac{(dx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}\,\text{ExpIntegralEi}\left(\frac{(m+1)(a+b\log(cx^n))}{bn}\right)}{bdn}$$

```
[In] Int[(d*x)^m/(a + b*Log[c*x^n]),x]
```

```
[Out] ((d*x)^(1 + m)*ExpIntegralEi[((1 + m)*(a + b*Log[c*x^n]))/(b*n)])/(b*d*E^((
a*(1 + m))/(b*n))*n*(c*x^n)^((1 + m)/n))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left((dx)^{1+m}\,(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{a+bx}\,dx, x, \log(cx^n)\right)}{dn}\\ &= \frac{e^{-\frac{a(1+m)}{bn}}(dx)^{1+m}\,(cx^n)^{-\frac{1+m}{n}}\,\text{Ei}\left(\frac{(1+m)(a+b\log(cx^n))}{bn}\right)}{bdn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{(dx)^m}{a+b\log(cx^n)}\,dx = \frac{e^{-\frac{(1+m)(a+b(-n\log(x)+\log(cx^n)))}{bn}}x^{-m}(dx)^m\,\text{ExpIntegralEi}\left(\frac{(1+m)(a+b\log(cx^n))}{bn}\right)}{bn}$$

```
[In] Integrate[(d*x)^m/(a + b*Log[c*x^n]),x]
[Out] ((d*x)^m*ExpIntegralEi[((1 + m)*(a + b*Log[c*x^n]))/(b*n)])/(b*E^(((1 + m)*
(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*n*x^m)
```

Maple [F]

$$\int \frac{(dx)^m}{a+b\ln(c\,x^n)}dx$$

```
[In] int((d*x)^m/(a+b*ln(c*x^n)),x)
[Out] int((d*x)^m/(a+b*ln(c*x^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{(dx)^m}{a+b\log(cx^n)}\,dx = \frac{\mathrm{Ei}\left(\frac{(bm+b)n\log(x)+am+(bm+b)\log(c)+a}{bn}\right)e^{\left(\frac{bmn\log(d)-am-(bm+b)\log(c)-a}{bn}\right)}}{bn}$$

```
[In] integrate((d*x)^m/(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] Ei(((b*m + b)*n*log(x) + a*m + (b*m + b)*log(c) + a)/(b*n))*e^((b*m*n*log(d
) - a*m - (b*m + b)*log(c) - a)/(b*n))/(b*n)
```

Sympy [F]

$$\int \frac{(dx)^m}{a+b\log(cx^n)}\,dx = \int \frac{(dx)^m}{a+b\log(cx^n)}\,dx$$

```
[In] integrate((d*x)**m/(a+b*ln(c*x**n)),x)
[Out] Integral((d*x)**m/(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \frac{(dx)^m}{a+b\log(cx^n)}\,dx = \int \frac{(dx)^m}{b\log(cx^n)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int \frac{(dx)^m}{a+b\log(cx^n)}\,dx = \int \frac{(dx)^m}{b\log(cx^n)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+b\log\left(cx^n\right)}\,dx = \int \frac{{\left(d\,x\right)}^m}{a+b\,\ln\left(c\,x^n\right)} \,d x$$

```
[In] int((d*x)^m/(a + b*log(c*x^n)),x)
[Out] int((d*x)^m/(a + b*log(c*x^n)), x)
```

3.154 $\int \frac{(dx)^m}{(a+b\log(cx^n))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 100

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^2}\,dx = \frac{e^{-\frac{a(1+m)}{bn}}(1+m)(dx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\,\text{ExpIntegralEi}\left(\frac{(1+m)(a+b\log(cx^n))}{bn}\right)}{b^2dn^2} - \frac{(dx)^{1+m}}{bdn(a+b\log(cx^n))}$$

```
[Out] (1+m)*(d*x)^(1+m)*Ei((1+m)*(a+b*ln(c*x^n))/b/n)/b^2/d/exp(a*(1+m)/b/n)/n^2/
((c*x^n)^((1+m)/n))-(d*x)^(1+m)/b/d/n/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2343, 2347, 2209}

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^2}\,dx = \frac{(m+1)(dx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}\,\text{ExpIntegralEi}\left(\frac{(m+1)(a+b\log(cx^n))}{bn}\right)}{b^2dn^2} - \frac{(dx)^{m+1}}{bdn(a+b\log(cx^n))}$$

```
[In] Int[(d*x)^m/(a + b*Log[c*x^n])^2,x]
```

```
[Out] ((1 + m)*(d*x)^(1 + m)*ExpIntegralEi[((1 + m)*(a + b*Log[c*x^n]))/(b*n)])/(
b^2*d*E^((a*(1 + m))/(b*n))*n^2*(c*x^n)^((1 + m)/n)) - (d*x)^(1 + m)/(b*d*n
*(a + b*Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(dx)^{1+m}}{bdn\left(a+b\log\left(cx^n\right)\right)}+\frac{(1+m)\int\frac{(dx)^m}{a+b\log(cx^n)}\,dx}{bn}\\
&= -\frac{(dx)^{1+m}}{bdn\left(a+b\log\left(cx^n\right)\right)}+\frac{\left((1+m)(dx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\frac{e^{\frac{(1+m)x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{bdn^2}\\
&= \frac{e^{-\frac{a(1+m)}{bn}}(1+m)(dx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\text{Ei}\left(\frac{(1+m)(a+b\log(cx^n))}{bn}\right)}{b^2dn^2}-\frac{(dx)^{1+m}}{bdn\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

$$\int\frac{(dx)^m}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx =\frac{(dx)^m\left(e^{-\frac{(1+m)(a-bn\log(x)+b\log(cx^n))}{bn}}(1+m)x^{-m}\,\text{ExpIntegralEi}\left(\frac{(1+m)(a+b\log(cx^n))}{bn}\right)-\frac{bnx}{a+b\log(cx^n)}\right)}{b^2n^2}$$

```
[In] Integrate[(d*x)^m/(a + b*Log[c*x^n])^2,x]
```

```
[Out] ((d*x)^m*(((1 + m)*ExpIntegralEi[((1 + m)*(a + b*Log[c*x^n]))/(b*n)])/(E^((
(1 + m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*x^m) - (b*n*x)/(a + b*Log[c
*x^n])))/(b^2*n^2)
```

Maple [F]

$$\int \frac{(dx)^m}{(a+b\ln(c\,x^n))^2}dx$$

```
[In] int((d*x)^m/(a+b*ln(c*x^n))^2,x)
```

```
[Out] int((d*x)^m/(a+b*ln(c*x^n))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.31

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^2}\,dx =$$
$$-\frac{bnxe^{(m\log(d)+m\log(x))} - ((bm+b)n\log(x) + am + (bm+b)\log(c) + a)\mathrm{Ei}\left(\frac{(bm+b)n\log(x)+am+(bm+b)\log(c)+a}{bn}\right.}{b^3n^3\log(x) + b^3n^2\log(c) + ab^2n^2}$$

```
[In] integrate((d*x)^m/(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] -(b*n*x*e^(m*log(d) + m*log(x)) - ((b*m + b)*n*log(x) + a*m + (b*m + b)*log
(c) + a)*Ei(((b*m + b)*n*log(x) + a*m + (b*m + b)*log(c) + a)/(b*n))*e^((b*
m*n*log(d) - a*m - (b*m + b)*log(c) - a)/(b*n)))/(b^3*n^3*log(x) + b^3*n^2*
log(c) + a*b^2*n^2)
```

Sympy [F]

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^2}\,dx = \int \frac{(dx)^m}{(a+b\log(cx^n))^2}\,dx$$

```
[In] integrate((d*x)**m/(a+b*ln(c*x**n))**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^2}\,dx = \int \frac{(dx)^m}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] d^m*(m + 1)*integrate(x^m/(b^2*n*log(c) + b^2*n*log(x^n) + a*b*n), x) - d^m
*x*x^m/(b^2*n*log(c) + b^2*n*log(x^n) + a*b*n)
```

Giac [F]

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^2}\,dx = \int \frac{(dx)^m}{(b\log(cx^n)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^2}\,dx = \int \frac{(d\,x)^m}{(a+b\,\ln(c\,x^n))^2}\,dx$$

```
[In] int((d*x)^m/(a + b*log(c*x^n))^2,x)
```

```
[Out] int((d*x)^m/(a + b*log(c*x^n))^2, x)
```

3.155 $\int \frac{(dx)^m}{(a+b\log(cx^n))^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^3}\,dx = \frac{e^{-\frac{a(1+m)}{bn}}(1+m)^2(dx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\,\text{ExpIntegralEi}\left(\frac{(1+m)(a+b\log(cx^n))}{bn}\right)}{2b^3dn^3} - \frac{(dx)^{1+m}}{2bdn(a+b\log(cx^n))^2} - \frac{(1+m)(dx)^{1+m}}{2b^2dn^2(a+b\log(cx^n))}$$

```
[Out] 1/2*(1+m)^2*(d*x)^(1+m)*Ei((1+m)*(a+b*ln(c*x^n))/b/n)/b^3/d/exp(a*(1+m)/b/n
)/n^3/((c*x^n)^((1+m)/n))-1/2*(d*x)^(1+m)/b/d/n/(a+b*ln(c*x^n))^2-1/2*(1+m)
*(d*x)^(1+m)/b^2/d/n^2/(a+b*ln(c*x^n))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2343, 2347, 2209}

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^3}\,dx = \frac{(m+1)^2(dx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}\,\text{ExpIntegralEi}\left(\frac{(m+1)(a+b\log(cx^n))}{bn}\right)}{2b^3dn^3} - \frac{(m+1)(dx)^{m+1}}{2b^2dn^2(a+b\log(cx^n))} - \frac{(dx)^{m+1}}{2bdn(a+b\log(cx^n))^2}$$

```
[In] Int[(d*x)^m/(a + b*Log[c*x^n])^3,x]
```

```
[Out] ((1 + m)^2*(d*x)^(1 + m)*ExpIntegralEi[((1 + m)*(a + b*Log[c*x^n]))/(b*n)])
/(2*b^3*d*E^((a*(1 + m))/(b*n))*n^3*(c*x^n)^((1 + m)/n)) - (d*x)^(1 + m)/(2
*b*d*n*(a + b*Log[c*x^n])^2) - ((1 + m)*(d*x)^(1 + m))/(2*b^2*d*n^2*(a + b*
Log[c*x^n]))
```

Rule 2209

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(dx)^{1+m}}{2bdn\left(a+b\log\left(cx^n\right)\right)^2}+\frac{(1+m)\int\frac{(dx)^m}{\left(a+b\log\left(cx^n\right)\right)^2}\,dx}{2bn}\\
&= -\frac{(dx)^{1+m}}{2bdn\left(a+b\log\left(cx^n\right)\right)^2}-\frac{(1+m)(dx)^{1+m}}{2b^2dn^2\left(a+b\log\left(cx^n\right)\right)}+\frac{(1+m)^2\int\frac{(dx)^m}{a+b\log\left(cx^n\right)}\,dx}{2b^2n^2}\\
&= -\frac{(dx)^{1+m}}{2bdn\left(a+b\log\left(cx^n\right)\right)^2}-\frac{(1+m)(dx)^{1+m}}{2b^2dn^2\left(a+b\log\left(cx^n\right)\right)}\\
&\quad+\frac{\left((1+m)^2(dx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\frac{e^{\frac{(1+m)x}{n}}}{a+bx}\,dx,x,\log\left(cx^n\right)\right)}{2b^2dn^3}\\
&= \frac{e^{-\frac{a(1+m)}{bn}}(1+m)^2(dx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\text{Ei}\left(\frac{(1+m)\left(a+b\log\left(cx^n\right)\right)}{bn}\right)}{2b^3dn^3}\\
&\quad-\frac{(dx)^{1+m}}{2bdn\left(a+b\log\left(cx^n\right)\right)^2}-\frac{(1+m)(dx)^{1+m}}{2b^2dn^2\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^3}\,dx = \frac{(dx)^m \left(e^{-\frac{(1+m)(a-bn\log(x)+b\log(cx^n))}{bn}}(1+m)^2 x^{-m}\,\text{ExpIntegralEi}\left(\frac{(1+m)(a+b\log(cx^n))}{bn}\right) - \frac{bnx(a+am+bn+b(1+m)\log(cx^n}{(a+b\log(cx^n))^2}\right.}{2b^3n^3}$$

[In] Integrate[(d*x)^m/(a + b*Log[c*x^n])^3,x]

[Out] ((d*x)^m*(((1 + m)^2*ExpIntegralEi[((1 + m)*(a + b*Log[c*x^n]))/(b*n)])/(E^(((1 + m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*x^m) - (b*n*x*(a + a*m + b*n + b*(1 + m)*Log[c*x^n]))/(a + b*Log[c*x^n])^2))/(2*b^3*n^3)

Maple [F]

$$\int \frac{(dx)^m}{(a+b\ln(c\,x^n))^3}dx$$

[In] int((d*x)^m/(a+b*ln(c*x^n))^3,x)

[Out] int((d*x)^m/(a+b*ln(c*x^n))^3,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(136) = 272.

Time = 0.32 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.27

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^3}\,dx = \frac{((b^2m^2+2\,b^2m+b^2)n^2\log(x)^2+a^2m^2+2\,a^2m+(b^2m^2+2\,b^2m+b^2)\log(c)^2+a^2+2\,(abm^2+2\,abm+}{}$$

[In] integrate((d*x)^m/(a+b*log(c*x^n))^3,x, algorithm="fricas")

[Out] 1/2*(((b^2*m^2 + 2*b^2*m + b^2)*n^2*log(x)^2 + a^2*m^2 + 2*a^2*m + (b^2*m^2 + 2*b^2*m + b^2)*log(c)^2 + a^2 + 2*(a*b*m^2 + 2*a*b*m + a*b)*log(c) + 2*((b^2*m^2 + 2*b^2*m + b^2)*n*log(c) + (a*b*m^2 + 2*a*b*m + a*b)*n)*log(x))*Ei(((b*m + b)*n*log(x) + a*m + (b*m + b)*log(c) + a)/(b*n))*e^((b*m*n*log(d) - a*m - (b*m + b)*log(c) - a)/(b*n)) - ((b^2*m + b^2)*n^2*x*log(x) + (b^2*m + b^2)*n*x*log(c) + (b^2*n^2 + (a*b*m + a*b)*n)*x)*e^(m*log(d) + m*log(x)))/(b^5*n^5*log(x)^2 + b^5*n^3*log(c)^2 + 2*a*b^4*n^3*log(c) + a^2*b^3*n^3 + 2*(b^5*n^4*log(c) + a*b^4*n^4)*log(x))

Sympy [F]

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^3}\,dx=\int \frac{(dx)^m}{(a+b\log(cx^n))^3}\,dx$$

```
[In] integrate((d*x)**m/(a+b*ln(c*x**n))**3,x)
```

```
[Out] Integral((d*x)**m/(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^3}\,dx=\int \frac{(dx)^m}{(b\log(cx^n)+a)^3}\,dx$$

```
[In] integrate((d*x)^m/(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] (m^2 + 2*m + 1)*d^m*integrate(1/2*x^m/(b^3*n^2*log(c) + b^3*n^2*log(x^n) +
a*b^2*n^2), x) - 1/2*(b*d^m*(m + 1)*x*x^m*log(x^n) + (a*d^m*(m + 1) + (d^m*
(m + 1)*log(c) + d^m*n)*b)*x*x^m)/(b^4*n^2*log(c)^2 + b^4*n^2*log(x^n)^2 +
2*a*b^3*n^2*log(c) + a^2*b^2*n^2 + 2*(b^4*n^2*log(c) + a*b^3*n^2)*log(x^n))
```

Giac [F]

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^3}\,dx=\int \frac{(dx)^m}{(b\log(cx^n)+a)^3}\,dx$$

```
[In] integrate((d*x)^m/(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*log(c*x^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+b\log(cx^n))^3}\,dx=\int \frac{(d\,x)^m}{(a+b\,\ln(c\,x^n))^3}\,dx$$

```
[In] int((d*x)^m/(a + b*log(c*x^n))^3,x)
```

```
[Out] int((d*x)^m/(a + b*log(c*x^n))^3, x)
```

3.156 $\int (dx)^{-1+n} \log^3(cx^n)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 74

$$\int (dx)^{-1+n} \log^3(cx^n)\, dx = -\frac{6(dx)^n}{dn} + \frac{6(dx)^n \log(cx^n)}{dn} - \frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{(dx)^n \log^3(cx^n)}{dn}$$

```
[Out] -6*(d*x)^n/d/n+6*(d*x)^n*ln(c*x^n)/d/n-3*(d*x)^n*ln(c*x^n)^2/d/n+(d*x)^n*ln
(c*x^n)^3/d/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2342, 2341}

$$\int (dx)^{-1+n} \log^3(cx^n)\, dx = \frac{(dx)^n \log^3(cx^n)}{dn} - \frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{6(dx)^n \log(cx^n)}{dn} - \frac{6(dx)^n}{dn}$$

```
[In] Int[(d*x)^(-1 + n)*Log[c*x^n]^3,x]
```

```
[Out] (-6*(d*x)^n)/(d*n) + (6*(d*x)^n*Log[c*x^n])/(d*n) - (3*(d*x)^n*Log[c*x^n]^2
)/(d*n) + ((d*x)^n*Log[c*x^n]^3)/(d*n)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(dx)^n \log^3(cx^n)}{dn} - 3\int (dx)^{-1+n} \log^2(cx^n)\,dx \\
&= -\frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{(dx)^n \log^3(cx^n)}{dn} + 6\int (dx)^{-1+n} \log(cx^n)\,dx \\
&= -\frac{6(dx)^n}{dn} + \frac{6(dx)^n \log(cx^n)}{dn} - \frac{3(dx)^n \log^2(cx^n)}{dn} + \frac{(dx)^n \log^3(cx^n)}{dn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int (dx)^{-1+n} \log^3(cx^n)\,dx = \frac{(dx)^n \left(-6 + 6\log(cx^n) - 3\log^2(cx^n) + \log^3(cx^n)\right)}{dn}$$

```
[In] Integrate[(d*x)^(-1 + n)*Log[c*x^n]^3,x]
[Out] ((d*x)^n*(-6 + 6*Log[c*x^n] - 3*Log[c*x^n]^2 + Log[c*x^n]^3))/(d*n)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{-(dx)^{n-1}\ln(c\,x^{n})^{3}x+3(dx)^{n-1}\ln(c\,x^{n})^{2}x-6(dx)^{n-1}x\ln(c\,x^{n})+6(dx)^{n-1}x}{n}$	69
risch	Expression too large to display	2008

```
[In] int((d*x)^(n-1)*ln(c*x^n)^3,x,method=_RETURNVERBOSE)
[Out] -(-(d*x)^(n-1)*ln(c*x^n)^3*x+3*(d*x)^(n-1)*ln(c*x^n)^2*x-6*(d*x)^(n-1)*x*ln
(c*x^n)+6*(d*x)^(n-1)*x)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int (dx)^{-1+n}\log^3(cx^n)\,dx = \frac{\left(n^3\log(x)^3 + \log(c)^3 + 3\left(n^2\log(c) - n^2\right)\log(x)^2 - 3\log(c)^2 + 3\left(n\log(c)^2 - 2n\log(c) + 2n\right)\log(x) + \right.}{n}$$

```
[In] integrate((d*x)^(-1+n)*log(c*x^n)^3,x, algorithm="fricas")
```

```
[Out] (n^3*log(x)^3 + log(c)^3 + 3*(n^2*log(c) - n^2)*log(x)^2 - 3*log(c)^2 + 3*(
n*log(c)^2 - 2*n*log(c) + 2*n)*log(x) + 6*log(c) - 6)*d^(n - 1)*x^n/n
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int (dx)^{-1+n}\log^3(cx^n)\,dx = \begin{cases} \frac{x(dx)^{n-1}\log(cx^n)^3}{n} - \frac{3x(dx)^{n-1}\log(cx^n)^2}{n} + \frac{6x(dx)^{n-1}\log(cx^n)}{n} - \frac{6x(dx)^{n-1}}{n} & \text{for } n \neq 0 \\ \frac{\log(c)^3\log(x)}{d} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x)**(-1+n)*ln(c*x**n)**3,x)
```

```
[Out] Piecewise((x*(d*x)**(n - 1)*log(c*x**n)**3/n - 3*x*(d*x)**(n - 1)*log(c*x**
n)**2/n + 6*x*(d*x)**(n - 1)*log(c*x**n)/n - 6*x*(d*x)**(n - 1)/n, Ne(n, 0)
), (log(c)**3*log(x)/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int (dx)^{-1+n}\log^3(cx^n)\,dx = -\frac{3\,d^{n-1}x^n\log(cx^n)^2}{n} + \frac{(dx)^n\log(cx^n)^3}{dn} + \frac{6\left(\frac{d^n x^n\log(cx^n)}{n} - \frac{d^n x^n}{n}\right)}{d}$$

```
[In] integrate((d*x)^(-1+n)*log(c*x^n)^3,x, algorithm="maxima")
```

```
[Out] -3*d^(n - 1)*x^n*log(c*x^n)^2/n + (d*x)^n*log(c*x^n)^3/(d*n) + 6*(d^n*x^n*l
og(c*x^n)/n - d^n*x^n/n)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(74) = 148.

Time = 0.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.19

$$\int (dx)^{-1+n}\log^3(cx^n)\,dx = \frac{d^n n^2 x^n \log(x)^3}{d} + \frac{3\,d^n n x^n \log(c)\log(x)^2}{d} + \frac{3\,d^n x^n \log(c)^2 \log(x)}{d} - \frac{3\,d^n n x^n \log(x)^2}{d} + \frac{d^n x^n \log(c)^3}{dn} - \frac{6\,d^n x^n \log(c)\log(x)}{d} - \frac{3\,d^n x^n \log(c)^2}{dn} + \frac{6\,d^n x^n \log(x)}{d} + \frac{6\,d^n x^n \log(c)}{dn} - \frac{6\,d^n x^n}{dn}$$

```
[In] integrate((d*x)^(-1+n)*log(c*x^n)^3,x, algorithm="giac")
```

```
[Out] d^n*n^2*x^n*log(x)^3/d + 3*d^n*n*x^n*log(c)*log(x)^2/d + 3*d^n*x^n*log(c)^2
*log(x)/d - 3*d^n*n*x^n*log(x)^2/d + d^n*x^n*log(c)^3/(d*n) - 6*d^n*x^n*log
(c)*log(x)/d - 3*d^n*x^n*log(c)^2/(d*n) + 6*d^n*x^n*log(x)/d + 6*d^n*x^n*lo
g(c)/(d*n) - 6*d^n*x^n/(d*n)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{-1+n}\log^3(cx^n)\,dx = \int \ln(c\,x^n)^3\,(d\,x)^{n-1}\,dx$$

```
[In] int(log(c*x^n)^3*(d*x)^(n - 1),x)
```

```
[Out] int(log(c*x^n)^3*(d*x)^(n - 1), x)
```

3.157 $\int (dx)^{-1+n} \log^2(cx^n)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 53

$$\int (dx)^{-1+n} \log^2(cx^n)\, dx = \frac{2(dx)^n}{dn} - \frac{2(dx)^n \log(cx^n)}{dn} + \frac{(dx)^n \log^2(cx^n)}{dn}$$

```
[Out] 2*(d*x)^n/d/n-2*(d*x)^n*ln(c*x^n)/d/n+(d*x)^n*ln(c*x^n)^2/d/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2342, 2341}

$$\int (dx)^{-1+n} \log^2(cx^n)\, dx = \frac{(dx)^n \log^2(cx^n)}{dn} - \frac{2(dx)^n \log(cx^n)}{dn} + \frac{2(dx)^n}{dn}$$

```
[In] Int[(d*x)^(-1 + n)*Log[c*x^n]^2,x]
```

```
[Out] (2*(d*x)^n)/(d*n) - (2*(d*x)^n*Log[c*x^n])/(d*n) + ((d*x)^n*Log[c*x^n]^2)/(
d*n)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
```

```
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(dx)^n \log^2(cx^n)}{dn} - 2\int (dx)^{-1+n}\log(cx^n)\, dx\\ &= \frac{2(dx)^n}{dn} - \frac{2(dx)^n\log(cx^n)}{dn} + \frac{(dx)^n\log^2(cx^n)}{dn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.57

$$\int (dx)^{-1+n}\log^2(cx^n)\, dx = \frac{(dx)^n\left(2 - 2\log(cx^n) + \log^2(cx^n)\right)}{dn}$$

```
[In] Integrate[(d*x)^(-1 + n)*Log[c*x^n]^2,x]
[Out] ((d*x)^n*(2 - 2*Log[c*x^n] + Log[c*x^n]^2))/(d*n)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$-\frac{-(dx)^{n-1}\ln(c\,x^n)^2 x+2(dx)^{n-1}x\ln(c\,x^n)-2(dx)^{n-1}x}{n}$	51
risch	Expression too large to display	750

```
[In] int((d*x)^(n-1)*ln(c*x^n)^2,x,method=_RETURNVERBOSE)
[Out] -(-(d*x)^(n-1)*ln(c*x^n)^2*x+2*(d*x)^(n-1)*x*ln(c*x^n)-2*(d*x)^(n-1)*x)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int (dx)^{-1+n}\log^2(cx^n)\,dx = \frac{\left(n^2\log(x)^2+\log(c)^2+2\,(n\log(c)-n)\log(x)-2\,\log(c)+2\right)d^{n-1}x^n}{n}$$

```
[In] integrate((d*x)^(-1+n)*log(c*x^n)^2,x, algorithm="fricas")
[Out] (n^2*log(x)^2 + log(c)^2 + 2*(n*log(c) - n)*log(x) - 2*log(c) + 2)*d^(n - 1
)*x^n/n
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int (dx)^{-1+n}\log^2(cx^n)\,dx = \begin{cases} \frac{x(dx)^{n-1}\log(cx^n)^2}{n} - \frac{2x(dx)^{n-1}\log(cx^n)}{n} + \frac{2x(dx)^{n-1}}{n} & \text{for } n \neq 0 \\ \frac{\log(c)^2\log(x)}{d} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x)**(-1+n)*ln(c*x**n)**2,x)
[Out] Piecewise((x*(d*x)**(n - 1)*log(c*x**n)**2/n - 2*x*(d*x)**(n - 1)*log(c*x**
n)/n + 2*x*(d*x)**(n - 1)/n, Ne(n, 0)), (log(c)**2*log(x)/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int (dx)^{-1+n}\log^2(cx^n)\,dx = -\frac{2\,d^{n-1}x^n\log(cx^n)}{n} + \frac{2\,d^{n-1}x^n}{n} + \frac{(dx)^n\log(cx^n)^2}{dn}$$

```
[In] integrate((d*x)^(-1+n)*log(c*x^n)^2,x, algorithm="maxima")
[Out] -2*d^(n - 1)*x^n*log(c*x^n)/n + 2*d^(n - 1)*x^n/n + (d*x)^n*log(c*x^n)^2/(d
*n)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

$$\int (dx)^{-1+n}\log^2(cx^n)\,dx = \frac{d^n n x^n \log(x)^2}{d} + \frac{2\,d^n x^n \log(c)\log(x)}{d} + \frac{d^n x^n \log(c)^2}{dn} - \frac{2\,d^n x^n \log(x)}{d} - \frac{2\,d^n x^n \log(c)}{dn} + \frac{2\,d^n x^n}{dn}$$

```
[In] integrate((d*x)^(-1+n)*log(c*x^n)^2,x, algorithm="giac")
```

```
[Out] d^n*n*x^n*log(x)^2/d + 2*d^n*x^n*log(c)*log(x)/d + d^n*x^n*log(c)^2/(d*n) -
 2*d^n*x^n*log(x)/d - 2*d^n*x^n*log(c)/(d*n) + 2*d^n*x^n/(d*n)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{-1+n}\log^2(cx^n)\,dx = \int \ln\left(c\,x^n\right)^2\left(d\,x\right)^{n-1}\,dx$$

```
[In] int(log(c*x^n)^2*(d*x)^(n - 1),x)
```

```
[Out] int(log(c*x^n)^2*(d*x)^(n - 1), x)
```

3.158 $\int (dx)^{-1+n}\log(cx^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int (dx)^{-1+n}\log(cx^n)\,dx=-\frac{(dx)^n}{dn}+\frac{(dx)^n\log(cx^n)}{dn}$$

```
[Out] -(d*x)^n/d/n+(d*x)^n*ln(c*x^n)/d/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2341}

$$\int (dx)^{-1+n}\log(cx^n)\,dx=\frac{(dx)^n\log(cx^n)}{dn}-\frac{(dx)^n}{dn}$$

```
[In] Int[(d*x)^(-1 + n)*Log[c*x^n],x]
```

```
[Out] -((d*x)^n/(d*n)) + ((d*x)^n*Log[c*x^n])/(d*n)
```

Rule 2341

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=-\frac{(dx)^n}{dn}+\frac{(dx)^n\log(cx^n)}{dn}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int (dx)^{-1+n} \log(cx^n)\, dx = \frac{(dx)^n(-1+\log(cx^n))}{dn}$$

```
[In] Integrate[(d*x)^(-1 + n)*Log[c*x^n],x]
[Out] ((d*x)^n*(-1 + Log[c*x^n]))/(d*n)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

method	result
parallelrisch	$-\frac{-(dx)^{n-1} x \ln(c\,x^{n})+(dx)^{n-1} x}{n}$
risch	$\frac{x\,\mathrm{e}^{\frac{(n-1)\left(-i\pi\ \mathrm{csgn}(id)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(idx)+i\pi\ \mathrm{csgn}(id)\,\mathrm{csgn}(idx)^{2}+i\pi\ \mathrm{csgn}(ix)\,\mathrm{csgn}(idx)^{2}-i\pi\,\mathrm{csgn}(idx)^{3}+2\ln(d)+2\ln(x)\right)}{2}}}{n} \ln(x^{n}) + \frac{(-$

```
[In] int((d*x)^(n-1)*ln(c*x^n),x,method=_RETURNVERBOSE)
[Out] -(-(d*x)^(n-1)*x*ln(c*x^n)+(d*x)^(n-1)*x)/n
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int (dx)^{-1+n} \log(cx^n)\, dx = \frac{(n\log(x)+\log(c)-1)d^{n-1}x^n}{n}$$

```
[In] integrate((d*x)^(-1+n)*log(c*x^n),x, algorithm="fricas")
[Out] (n*log(x) + log(c) - 1)*d^(n - 1)*x^n/n
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int (dx)^{-1+n}\log (cx^n)\,dx = \begin{cases} \frac{x(dx)^{n-1}\log (cx^n)}{n} - \frac{x(dx)^{n-1}}{n} & \text{for } n \neq 0 \\ \frac{\log (c)\log (x)}{d} & \text{otherwise} \end{cases}$$

```
[In] integrate((d*x)**(-1+n)*ln(c*x**n),x)
```

```
[Out] Piecewise((x*(d*x)**(n - 1)*log(c*x**n)/n - x*(d*x)**(n - 1)/n, Ne(n, 0)),
(log(c)*log(x)/d, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int (dx)^{-1+n}\log (cx^n)\,dx = -\frac{d^{n-1}x^n}{n} + \frac{(dx)^n\log (cx^n)}{dn}$$

```
[In] integrate((d*x)^(-1+n)*log(c*x^n),x, algorithm="maxima")
```

```
[Out] -d^(n - 1)*x^n/n + (d*x)^n*log(c*x^n)/(d*n)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int (dx)^{-1+n}\log (cx^n)\,dx = \frac{d^n x^n\log (x)}{d} + \frac{d^n x^n\log (c)}{dn} - \frac{d^n x^n}{dn}$$

```
[In] integrate((d*x)^(-1+n)*log(c*x^n),x, algorithm="giac")
```

```
[Out] d^n*x^n*log(x)/d + d^n*x^n*log(c)/(d*n) - d^n*x^n/(d*n)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{-1+n}\log(cx^n)\,dx=\int \ln\left(c\,x^n\right)\,\left(d\,x\right)^{n-1}\,dx$$

```
[In] int(log(c*x^n)*(d*x)^(n - 1),x)
[Out] int(log(c*x^n)*(d*x)^(n - 1), x)
```

3.159 $\int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx = \frac{x^{1-n}(dx)^{-1+n}\,\text{LogIntegral}(cx^n)}{cn}$$

```
[Out] x^(1-n)*(d*x)^(-1+n)*Li(c*x^n)/c/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2345, 2344, 2335}

$$\int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx = \frac{x^{1-n}(dx)^{n-1}\,\text{LogIntegral}(cx^n)}{cn}$$

```
[In] Int[(d*x)^(-1 + n)/Log[c*x^n],x]
[Out] (x^(1 - n)*(d*x)^(-1 + n)*LogIntegral[c*x^n])/(c*n)
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2344

```
Int[(x_)^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Dist[1/n, Subst[Int[1/Log
[c*x], x], x, x^n], x] /; FreeQ[{c, m, n}, x] && EqQ[m, n - 1]
```

Rule 2345

```
Int[((d_)*(x_))^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Dist[(d*x)^m/x^m,
Int[x^m/Log[c*x^n], x], x] /; FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(x^{1-n}(dx)^{-1+n}\right)\int \frac{x^{-1+n}}{\log(cx^n)}\,dx\\ &= \frac{\left(x^{1-n}(dx)^{-1+n}\right)\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, x^n\right)}{n}\\ &= \frac{x^{1-n}(dx)^{-1+n}\text{li}(cx^n)}{cn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx = \frac{x^{1-n}(dx)^{-1+n}\,\text{LogIntegral}(cx^n)}{cn}$$

```
[In] Integrate[(d*x)^(-1 + n)/Log[c*x^n],x]
[Out] (x^(1 - n)*(d*x)^(-1 + n)*LogIntegral[c*x^n])/(c*n)
```

Maple [F]

$$\int \frac{(dx)^{n-1}}{\ln(c\,x^n)}dx$$

```
[In] int((d*x)^(n-1)/ln(c*x^n),x)
[Out] int((d*x)^(n-1)/ln(c*x^n),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx = \frac{d^{n-1}\text{Ei}(n\log(x) + \log(c))}{cn}$$

```
[In] integrate((d*x)^(-1+n)/log(c*x^n),x, algorithm="fricas")
[Out] d^(n - 1)*Ei(n*log(x) + log(c))/(c*n)
```

Sympy [F]

$$\int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx = \int \frac{(dx)^{n-1}}{\log(cx^n)}\,dx$$

```
[In] integrate((d*x)**(-1+n)/ln(c*x**n),x)
[Out] Integral((d*x)**(n - 1)/log(c*x**n), x)
```

Maxima [F]

$$\int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx = \int \frac{(dx)^{n-1}}{\log(cx^n)}\,dx$$

```
[In] integrate((d*x)^(-1+n)/log(c*x^n),x, algorithm="maxima")
[Out] integrate((d*x)^(n - 1)/log(c*x^n), x)
```

Giac [F]

$$\int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx = \int \frac{(dx)^{n-1}}{\log(cx^n)}\,dx$$

```
[In] integrate((d*x)^(-1+n)/log(c*x^n),x, algorithm="giac")
[Out] integrate((d*x)^(n - 1)/log(c*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx = \int \frac{(d\,x)^{n-1}}{\ln(c\,x^n)}\,dx$$

```
[In] int((d*x)^(n - 1)/log(c*x^n),x)
[Out] int((d*x)^(n - 1)/log(c*x^n), x)
```

3.160 $\int \frac{(dx)^{-1+n}}{\log^2(cx^n)} dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{(dx)^{-1+n}}{\log^2(cx^n)} dx = -\frac{(dx)^n}{dn\log(cx^n)} + \frac{x^{1-n}(dx)^{-1+n}\,\text{LogIntegral}(cx^n)}{cn}$$

```
[Out] x^(1-n)*(d*x)^(-1+n)*Li(c*x^n)/c/n-(d*x)^n/d/n/ln(c*x^n)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2343, 2345, 2344, 2335}

$$\int \frac{(dx)^{-1+n}}{\log^2(cx^n)} dx = \frac{x^{1-n}(dx)^{n-1}\,\text{LogIntegral}(cx^n)}{cn} - \frac{(dx)^n}{dn\log(cx^n)}$$

```
[In] Int[(d*x)^(-1 + n)/Log[c*x^n]^2,x]
```

```
[Out] -((d*x)^n/(d*n*Log[c*x^n])) + (x^(1 - n)*(d*x)^(-1 + n)*LogIntegral[c*x^n])
/(c*n)
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
```

```
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2344

```
Int[(x_)^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Dist[1/n, Subst[Int[1/Log
[c*x], x], x, x^n], x] /; FreeQ[{c, m, n}, x] && EqQ[m, n - 1]
```

Rule 2345

```
Int[((d_)*(x_))^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Dist[(d*x)^m/x^m,
Int[x^m/Log[c*x^n], x], x] /; FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(dx)^n}{dn\log(cx^n)} + \int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx \\
&= -\frac{(dx)^n}{dn\log(cx^n)} + \left(x^{1-n}(dx)^{-1+n}\right)\int \frac{x^{-1+n}}{\log(cx^n)}\,dx \\
&= -\frac{(dx)^n}{dn\log(cx^n)} + \frac{\left(x^{1-n}(dx)^{-1+n}\right)\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, x^n\right)}{n} \\
&= -\frac{(dx)^n}{dn\log(cx^n)} + \frac{x^{1-n}(dx)^{-1+n}\text{li}(cx^n)}{cn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{-1+n}}{\log^2(cx^n)}\,dx = -\frac{x(dx)^{-1+n}}{n\log(cx^n)} + \frac{x^{1-n}(dx)^{-1+n}\,\text{LogIntegral}(cx^n)}{cn}$$

```
[In] Integrate[(d*x)^(-1 + n)/Log[c*x^n]^2,x]
[Out] -((x*(d*x)^(-1 + n))/(n*Log[c*x^n])) + (x^(1 - n)*(d*x)^(-1 + n)*LogIntegra
l[c*x^n])/(c*n)
```

Maple [F]

$$\int \frac{(dx)^{n-1}}{\ln (c\,x^{n})^{2}}dx$$

[In] `int((d*x)^(n-1)/ln(c*x^n)^2,x)`

[Out] `int((d*x)^(n-1)/ln(c*x^n)^2,x)`

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{(dx)^{-1+n}}{\log^2(cx^n)}\,dx = -\frac{d^{n-1}x^n - \frac{(n\log(x)+\log(c))d^{n-1}\mathrm{Ei}(n\log(x)+\log(c))}{c}}{n^2\log(x) + n\log(c)}$$

```
[In] integrate((d*x)^(-1+n)/log(c*x^n)^2,x, algorithm="fricas")
[Out] -(d^(n - 1)*x^n - (n*log(x) + log(c))*d^(n - 1)*Ei(n*log(x) + log(c))/c)/(n
^2*log(x) + n*log(c))
```

Sympy [F]

$$\int \frac{(dx)^{-1+n}}{\log^2(cx^n)}\,dx = \int \frac{(dx)^{n-1}}{\log(cx^n)^2}\,dx$$

```
[In] integrate((d*x)**(-1+n)/ln(c*x**n)**2,x)
[Out] Integral((d*x)**(n - 1)/log(c*x**n)**2, x)
```

Maxima [F]

$$\int \frac{(dx)^{-1+n}}{\log^2(cx^n)}\,dx = \int \frac{(dx)^{n-1}}{\log(cx^n)^2}\,dx$$

```
[In] integrate((d*x)^(-1+n)/log(c*x^n)^2,x, algorithm="maxima")
[Out] d^n*integrate(x^n/(d*x*log(c) + d*x*log(x^n)), x) - d^n*x^n/(d*n*log(c) + d
*n*log(x^n))
```

Giac [F]

$$\int \frac{(dx)^{-1+n}}{\log^2(cx^n)}\,dx = \int \frac{(dx)^{n-1}}{\log(cx^n)^2}\,dx$$

```
[In] integrate((d*x)^(-1+n)/log(c*x^n)^2,x, algorithm="giac")
[Out] integrate((d*x)^(n - 1)/log(c*x^n)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{-1+n}}{\log^2(cx^n)}\,dx = \int \frac{(d\,x)^{n-1}}{\ln(c\,x^n)^2}\,dx$$

```
[In] int((d*x)^(n - 1)/log(c*x^n)^2,x)
[Out] int((d*x)^(n - 1)/log(c*x^n)^2, x)
```

3.161 $\int \frac{(dx)^{-1+n}}{\log^3(cx^n)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{(dx)^{-1+n}}{\log^3(cx^n)}\,dx = -\frac{(dx)^n}{2dn\log^2(cx^n)} - \frac{(dx)^n}{2dn\log(cx^n)} + \frac{x^{1-n}(dx)^{-1+n}\,\mathrm{LogIntegral}(cx^n)}{2cn}$$

```
[Out] 1/2*x^(1-n)*(d*x)^(-1+n)*Li(c*x^n)/c/n-1/2*(d*x)^n/d/n/ln(c*x^n)^2-1/2*(d*x
)^n/d/n/ln(c*x^n)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2343, 2345, 2344, 2335}

$$\int \frac{(dx)^{-1+n}}{\log^3(cx^n)}\,dx = \frac{x^{1-n}(dx)^{n-1}\,\mathrm{LogIntegral}(cx^n)}{2cn} - \frac{(dx)^n}{2dn\log^2(cx^n)} - \frac{(dx)^n}{2dn\log(cx^n)}$$

```
[In] Int[(d*x)^(-1 + n)/Log[c*x^n]^3,x]
```

```
[Out] -1/2*(d*x)^n/(d*n*Log[c*x^n]^2) - (d*x)^n/(2*d*n*Log[c*x^n]) + (x^(1 - n)*(
d*x)^(-1 + n)*LogIntegral[c*x^n])/(2*c*n)
```

Rule 2335

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
```

```
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2344

```
Int[(x_)^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Dist[1/n, Subst[Int[1/Log
[c*x], x], x, x^n], x] /; FreeQ[{c, m, n}, x] && EqQ[m, n - 1]
```

Rule 2345

```
Int[((d_)*(x_))^(m_.)/Log[(c_.)*(x_)^(n_)], x_Symbol] :> Dist[(d*x)^m/x^m,
Int[x^m/Log[c*x^n], x], x] /; FreeQ[{c, d, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(dx)^n}{2dn\log^2(cx^n)} + \frac{1}{2}\int \frac{(dx)^{-1+n}}{\log^2(cx^n)}\,dx \\
&= -\frac{(dx)^n}{2dn\log^2(cx^n)} - \frac{(dx)^n}{2dn\log(cx^n)} + \frac{1}{2}\int \frac{(dx)^{-1+n}}{\log(cx^n)}\,dx \\
&= -\frac{(dx)^n}{2dn\log^2(cx^n)} - \frac{(dx)^n}{2dn\log(cx^n)} + \frac{1}{2}\left(x^{1-n}(dx)^{-1+n}\right)\int \frac{x^{-1+n}}{\log(cx^n)}\,dx \\
&= -\frac{(dx)^n}{2dn\log^2(cx^n)} - \frac{(dx)^n}{2dn\log(cx^n)} + \frac{\left(x^{1-n}(dx)^{-1+n}\right)\text{Subst}\left(\int \frac{1}{\log(cx)}\,dx, x, x^n\right)}{2n} \\
&= -\frac{(dx)^n}{2dn\log^2(cx^n)} - \frac{(dx)^n}{2dn\log(cx^n)} + \frac{x^{1-n}(dx)^{-1+n}\text{li}(cx^n)}{2cn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{(dx)^{-1+n}}{\log^3(cx^n)}\,dx = \frac{x^{-n}(dx)^n\left(-cx^n(1+\log(cx^n)) + \log^2(cx^n)\,\text{LogIntegral}(cx^n)\right)}{2cdn\log^2(cx^n)}$$

```
[In] Integrate[(d*x)^(-1 + n)/Log[c*x^n]^3,x]
[Out] ((d*x)^n*(-(c*x^n*(1 + Log[c*x^n])) + Log[c*x^n]^2*LogIntegral[c*x^n]))/(2*
c*d*n*x^n*Log[c*x^n]^2)
```

Maple [F]

$$\int \frac{(dx)^{n-1}}{\ln (c\,x^{n})^{3}}dx$$

```
[In] int((d*x)^(n-1)/ln(c*x^n)^3,x)
[Out] int((d*x)^(n-1)/ln(c*x^n)^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \frac{(dx)^{-1+n}}{\log^3(cx^n)}\,dx = -\frac{(n\log(x)+\log(c)+1)d^{n-1}x^n - \frac{\left(n^2\log(x)^2+2\,n\log(c)\log(x)+\log(c)^2\right)d^{n-1}\mathrm{Ei}(n\log(x)+\log(c))}{c}}{2\left(n^3\log(x)^2+2\,n^2\log(c)\log(x)+n\log(c)^2\right)}$$

```
[In] integrate((d*x)^(-1+n)/log(c*x^n)^3,x, algorithm="fricas")
[Out] -1/2*((n*log(x) + log(c) + 1)*d^(n - 1)*x^n - (n^2*log(x)^2 + 2*n*log(c)*lo
g(x) + log(c)^2)*d^(n - 1)*Ei(n*log(x) + log(c))/c)/(n^3*log(x)^2 + 2*n^2*l
og(c)*log(x) + n*log(c)^2)
```

Sympy [F]

$$\int \frac{(dx)^{-1+n}}{\log^3(cx^n)}\,dx = \int \frac{(dx)^{n-1}}{\log(cx^n)^3}\,dx$$

```
[In] integrate((d*x)**(-1+n)/ln(c*x**n)**3,x)
[Out] Integral((d*x)**(n - 1)/log(c*x**n)**3, x)
```

Maxima [F]

$$\int \frac{(dx)^{-1+n}}{\log^3(cx^n)}\,dx = \int \frac{(dx)^{n-1}}{\log(cx^n)^3}\,dx$$

```
[In] integrate((d*x)^(-1+n)/log(c*x^n)^3,x, algorithm="maxima")
```

```
[Out] d^n*integrate(1/2*x^n/(d*x*log(c) + d*x*log(x^n)), x) - 1/2*(d^n*x^n*log(x^
n) + (d^n*log(c) + d^n)*x^n)/(d*n*log(c)^2 + 2*d*n*log(c)*log(x^n) + d*n*lo
g(x^n)^2)
```

Giac [F]

$$\int \frac{(dx)^{-1+n}}{\log^3(cx^n)}\,dx = \int \frac{(dx)^{n-1}}{\log(cx^n)^3}\,dx$$

```
[In] integrate((d*x)^(-1+n)/log(c*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(n - 1)/log(c*x^n)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(dx)^{-1+n}}{\log^3(cx^n)}\,dx = \int \frac{(d\,x)^{n-1}}{\ln(c\,x^n)^3}\,dx$$

```
[In] int((d*x)^(n - 1)/log(c*x^n)^3,x)
```

```
[Out] int((d*x)^(n - 1)/log(c*x^n)^3, x)
```

3.162 $\int x^m \log^{\frac{3}{2}}(ax^n)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 111

$$\int x^m \log^{\frac{3}{2}}(ax^n)\,dx = \frac{3n^{3/2}\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4(1+m)^{5/2}} - \frac{3nx^{1+m}\sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m}\log^{\frac{3}{2}}(ax^n)}{1+m}$$

```
[Out] x^(1+m)*ln(a*x^n)^(3/2)/(1+m)+3/4*n^(3/2)*x^(1+m)*erfi((1+m)^(1/2)*ln(a*x^n
)^(1/2)/n^(1/2))*Pi^(1/2)/(1+m)^(5/2)/((a*x^n)^((1+m)/n))-3/2*n*x^(1+m)*ln(
a*x^n)^(1/2)/(1+m)^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2342, 2347, 2211, 2235}

$$\int x^m \log^{\frac{3}{2}}(ax^n)\,dx = \frac{3\sqrt{\pi}n^{3/2}x^{m+1}(ax^n)^{-\frac{m+1}{n}}\operatorname{erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4(m+1)^{5/2}} + \frac{x^{m+1}\log^{\frac{3}{2}}(ax^n)}{m+1} - \frac{3nx^{m+1}\sqrt{\log(ax^n)}}{2(m+1)^2}$$

```
[In] Int[x^m*Log[a*x^n]^(3/2),x]
```

```
[Out] (3*n^(3/2)*Sqrt[Pi]*x^(1 + m)*Erfi[(Sqrt[1 + m]*Sqrt[Log[a*x^n]])/Sqrt[n]])
/(4*(1 + m)^(5/2)*(a*x^n)^((1 + m)/n)) - (3*n*x^(1 + m)*Sqrt[Log[a*x^n]])/(
2*(1 + m)^2) + (x^(1 + m)*Log[a*x^n]^(3/2))/(1 + m)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^{1+m}\log^{\frac{3}{2}}(ax^n)}{1+m} - \frac{(3n)\int x^m\sqrt{\log(ax^n)}\,dx}{2(1+m)} \\
&= -\frac{3nx^{1+m}\sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m}\log^{\frac{3}{2}}(ax^n)}{1+m} + \frac{(3n^2)\int \frac{x^m}{\sqrt{\log(ax^n)}}\,dx}{4(1+m)^2} \\
&= -\frac{3nx^{1+m}\sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m}\log^{\frac{3}{2}}(ax^n)}{1+m} \\
&\quad + \frac{\left(3nx^{1+m}(ax^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{4(1+m)^2} \\
&= -\frac{3nx^{1+m}\sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m}\log^{\frac{3}{2}}(ax^n)}{1+m} \\
&\quad + \frac{\left(3nx^{1+m}(ax^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{2(1+m)^2} \\
&= \frac{3n^{3/2}\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{4(1+m)^{5/2}} - \frac{3nx^{1+m}\sqrt{\log(ax^n)}}{2(1+m)^2} + \frac{x^{1+m}\log^{\frac{3}{2}}(ax^n)}{1+m}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int x^m \log^{\frac{3}{2}}(ax^n)\, dx = \frac{x^{1+m}\left(3n^{3/2}\sqrt{\pi}(ax^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)+2\sqrt{1+m}\sqrt{\log(ax^n)}(-3n+2(1+m)\log(ax^n))\right)}{4(1+m)^{5/2}}$$

```
[In] Integrate[x^m*Log[a*x^n]^(3/2),x]
[Out] (x^(1 + m)*((3*n^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[1 + m]*Sqrt[Log[a*x^n]])/Sqrt[n]
])/(a*x^n)^((1 + m)/n) + 2*Sqrt[1 + m]*Sqrt[Log[a*x^n]]*(-3*n + 2*(1 + m)*L
og[a*x^n])))/(4*(1 + m)^(5/2))
```

Maple [F]

$$\int x^m \ln(a x^n)^{\frac{3}{2}}\, dx$$

```
[In] int(x^m*ln(a*x^n)^(3/2),x)
[Out] int(x^m*ln(a*x^n)^(3/2),x)
```

Fricas [F]

$$\int x^m \log^{\frac{3}{2}}(ax^n)\, dx = \int x^m \log(ax^n)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x^m*log(a*x^n)^(3/2),x, algorithm="fricas")
[Out] integral(x^m*log(a*x^n)^(3/2), x)
```

Sympy [F]

$$\int x^m \log^{\frac{3}{2}}(ax^n)\, dx = \int x^m \log(ax^n)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x**m*ln(a*x**n)**(3/2),x)
[Out] Integral(x**m*log(a*x**n)**(3/2), x)
```

Maxima [F]

$$\int x^m \log^{\frac{3}{2}}(ax^n)\, dx = \int x^m \log(ax^n)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x^m*log(a*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(x^m*log(a*x^n)^(3/2), x)
```

Giac [F]

$$\int x^m \log^{\frac{3}{2}}(ax^n)\, dx = \int x^m \log(ax^n)^{\frac{3}{2}}\, dx$$

```
[In] integrate(x^m*log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(x^m*log(a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \log^{\frac{3}{2}}(ax^n)\, dx = \int x^m \ln(a\,x^n)^{3/2}\, dx$$

```
[In] int(x^m*log(a*x^n)^(3/2),x)
[Out] int(x^m*log(a*x^n)^(3/2), x)
```

3.163 $\int x^m \sqrt{\log(ax^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 86

$$\int x^m \sqrt{\log(ax^n)}\,dx = -\frac{\sqrt{n}\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(1+m)^{3/2}} + \frac{x^{1+m}\sqrt{\log(ax^n)}}{1+m}$$

```
[Out] -1/2*x^(1+m)*erfi((1+m)^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*n^(1/2)*Pi^(1/2)/(1+
m)^(3/2)/((a*x^n)^((1+m)/n))+x^(1+m)*ln(a*x^n)^(1/2)/(1+m)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2342, 2347, 2211, 2235}

$$\int x^m \sqrt{\log(ax^n)}\,dx = \frac{x^{m+1}\sqrt{\log(ax^n)}}{m+1} - \frac{\sqrt{\pi}\sqrt{n}x^{m+1}(ax^n)^{-\frac{m+1}{n}}\operatorname{erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(m+1)^{3/2}}$$

```
[In] Int[x^m*Sqrt[Log[a*x^n]],x]
```

```
[Out] -1/2*(Sqrt[n]*Sqrt[Pi]*x^(1 + m)*Erfi[(Sqrt[1 + m]*Sqrt[Log[a*x^n]])/Sqrt[n
]])/((1 + m)^(3/2)*(a*x^n)^((1 + m)/n)) + (x^(1 + m)*Sqrt[Log[a*x^n]])/(1 +
 m)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2342

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Dist[b*n*
(p/(m + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^{1+m}\sqrt{\log(ax^n)}}{1+m} - \frac{n\int\frac{x^m}{\sqrt{\log(ax^n)}}\,dx}{2(1+m)} \\
&= \frac{x^{1+m}\sqrt{\log(ax^n)}}{1+m} - \frac{\left(x^{1+m}(ax^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{2(1+m)} \\
&= \frac{x^{1+m}\sqrt{\log(ax^n)}}{1+m} - \frac{\left(x^{1+m}(ax^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{1+m} \\
&= -\frac{\sqrt{n}\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\,\text{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(1+m)^{3/2}} + \frac{x^{1+m}\sqrt{\log(ax^n)}}{1+m}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$
\int x^m\sqrt{\log(ax^n)}\,dx = -\frac{\sqrt{n}\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\,\text{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{2(1+m)^{3/2}} + \frac{x^{1+m}\sqrt{\log(ax^n)}}{1+m}
$$

```
[In] Integrate[x^m*Sqrt[Log[a*x^n]],x]
```

```
[Out] -1/2*(Sqrt[n]*Sqrt[Pi]*x^(1 + m)*Erfi[(Sqrt[1 + m]*Sqrt[Log[a*x^n]])/Sqrt[n
]])/((1 + m)^(3/2)*(a*x^n)^((1 + m)/n)) + (x^(1 + m)*Sqrt[Log[a*x^n]])/(1 +
 m)
```

Maple [F]

$$\int x^m \sqrt{\ln (a\, x^n)} dx$$

```
[In] int(x^m*ln(a*x^n)^(1/2),x)
[Out] int(x^m*ln(a*x^n)^(1/2),x)
```

Fricas [F]

$$\int x^m \sqrt{\log (ax^n)}\, dx = \int x^m \sqrt{\log (ax^n)}\, dx$$

```
[In] integrate(x^m*log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] integral(x^m*sqrt(log(a*x^n)), x)
```

Sympy [F]

$$\int x^m \sqrt{\log (ax^n)}\, dx = \int x^m \sqrt{\log (ax^n)}\, dx$$

```
[In] integrate(x**m*ln(a*x**n)**(1/2),x)
[Out] Integral(x**m*sqrt(log(a*x**n)), x)
```

Maxima [F]

$$\int x^m \sqrt{\log (ax^n)}\, dx = \int x^m \sqrt{\log (ax^n)}\, dx$$

```
[In] integrate(x^m*log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*sqrt(log(a*x^n)), x)
```

Giac [F]

$$\int x^m \sqrt{\log(ax^n)}\,dx = \int x^m \sqrt{\log(ax^n)}\,dx$$

```
[In] integrate(x^m*log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^m*sqrt(log(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \sqrt{\log(ax^n)}\,dx = \int x^m\,\sqrt{\ln(a\,x^n)}\,dx$$

```
[In] int(x^m*log(a*x^n)^(1/2),x)
[Out] int(x^m*log(a*x^n)^(1/2), x)
```

3.164 $\int \frac{x^m}{\sqrt{\log(ax^n)}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \frac{x^m}{\sqrt{\log(ax^n)}} \, dx = \frac{\sqrt{\pi} x^{1+m} (ax^n)^{-\frac{1+m}{n}} \operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{1+m}\sqrt{n}}$$

```
[Out] x^(1+m)*erfi((1+m)^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*Pi^(1/2)/((a*x^n)^((1+m)/
n))/(1+m)^(1/2)/n^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2347, 2211, 2235}

$$\int \frac{x^m}{\sqrt{\log(ax^n)}} \, dx = \frac{\sqrt{\pi} x^{m+1} (ax^n)^{-\frac{m+1}{n}} \operatorname{erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{m+1}\sqrt{n}}$$

```
[In] Int[x^m/Sqrt[Log[a*x^n]],x]
```

```
[Out] (Sqrt[Pi]*x^(1 + m)*Erfi[(Sqrt[1 + m]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(Sqrt[1 +
 m]*Sqrt[n]*(a*x^n)^((1 + m)/n))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{1+m}(ax^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}}\,dx, x, \log(ax^n)\right)}{n} \\
&= \frac{\left(2x^{1+m}(ax^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}}\,dx, x, \sqrt{\log(ax^n)}\right)}{n} \\
&= \frac{\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\,\text{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{1+m}\sqrt{n}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{\log(ax^n)}}\,dx = \frac{\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\,\text{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{\sqrt{1+m}\sqrt{n}}$$

```
[In] Integrate[x^m/Sqrt[Log[a*x^n]],x]
```

```
[Out] (Sqrt[Pi]*x^(1 + m)*Erfi[(Sqrt[1 + m]*Sqrt[Log[a*x^n]])/Sqrt[n]])/(Sqrt[1 +
 m]*Sqrt[n]*(a*x^n)^((1 + m)/n))
```

Maple [F]

$$\int \frac{x^m}{\sqrt{\ln (a\, x^n)}}dx$$

```
[In] int(x^m/ln(a*x^n)^(1/2),x)
[Out] int(x^m/ln(a*x^n)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{\log (ax^n)}}\, dx = \int \frac{x^m}{\sqrt{\log (ax^n)}}\, dx$$

```
[In] integrate(x^m/log(a*x^n)^(1/2),x, algorithm="fricas")
[Out] integral(x^m/sqrt(log(a*x^n)), x)
```

Sympy [F]

$$\int \frac{x^m}{\sqrt{\log (ax^n)}}\, dx = \int \frac{x^m}{\sqrt{\log (ax^n)}}\, dx$$

```
[In] integrate(x**m/ln(a*x**n)**(1/2),x)
[Out] Integral(x**m/sqrt(log(a*x**n)), x)
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{\log (ax^n)}}\, dx = \int \frac{x^m}{\sqrt{\log (ax^n)}}\, dx$$

```
[In] integrate(x^m/log(a*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m/sqrt(log(a*x^n)), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{\log(ax^n)}}\,dx = \int \frac{x^m}{\sqrt{\log(ax^n)}}\,dx$$

```
[In] integrate(x^m/log(a*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^m/sqrt(log(a*x^n)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{\log(ax^n)}}\,dx = \int \frac{x^m}{\sqrt{\ln(a\,x^n)}}\,dx$$

```
[In] int(x^m/log(a*x^n)^(1/2),x)
[Out] int(x^m/log(a*x^n)^(1/2), x)
```

3.165 $\int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)}\,dx = \frac{2\sqrt{1+m}\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^{1+m}}{n\sqrt{\log(ax^n)}}$$

```
[Out] 2*x^(1+m)*erfi((1+m)^(1/2)*ln(a*x^n)^(1/2)/n^(1/2))*(1+m)^(1/2)*Pi^(1/2)/n^
(3/2)/((a*x^n)^((1+m)/n))-2*x^(1+m)/n/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2343, 2347, 2211, 2235}

$$\int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)}\,dx = \frac{2\sqrt{\pi}\sqrt{m+1}x^{m+1}(ax^n)^{-\frac{m+1}{n}}\operatorname{erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}} - \frac{2x^{m+1}}{n\sqrt{\log(ax^n)}}$$

```
[In] Int[x^m/Log[a*x^n]^(3/2),x]
```

```
[Out] (2*Sqrt[1 + m]*Sqrt[Pi]*x^(1 + m)*Erfi[(Sqrt[1 + m]*Sqrt[Log[a*x^n]])/Sqrt[
n]])/(n^(3/2)*(a*x^n)^((1 + m)/n)) - (2*x^(1 + m))/(n*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x^{1+m}}{n\sqrt{\log\left(ax^n\right)}}+\frac{(2(1+m))\int\frac{x^m}{\sqrt{\log(ax^n)}}\,dx}{n}\\
&= -\frac{2x^{1+m}}{n\sqrt{\log\left(ax^n\right)}}+\frac{\left(2(1+m)x^{1+m}(ax^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}}\,dx,x,\log\left(ax^n\right)\right)}{n^2}\\
&= -\frac{2x^{1+m}}{n\sqrt{\log\left(ax^n\right)}}+\frac{\left(4(1+m)x^{1+m}(ax^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}}\,dx,x,\sqrt{\log\left(ax^n\right)}\right)}{n^2}\\
&= \frac{2\sqrt{1+m}\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\,\text{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{n^{3/2}}-\frac{2x^{1+m}}{n\sqrt{\log\left(ax^n\right)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$
\int\frac{x^m}{\log^{\frac{3}{2}}\left(ax^n\right)}\,dx=\frac{2x^{1+m}\left(-1+(ax^n)^{-\frac{1+m}{n}}\,\Gamma\left(\frac{1}{2},-\frac{(1+m)\log(ax^n)}{n}\right)\sqrt{-\frac{(1+m)\log(ax^n)}{n}}\right)}{n\sqrt{\log\left(ax^n\right)}}
$$

```
[In] Integrate[x^m/Log[a*x^n]^(3/2),x]
[Out] (2*x^(1 + m)*(-1 + (Gamma[1/2, -(((1 + m)*Log[a*x^n])/n)]*Sqrt[-(((1 + m)*L
og[a*x^n])/n)])/(a*x^n)^((1 + m)/n)))/(n*Sqrt[Log[a*x^n]])
```

Maple [F]

$$\int \frac{x^m}{\ln\left(a\,x^n\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^m/ln(a*x^n)^(3/2),x)
[Out] int(x^m/ln(a*x^n)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^m}{\log\left(ax^n\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^m/log(a*x^n)^(3/2),x, algorithm="fricas")
[Out] integral(x^m/log(a*x^n)^(3/2), x)
```

Sympy [F]

$$\int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^m}{\log\left(ax^n\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**m/ln(a*x**n)**(3/2),x)
[Out] Integral(x**m/log(a*x**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^m}{\log\left(ax^n\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^m/log(a*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(x^m/log(a*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^m}{\log(ax^n)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^m/log(a*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(x^m/log(a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\log^{\frac{3}{2}}(ax^n)}\,dx = \int \frac{x^m}{\ln(a\,x^n)^{3/2}}\,dx$$

```
[In] int(x^m/log(a*x^n)^(3/2),x)
[Out] int(x^m/log(a*x^n)^(3/2), x)
```

3.166 $\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 112

$$\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)}\,dx = \frac{4(1+m)^{3/2}\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{2x^{1+m}}{3n\log^{\frac{3}{2}}(ax^n)} - \frac{4(1+m)x^{1+m}}{3n^2\sqrt{\log(ax^n)}}$$

```
[Out] -2/3*x^(1+m)/n/ln(a*x^n)^(3/2)+4/3*(1+m)^(3/2)*x^(1+m)*erfi((1+m)^(1/2)*ln(
a*x^n)^(1/2)/n^(1/2))*Pi^(1/2)/n^(5/2)/((a*x^n)^((1+m)/n))-4/3*(1+m)*x^(1+m
)/n^2/ln(a*x^n)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2343, 2347, 2211, 2235}

$$\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)}\,dx = \frac{4\sqrt{\pi}(m+1)^{3/2}x^{m+1}(ax^n)^{-\frac{m+1}{n}}\operatorname{erfi}\left(\frac{\sqrt{m+1}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}} - \frac{4(m+1)x^{m+1}}{3n^2\sqrt{\log(ax^n)}} - \frac{2x^{m+1}}{3n\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Int[x^m/Log[a*x^n]^(5/2),x]
```

```
[Out] (4*(1 + m)^(3/2)*Sqrt[Pi]*x^(1 + m)*Erfi[(Sqrt[1 + m]*Sqrt[Log[a*x^n]])/Sqr
t[n]])/(3*n^(5/2)*(a*x^n)^((1 + m)/n)) - (2*x^(1 + m))/(3*n*Log[a*x^n]^(3/2
)) - (4*(1 + m)*x^(1 + m))/(3*n^2*Sqrt[Log[a*x^n]])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^{1+m}}{3n\log^{\frac{3}{2}}(ax^n)}+\frac{(2(1+m))\int\frac{x^m}{\log^{\frac{3}{2}}(ax^n)}\,dx}{3n}\\
&= -\frac{2x^{1+m}}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{4(1+m)x^{1+m}}{3n^2\sqrt{\log(ax^n)}}+\frac{(4(1+m)^2)\int\frac{x^m}{\sqrt{\log(ax^n)}}\,dx}{3n^2}\\
&= -\frac{2x^{1+m}}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{4(1+m)x^{1+m}}{3n^2\sqrt{\log(ax^n)}}\\
&\quad+\frac{\left(4(1+m)^2x^{1+m}(ax^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\frac{e^{\frac{(1+m)x}{n}}}{\sqrt{x}}\,dx,x,\log(ax^n)\right)}{3n^3}\\
&= -\frac{2x^{1+m}}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{4(1+m)x^{1+m}}{3n^2\sqrt{\log(ax^n)}}\\
&\quad+\frac{\left(8(1+m)^2x^{1+m}(ax^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m)x^2}{n}}\,dx,x,\sqrt{\log(ax^n)}\right)}{3n^3}\\
&= \frac{4(1+m)^{3/2}\sqrt{\pi}x^{1+m}(ax^n)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{\sqrt{1+m}\sqrt{\log(ax^n)}}{\sqrt{n}}\right)}{3n^{5/2}}-\frac{2x^{1+m}}{3n\log^{\frac{3}{2}}(ax^n)}-\frac{4(1+m)x^{1+m}}{3n^2\sqrt{\log(ax^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

$$\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)}\,dx = -\frac{2e^{-\frac{(1+m)(-n\log(x)+\log(ax^n))}{n}}\left(2n\Gamma\left(\frac{1}{2},-\frac{(1+m)\log(ax^n)}{n}\right)\left(-\frac{(1+m)\log(ax^n)}{n}\right)^{3/2}+(ax^n)^{\frac{1+m}{n}}(n+2(1+m)\log(ax\right.}{3n^2\log^{\frac{3}{2}}(ax^n)}$$

```
[In] Integrate[x^m/Log[a*x^n]^(5/2),x]
```

```
[Out] (-2*(2*n*Gamma[1/2, -(((1 + m)*Log[a*x^n])/n)]*(-(((1 + m)*Log[a*x^n])/n))^
(3/2) + (a*x^n)^((1 + m)/n)*(n + 2*(1 + m)*Log[a*x^n])))/(3*E^(((1 + m)*(-(
n*Log[x]) + Log[a*x^n]))/n)*n^2*Log[a*x^n]^(3/2))
```

Maple [F]

$$\int \frac{x^m}{\ln(a\,x^n)^{\frac{5}{2}}}dx$$

```
[In] int(x^m/ln(a*x^n)^(5/2),x)
```

```
[Out] int(x^m/ln(a*x^n)^(5/2),x)
```

Fricas [F]

$$\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^m}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^m/log(a*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(x^m/log(a*x^n)^(5/2), x)
```

Sympy [F]

$$\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^m}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x**m/ln(a*x**n)**(5/2),x)
```

```
[Out] Integral(x**m/log(a*x**n)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^m}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^m/log(a*x^n)^(5/2),x, algorithm="maxima")
[Out] integrate(x^m/log(a*x^n)^(5/2), x)
```

Giac [F]

$$\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^m}{\log(ax^n)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^m/log(a*x^n)^(5/2),x, algorithm="giac")
[Out] integrate(x^m/log(a*x^n)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\log^{\frac{5}{2}}(ax^n)}\,dx = \int \frac{x^m}{\ln(a\,x^n)^{5/2}}\,dx$$

```
[In] int(x^m/log(a*x^n)^(5/2),x)
[Out] int(x^m/log(a*x^n)^(5/2), x)
```

3.167 $\int (dx)^m (a+b\log(cx^n))^p \, dx$

Optimal result

Integrand size = 18, antiderivative size = 106

$$\int (dx)^m (a+b\log(cx^n))^p \, dx = \frac{e^{-\frac{a(1+m)}{bn}}(dx)^{1+m}(cx^n)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{d(1+m)}$$

```
[Out] (d*x)^(1+m)*GAMMA(p+1,-(1+m)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/d/exp(a
*(1+m)/b/n)/(1+m)/((c*x^n)^((1+m)/n))/((-(1+m)*(a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2347, 2212}

$$\int (dx)^m (a+b\log(cx^n))^p \, dx = \frac{(dx)^{m+1}e^{-\frac{a(m+1)}{bn}}(cx^n)^{-\frac{m+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx^n))}{bn}\right)}{d(m+1)}$$

```
[In] Int[(d*x)^m*(a + b*Log[c*x^n])^p,x]
```

```
[Out] ((d*x)^(1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*L
og[c*x^n])^p)/(d*E^((a*(1 + m))/(b*n))*(1 + m)*(c*x^n)^((1 + m)/n)*(-(((1 +
 m)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left((dx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int e^{\frac{(1+m)x}{n}}(a+bx)^p\,dx,x,\log\left(cx^n\right)\right)}{dn}$$

$$= \frac{e^{-\frac{a(1+m)}{bn}}(dx)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{d(1+m)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.01

$$\int (dx)^m\left(a+b\log\left(cx^n\right)\right)^p\,dx$$

$$= \frac{e^{-\frac{(1+m)(a+b(-n\log(x)+\log(cx^n)))}{bn}}x^{-m}(dx)^m\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)\left(a+b\log\left(cx^n\right)\right)^p\left(-\frac{(1+m)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+m}$$

```
[In] Integrate[(d*x)^m*(a + b*Log[c*x^n])^p,x]
```

```
[Out] ((d*x)^m*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log[c*x
^n])^p)/(E^(((1 + m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*x^m
*(-(((1 + m)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Maple [F]

$$\int (dx)^m (a+b\ln(c\,x^n))^p dx$$

```
[In] int((d*x)^m*(a+b*ln(c*x^n))^p,x)
[Out] int((d*x)^m*(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int (dx)^m (a+b\log(cx^n))^p \, dx = \int (dx)^m (b\log(cx^n)+a)^p \, dx$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral((d*x)^m*(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int (dx)^m (a+b\log(cx^n))^p \, dx = \int (dx)^m (a+b\log(cx^n))^p \, dx$$

```
[In] integrate((d*x)**m*(a+b*ln(c*x**n))**p,x)
[Out] Integral((d*x)**m*(a + b*log(c*x**n))**p, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (dx)^m (a+b\log(cx^n))^p \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^p,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int (dx)^m (a + b \log (cx^n))^p \, dx = \int (dx)^m (b \log (cx^n) + a)^p \, dx$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate((d*x)^m*(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m (a + b \log (cx^n))^p \, dx = \int (d\,x)^m \,(a + b\,\ln (c\,x^n))^p \, dx$$

```
[In] int((d*x)^m*(a + b*log(c*x^n))^p,x)
[Out] int((d*x)^m*(a + b*log(c*x^n))^p, x)
```

3.168 $\int x^2(a+b\log(cx^n))^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int x^2(a+b\log(cx^n))^p\,dx = 3^{-1-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}$$

```
[Out] 3^(-1-p)*x^3*GAMMA(p+1,-3*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/exp(3*a/b/
n)/((c*x^n)^(3/n))/(((-a-b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2347, 2212}

$$\int x^2(a+b\log(cx^n))^p\,dx = 3^{-p-1}x^3e^{-\frac{3a}{bn}}(cx^n)^{-3/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(cx^n))}{bn}\right)$$

```
[In] Int[x^2*(a + b*Log[c*x^n])^p,x]
```

```
[Out] (3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*x^n]))/(b*n)]*(a + b*Log[c*x^
n])^p)/(E^((3*a)/(b*n))*(c*x^n)^(3/n)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int e^{\frac{3x}{n}}(a+bx)^p\,dx,x,\log(cx^n)\right)}{n}$$
$$=3^{-1-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\,\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x^2(a+b\log(cx^n))^p\,dx=3^{-1-p}e^{-\frac{3a}{bn}}x^3(cx^n)^{-3/n}\,\Gamma\left(1+p,-\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}$$

```
[In] Integrate[x^2*(a + b*Log[c*x^n])^p,x]
[Out] (3^(-1 - p)*x^3*Gamma[1 + p, (-3*(a + b*Log[c*x^n]))/(b*n)]*(a + b*Log[c*x^
n])^p)/(E^((3*a)/(b*n))*(c*x^n)^(3/n)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Maple [F]

$$\int x^2(a+b\ln(c\,x^n))^p dx$$

```
[In] int(x^2*(a+b*ln(c*x^n))^p,x)
[Out] int(x^2*(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int x^2(a+b\log(cx^n))^p\,dx = \int (b\log(cx^n)+a)^px^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)^p*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\log(cx^n))^p\,dx = \int x^2(a+b\log(cx^n))^p\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**n))**p,x)
[Out] Integral(x**2*(a + b*log(c*x**n))**p, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2(a+b\log(cx^n))^p\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^p,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int x^2(a+b\log(cx^n))^p\,dx = \int (b\log(cx^n)+a)^px^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx^n))^p\,dx=\int x^2\,(a+b\,\ln(c\,x^n))^p\,dx$$

```
[In] int(x^2*(a + b*log(c*x^n))^p,x)
[Out] int(x^2*(a + b*log(c*x^n))^p, x)
```

3.169 $\int x(a+b\log(cx^n))^p\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int x(a+b\log(cx^n))^p\,dx=2^{-1-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\Gamma\left(1+p,-\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}$$

```
[Out] 2^(-1-p)*x^2*GAMMA(p+1,-2*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/exp(2*a/b/
n)/((c*x^n)^(2/n))/(((-a-b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2347, 2212}

$$\int x(a+b\log(cx^n))^p\,dx=2^{-p-1}x^2e^{-\frac{2a}{bn}}(cx^n)^{-2/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(cx^n))}{bn}\right)$$

```
[In] Int[x*(a + b*Log[c*x^n])^p,x]
```

```
[Out] (2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*x^n]))/(b*n)]*(a + b*Log[c*x^
n])^p)/(E^((2*a)/(b*n))*(c*x^n)^(2/n)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int e^{\frac{2x}{n}}(a+bx)^p\,dx, x, \log(cx^n)\right)}{n}$$

$$= 2^{-1-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\,\Gamma\left(1+p, -\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int x(a+b\log(cx^n))^p\,dx = 2^{-1-p}e^{-\frac{2a}{bn}}x^2(cx^n)^{-2/n}\,\Gamma\left(1+p, -\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}$$

```
[In] Integrate[x*(a + b*Log[c*x^n])^p,x]
[Out] (2^(-1 - p)*x^2*Gamma[1 + p, (-2*(a + b*Log[c*x^n]))/(b*n)]*(a + b*Log[c*x^
n])^p)/(E^((2*a)/(b*n))*(c*x^n)^(2/n)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Maple [F]

$$\int x(a+b\ln(c\,x^n))^p dx$$

```
[In] int(x*(a+b*ln(c*x^n))^p,x)
[Out] int(x*(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int x(a+b\log(cx^n))^p\,dx=\int(b\log(cx^n)+a)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)^p*x, x)
```

Sympy [F]

$$\int x(a+b\log(cx^n))^p\,dx=\int x(a+b\log(cx^n))^p\,dx$$

```
[In] integrate(x*(a+b*ln(c*x**n))**p,x)
[Out] Integral(x*(a + b*log(c*x**n))**p, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x(a+b\log(cx^n))^p\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(a+b*log(c*x^n))^p,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int x(a+b\log(cx^n))^p\,dx=\int(b\log(cx^n)+a)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx^n))^p\,dx = \int x\,(a+b\,\ln(c\,x^n))^p\,dx$$

```
[In] int(x*(a + b*log(c*x^n))^p,x)
[Out] int(x*(a + b*log(c*x^n))^p, x)
```

3.170 $\int (a+b\log(cx^n))^p\,dx$

Optimal result

Integrand size = 12, antiderivative size = 80

$$\int (a+b\log(cx^n))^p\,dx = e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}$$

```
[Out] x*GAMMA(p+1,(-a-b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/exp(a/b/n)/((c*x^n)^(1/
n))/(((-a-b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2337, 2212}

$$\int (a+b\log(cx^n))^p\,dx = xe^{-\frac{a}{bn}}(cx^n)^{-1/n}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{bn}\right)$$

```
[In] Int[(a + b*Log[c*x^n])^p,x]
```

```
[Out] (x*Gamma[1 + p, -((a + b*Log[c*x^n])/(b*n))]*(a + b*Log[c*x^n])^p)/(E^(a/(b
*n))*(c*x^n)^n^(-1)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

<u>Rule 2212</u>

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2337

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[x/(n*(c*x
^n)^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[
{a, b, c, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int e^{\frac{x}{n}}(a+bx)^p\,dx, x, \log(cx^n)\right)}{n}\\ &= e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int (a+b\log(cx^n))^p\,dx = e^{-\frac{a}{bn}}x(cx^n)^{-1/n}\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{bn}\right)^{-p}$$

```
[In] Integrate[(a + b*Log[c*x^n])^p,x]
[Out] (x*Gamma[1 + p, -((a + b*Log[c*x^n])/(b*n))]*(a + b*Log[c*x^n])^p)/(E^(a/(b
*n))*(c*x^n)^n^(-1)*(-((a + b*Log[c*x^n])/(b*n)))^p)
```

Maple [F]

$$\int (a+b\ln(c\,x^n))^p dx$$

```
[In] int((a+b*ln(c*x^n))^p,x)
[Out] int((a+b*ln(c*x^n))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.65

$$\int (a + b\log(cx^n))^p \, dx = e^{\left(-\frac{bnp\log\left(-\frac{1}{bn}\right)+b\log(c)+a}{bn}\right)}\Gamma\left(p+1, -\frac{bn\log(x)+b\log(c)+a}{bn}\right)$$

```
[In] integrate((a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] e^(-(b*n*p*log(-1/(b*n)) + b*log(c) + a)/(b*n))*gamma(p + 1, -(b*n*log(x) +
 b*log(c) + a)/(b*n))
```

Sympy [F]

$$\int (a + b\log(cx^n))^p \, dx = \int (a + b\log(cx^n))^p \, dx$$

```
[In] integrate((a+b*ln(c*x**n))**p,x)
[Out] Integral((a + b*log(c*x**n))**p, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a + b\log(cx^n))^p \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*log(c*x^n))^p,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int (a + b\log(cx^n))^p \, dx = \int (b\log(cx^n) + a)^p \, dx$$

```
[In] integrate((a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx^n))^p\,dx = \int (a+b\,\ln(c\,x^n))^p\,dx$$

```
[In] int((a + b*log(c*x^n))^p,x)
[Out] int((a + b*log(c*x^n))^p, x)
```

3.171 $\int \frac{(a+b\log(cx^n))^p}{x} dx$

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{(a+b\log(cx^n))^p}{x} dx = \frac{(a+b\log(cx^n))^{1+p}}{bn(1+p)}$$

```
[Out] (a+b*ln(c*x^n))^(p+1)/b/n/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2339, 30}

$$\int \frac{(a+b\log(cx^n))^p}{x} dx = \frac{(a+b\log(cx^n))^{p+1}}{bn(p+1)}$$

```
[In] Int[(a + b*Log[c*x^n])^p/x,x]
```

```
[Out] (a + b*Log[c*x^n])^(1 + p)/(b*n*(1 + p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^p\,dx, x, a+b\log\left(cx^n\right)\right)}{bn} \\
&= \frac{\left(a+b\log\left(cx^n\right)\right)^{1+p}}{bn(1+p)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(cx^n\right)\right)^p}{x}\,dx = \frac{\left(a+b\log\left(cx^n\right)\right)^{1+p}}{bn(1+p)}$$

```
[In] Integrate[(a + b*Log[c*x^n])^p/x,x]
[Out] (a + b*Log[c*x^n])^(1 + p)/(b*n*(1 + p))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{p+1}}{bn\left(p+1\right)}$	27
default	$\frac{\left(a+b\ln\left(c\,x^{n}\right)\right)^{p+1}}{bn\left(p+1\right)}$	27
parallelrisch	$-\frac{-\ln\left(c\,x^{n}\right)\left(a+b\ln\left(c\,x^{n}\right)\right)^{p} b^{2}-\left(a+b\ln\left(c\,x^{n}\right)\right)^{p} a b}{b^{2} n\left(p+1\right)}$	54
risch	$\frac{\left(\ln\left(x^{n}\right)b+a+b\left(\ln\left(c\right)-\frac{i\pi\,\operatorname{csgn}\left(i c\,x^{n}\right)\left(-\operatorname{csgn}\left(i c\,x^{n}\right)+\operatorname{csgn}\left(i c\right)\right)\left(-\operatorname{csgn}\left(i c\,x^{n}\right)+\operatorname{csgn}\left(i x^{n}\right)\right)}{2}\right)\right)^{p+1}}{n b\left(p+1\right)}$	76

```
[In] int((a+b*ln(c*x^n))^p/x,x,method=_RETURNVERBOSE)
[Out] (a+b*ln(c*x^n))^(p+1)/b/n/(p+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\log(cx^n))^p}{x}\,dx = \frac{(bn\log(x)+b\log(c)+a)(bn\log(x)+b\log(c)+a)^p}{bnp+bn}$$

```
[In] integrate((a+b*log(c*x^n))^p/x,x, algorithm="fricas")
[Out] (b*n*log(x) + b*log(c) + a)*(b*n*log(x) + b*log(c) + a)^p/(b*n*p + b*n)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int \frac{(a+b\log(cx^n))^p}{x}\,dx = -\begin{cases} -a^p\log(x) & \text{for } b=0 \\ -(a+b\log(c))^p\log(x) & \text{for } n=0 \\ -\frac{\begin{cases} \frac{(a+b\log(cx^n))^{p+1}}{p+1} & \text{for } p\neq -1 \\ \log(a+b\log(cx^n)) & \text{otherwise} \end{cases}}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln(c*x**n))**p/x,x)
[Out] -Piecewise((-a**p*log(x), Eq(b, 0)), (-(a + b*log(c))**p*log(x), Eq(n, 0)),
 (-Piecewise(((a + b*log(c*x**n))**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*
log(c*x**n)), True))/(b*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))^p}{x}\,dx = \frac{(b\log(cx^n)+a)^{p+1}}{bn(p+1)}$$

```
[In] integrate((a+b*log(c*x^n))^p/x,x, algorithm="maxima")
[Out] (b*log(c*x^n) + a)^(p + 1)/(b*n*(p + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{(a+b\log(cx^n))^p}{x}\,dx = \frac{(bn\log(x)+b\log(c)+a)^{p+1}}{bn(p+1)}$$

```
[In] integrate((a+b*log(c*x^n))^p/x,x, algorithm="giac")
[Out] (b*n*log(x) + b*log(c) + a)^(p + 1)/(b*n*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))^p}{x}\,dx = \frac{(a+b\,\ln(c\,x^n))^{p+1}}{b\,n\,(p+1)}$$

```
[In] int((a + b*log(c*x^n))^p/x,x)
[Out] (a + b*log(c*x^n))^(p + 1)/(b*n*(p + 1))
```

3.172 $\int \frac{(a+b\log(cx^n))^p}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{(a+b\log(cx^n))^p}{x^2}\,dx = -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\Gamma\left(1+p,\frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x}$$

```
[Out] -exp(a/b/n)*(c*x^n)^(1/n)*GAMMA(p+1,(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/
x/(((a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2347, 2212}

$$\int \frac{(a+b\log(cx^n))^p}{x^2}\,dx = -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx^n)}{bn}\right)}{x}$$

```
[In] Int[(a + b*Log[c*x^n])^p/x^2,x]
```

```
[Out] -((E^(a/(b*n))*(c*x^n)^n^(-1)*Gamma[1 + p, (a + b*Log[c*x^n])/(b*n)]*(a + b
*Log[c*x^n])^p)/(x*((a + b*Log[c*x^n])/(b*n))^p))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int e^{-\frac{x}{n}}(a+bx)^p\,dx, x, \log(cx^n)\right)}{nx}$$
$$= -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\,\Gamma\left(1+p, \frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))^p}{x^2}\,dx = -\frac{e^{\frac{a}{bn}}(cx^n)^{\frac{1}{n}}\,\Gamma\left(1+p, \frac{a+b\log(cx^n)}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x}$$

```
[In] Integrate[(a + b*Log[c*x^n])^p/x^2,x]
[Out] -((E^(a/(b*n))*(c*x^n)^n^(-1)*Gamma[1 + p, (a + b*Log[c*x^n])/(b*n)]*(a + b
*Log[c*x^n])^p)/(x*((a + b*Log[c*x^n])/(b*n))^p))
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^p}{x^2}dx$$

```
[In] int((a+b*ln(c*x^n))^p/x^2,x)
[Out] int((a+b*ln(c*x^n))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^p}{x^2}\,dx = \int \frac{(b\log(cx^n)+a)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\log{(cx^n)})^p}{x^2}\,dx = \int \frac{(a+b\log{(cx^n)})^p}{x^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**p/x**2,x)
[Out] Integral((a + b*log(c*x**n))**p/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log{(cx^n)})^p}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*log(c*x^n))^p/x^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(a+b\log{(cx^n)})^p}{x^2}\,dx = \int \frac{(b\log{(cx^n)}+a)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log{(cx^n)})^p}{x^2}\,dx = \int \frac{(a+b\,\ln{(c\,x^n)})^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*x^n))^p/x^2,x)
[Out] int((a + b*log(c*x^n))^p/x^2, x)
```

3.173 $\int \frac{(a+b\log(cx^n))^p}{x^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{(a+b\log(cx^n))^p}{x^3}\,dx = -\frac{2^{-1-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(1+p,\frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^2}$$

```
[Out] -2^(-1-p)*exp(2*a/b/n)*(c*x^n)^(2/n)*GAMMA(p+1,2*(a+b*ln(c*x^n))/b/n)*(a+b*
ln(c*x^n))^p/x^2/(((a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2347, 2212}

$$\int \frac{(a+b\log(cx^n))^p}{x^3}\,dx = -\frac{2^{-p-1}e^{\frac{2a}{bn}}(cx^n)^{2/n}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx^n))}{bn}\right)}{x^2}$$

```
[In] Int[(a + b*Log[c*x^n])^p/x^3,x]
```

```
[Out] -((2^(-1 - p)*E^((2*a)/(b*n))*(c*x^n)^(2/n)*Gamma[1 + p, (2*(a + b*Log[c*x^
n]))/(b*n)]*(a + b*Log[c*x^n])^p)/(x^2*((a + b*Log[c*x^n])/(b*n))^p))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{(cx^n)^{2/n} \operatorname{Subst}\left(\int e^{-\frac{2x}{n}}(a+bx)^p\,dx, x, \log(cx^n)\right)}{nx^2}$$
$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(1+p, \frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))^p}{x^3}\,dx$$
$$= -\frac{2^{-1-p}e^{\frac{2a}{bn}}(cx^n)^{2/n}\,\Gamma\left(1+p, \frac{2(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^2}$$

```
[In] Integrate[(a + b*Log[c*x^n])^p/x^3,x]
[Out] -((2^(-1 - p)*E^((2*a)/(b*n))*(c*x^n)^(2/n)*Gamma[1 + p, (2*(a + b*Log[c*x^
n]))/(b*n)]*(a + b*Log[c*x^n])^p)/(x^2*((a + b*Log[c*x^n])/(b*n))^p))
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^p}{x^3}dx$$

```
[In] int((a+b*ln(c*x^n))^p/x^3,x)
[Out] int((a+b*ln(c*x^n))^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a + b\log(cx^n))^p}{x^3}\,dx = \int \frac{(b\log(cx^n) + a)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^p/x^3,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)^p/x^3, x)
```

Sympy [F]

$$\int \frac{(a + b\log(cx^n))^p}{x^3}\,dx = \int \frac{(a + b\log(cx^n))^p}{x^3}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**p/x**3,x)
[Out] Integral((a + b*log(c*x**n))**p/x**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + b\log(cx^n))^p}{x^3}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*log(c*x^n))^p/x^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(a + b\log(cx^n))^p}{x^3}\,dx = \int \frac{(b\log(cx^n) + a)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^p/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\log(cx^n))^p}{x^3}\,dx = \int \frac{(a + b\,\ln(c\,x^n))^p}{x^3}\,dx$$

```
[In] int((a + b*log(c*x^n))^p/x^3,x)
```

```
[Out] int((a + b*log(c*x^n))^p/x^3, x)
```

3.174 $\int \frac{(a+b\log(cx^n))^p}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{(a+b\log(cx^n))^p}{x^4}\,dx = -\frac{3^{-1-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\Gamma\left(1+p,\frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^3}$$

```
[Out] -3^(-1-p)*exp(3*a/b/n)*(c*x^n)^(3/n)*GAMMA(p+1,3*(a+b*ln(c*x^n))/b/n)*(a+b*
ln(c*x^n))^p/x^3/(((a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2347, 2212}

$$\int \frac{(a+b\log(cx^n))^p}{x^4}\,dx = -\frac{3^{-p-1}e^{\frac{3a}{bn}}(cx^n)^{3/n}(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx^n))}{bn}\right)}{x^3}$$

```
[In] Int[(a + b*Log[c*x^n])^p/x^4,x]
```

```
[Out] -((3^(-1 - p)*E^((3*a)/(b*n))*(c*x^n)^(3/n)*Gamma[1 + p, (3*(a + b*Log[c*x^
n]))/(b*n)]*(a + b*Log[c*x^n])^p)/(x^3*((a + b*Log[c*x^n])/(b*n))^p))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(cx^n)^{3/n}\,\text{Subst}\left(\int e^{-\frac{3x}{n}}(a+bx)^p\,dx, x, \log(cx^n)\right)}{nx^3}\\ &= -\frac{3^{-1-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\Gamma\left(1+p, \frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx^n))^p}{x^4}\,dx = -\frac{3^{-1-p}e^{\frac{3a}{bn}}(cx^n)^{3/n}\,\Gamma\left(1+p, \frac{3(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(\frac{a+b\log(cx^n)}{bn}\right)^{-p}}{x^3}$$

```
[In] Integrate[(a + b*Log[c*x^n])^p/x^4,x]
[Out] -((3^(-1 - p)*E^((3*a)/(b*n))*(c*x^n)^(3/n)*Gamma[1 + p, (3*(a + b*Log[c*x^
n]))/(b*n)]*(a + b*Log[c*x^n])^p)/(x^3*((a + b*Log[c*x^n])/(b*n))^p))
```

Maple [F]

$$\int \frac{(a+b\ln(c\,x^n))^p}{x^4}dx$$

```
[In] int((a+b*ln(c*x^n))^p/x^4,x)
[Out] int((a+b*ln(c*x^n))^p/x^4,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx^n))^p}{x^4}\,dx = \int \frac{(b\log(cx^n)+a)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)^p/x^4, x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx^n))^p}{x^4}\,dx = \int \frac{(a+b\log(cx^n))^p}{x^4}\,dx$$

```
[In] integrate((a+b*ln(c*x**n))**p/x**4,x)
[Out] Integral((a + b*log(c*x**n))**p/x**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\log(cx^n))^p}{x^4}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*log(c*x^n))^p/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int \frac{(a+b\log(cx^n))^p}{x^4}\,dx = \int \frac{(b\log(cx^n)+a)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^n))^p/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx^n))^p}{x^4}\,dx=\int \frac{(a+b\,\ln(c\,x^n))^p}{x^4}\,dx$$

```
[In] int((a + b*log(c*x^n))^p/x^4,x)
```

```
[Out] int((a + b*log(c*x^n))^p/x^4, x)
```

3.175 $\int (dx)^m (a+b\log(cx))^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 86

$$\int (dx)^m (a+b\log(cx))^p\,dx = \frac{e^{-\frac{a(1+m)}{b}}(cx)^{-1-m}(dx)^{1+m}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(-\frac{(1+m)(a+b\log(cx))}{b}\right)^{-p}}{d(1+m)}$$

```
[Out] (c*x)^(-1-m)*(d*x)^(1+m)*GAMMA(p+1,-(1+m)*(a+b*ln(c*x))/b)*(a+b*ln(c*x))^p/
d/exp(a*(1+m)/b)/(1+m)/((-(1+m)*(a+b*ln(c*x))/b)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2347, 2212}

$$\int (dx)^m (a+b\log(cx))^p\,dx = \frac{e^{-\frac{a(m+1)}{b}}(cx)^{-m-1}(dx)^{m+1}(a+b\log(cx))^p\left(-\frac{(m+1)(a+b\log(cx))}{b}\right)^{-p}\Gamma\left(p+1,-\frac{(m+1)(a+b\log(cx))}{b}\right)}{d(m+1)}$$

```
[In] Int[(d*x)^m*(a + b*Log[c*x])^p,x]
```

```
[Out] ((c*x)^(-1 - m)*(d*x)^(1 + m)*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x]))/b)]
*(a + b*Log[c*x])^p)/(d*E^((a*(1 + m))/b)*(1 + m)*(-(((1 + m)*(a + b*Log[c*
x]))/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left((cx)^{-1-m}(dx)^{1+m}\right)\text{Subst}\left(\int e^{(1+m)x}(a+bx)^p\,dx, x, \log(cx)\right)}{d}$$

$$= \frac{e^{-\frac{a(1+m)}{b}}(cx)^{-1-m}(dx)^{1+m}\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(-\frac{(1+m)(a+b\log(cx))}{b}\right)^{-p}}{d(1+m)}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int (dx)^m(a+b\log(cx))^p\,dx$$

$$= \frac{e^{-\frac{a(1+m)}{b}}(cx)^{-m}(dx)^m\Gamma\left(1+p,-\frac{(1+m)(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(-\frac{(1+m)(a+b\log(cx))}{b}\right)^{-p}}{c(1+m)}$$

```
[In] Integrate[(d*x)^m*(a + b*Log[c*x])^p,x]
```

```
[Out] ((d*x)^m*Gamma[1 + p, -(((1 + m)*(a + b*Log[c*x]))/b)]*(a + b*Log[c*x])^p)/
(c*E^((a*(1 + m))/b)*(1 + m)*(c*x)^m*(-(((1 + m)*(a + b*Log[c*x]))/b))^p)
```

Maple [F]

$$\int (dx)^m\,(a+b\ln(xc))^p\,dx$$

```
[In] int((d*x)^m*(a+b*ln(x*c))^p,x)
```

```
[Out] int((d*x)^m*(a+b*ln(x*c))^p,x)
```

Fricas [F]

$$\int (dx)^m(a+b\log(cx))^p\,dx=\int (dx)^m\,(b\log{(cx)}+a)^p\,dx$$

```
[In] integrate((d*x)^m*(a+b*log(c*x))^p,x, algorithm="fricas")
[Out] integral((d*x)^m*(b*log(c*x) + a)^p, x)
```

Sympy [F]

$$\int (dx)^m(a+b\log(cx))^p\,dx=\int (dx)^m\,(a+b\log{(cx)})^p\;dx$$

```
[In] integrate((d*x)**m*(a+b*ln(c*x))**p,x)
[Out] Integral((d*x)**m*(a + b*log(c*x))**p, x)
```

Maxima [F]

$$\int (dx)^m(a+b\log(cx))^p\,dx=\int (dx)^m\,(b\log{(cx)}+a)^p\,dx$$

```
[In] integrate((d*x)^m*(a+b*log(c*x))^p,x, algorithm="maxima")
[Out] integrate((d*x)^m*(b*log(c*x) + a)^p, x)
```

Giac [F]

$$\int (dx)^m(a+b\log(cx))^p\,dx=\int (dx)^m\,(b\log{(cx)}+a)^p\,dx$$

```
[In] integrate((d*x)^m*(a+b*log(c*x))^p,x, algorithm="giac")
[Out] integrate((d*x)^m*(b*log(c*x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m (a + b\log(cx))^p \, dx = \int (a + b\,\ln(c\,x))^p\,(d\,x)^m \,dx$$

```
[In] int((a + b*log(c*x))^p*(d*x)^m,x)
[Out] int((a + b*log(c*x))^p*(d*x)^m, x)
```

3.176 $\int x^2(a+b\log(cx))^p\,dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int x^2(a+b\log(cx))^p\,dx=\frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p,-\frac{3(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}}{c^3}$$

```
[Out] 3^(-1-p)*GAMMA(p+1,-3*(a+b*ln(c*x))/b)*(a+b*ln(c*x))^p/c^3/exp(3*a/b)/(((-a
-b*ln(c*x))/b)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {2346, 2212}

$$\int x^2(a+b\log(cx))^p\,dx=\frac{3^{-p-1}e^{-\frac{3a}{b}}(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{3(a+b\log(cx))}{b}\right)}{c^3}$$

```
[In] Int[x^2*(a + b*Log[c*x])^p,x]
```

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*x]))/b]*(a + b*Log[c*x])^p)/(c^3*
E^((3*a)/b)*(-((a + b*Log[c*x])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int e^{3x}(a+bx)^p\,dx, x, \log(cx)\right)}{c^3}$$
$$= \frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}}{c^3}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int x^2(a+b\log(cx))^p\,dx = \frac{3^{-1-p}e^{-\frac{3a}{b}}\Gamma\left(1+p, -\frac{3(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}}{c^3}$$

```
[In] Integrate[x^2*(a + b*Log[c*x])^p,x]
[Out] (3^(-1 - p)*Gamma[1 + p, (-3*(a + b*Log[c*x]))/b]*(a + b*Log[c*x])^p)/(c^3*
E^((3*a)/b)*(-((a + b*Log[c*x])/b))^p)
```

Maple [F]

$$\int x^2(a+b\ln(xc))^p\,dx$$

```
[In] int(x^2*(a+b*ln(x*c))^p,x)
[Out] int(x^2*(a+b*ln(x*c))^p,x)
```

Fricas [F]

$$\int x^2(a+b\log(cx))^p\,dx = \int (b\log(cx)+a)^p x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x) + a)^p*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\log(cx))^p\,dx = \int x^2(a+b\log{(cx)})^p\,dx$$

```
[In] integrate(x**2*(a+b*ln(c*x))**p,x)
[Out] Integral(x**2*(a + b*log(c*x))**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int x^2(a+b\log(cx))^p\,dx = -\frac{(b\log{(cx)}+a)^{p+1}e^{\left(-\frac{3a}{b}\right)}E_{-p}\left(-\frac{3(b\log(cx)+a)}{b}\right)}{bc^3}$$

```
[In] integrate(x^2*(a+b*log(c*x))^p,x, algorithm="maxima")
[Out] -(b*log(c*x) + a)^(p + 1)*e^(-3*a/b)*exp_integral_e(-p, -3*(b*log(c*x) + a)
/b)/(b*c^3)
```

Giac [F]

$$\int x^2(a+b\log(cx))^p\,dx = \int (b\log{(cx)}+a)^p x^2\,dx$$

```
[In] integrate(x^2*(a+b*log(c*x))^p,x, algorithm="giac")
[Out] integrate((b*log(c*x) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\log(cx))^p\,dx = \int x^2\,{\left(a+b\,\ln\left(c\,x\right)\right)}^p\,dx$$

```
[In] int(x^2*(a + b*log(c*x))^p,x)
[Out] int(x^2*(a + b*log(c*x))^p, x)
```

3.177 $\int x(a+b\log(cx))^p\,dx$

Optimal result

Integrand size = 12, antiderivative size = 63

$$\int x(a+b\log(cx))^p\,dx=\frac{2^{-1-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}}{c^2}$$

```
[Out] 2^(-1-p)*GAMMA(p+1,-2*(a+b*ln(c*x))/b)*(a+b*ln(c*x))^p/c^2/exp(2*a/b)/(((-a
-b*ln(c*x))/b)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2346, 2212}

$$\int x(a+b\log(cx))^p\,dx=\frac{2^{-p-1}e^{-\frac{2a}{b}}(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2(a+b\log(cx))}{b}\right)}{c^2}$$

```
[In] Int[x*(a + b*Log[c*x])^p,x]
```

```
[Out] (2^(-1 - p)*Gamma[1 + p, (-2*(a + b*Log[c*x]))/b]*(a + b*Log[c*x])^p)/(c^2*
E^((2*a)/b)*(-((a + b*Log[c*x])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log(cx)\right)}{c^2}\\ &= \frac{2^{-1-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}}{c^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int x(a+b\log(cx))^p\,dx = \frac{2^{-1-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}}{c^2}$$

```
[In] Integrate[x*(a + b*Log[c*x])^p,x]
[Out] (2^(-1 - p)*Gamma[1 + p, (-2*(a + b*Log[c*x]))/b]*(a + b*Log[c*x])^p)/(c^2*
E^((2*a)/b)*(-((a + b*Log[c*x])/b))^p)
```

Maple [F]

$$\int x(a+b\ln(xc))^p\,dx$$

```
[In] int(x*(a+b*ln(x*c))^p,x)
[Out] int(x*(a+b*ln(x*c))^p,x)
```

Fricas [F]

$$\int x(a+b\log(cx))^p\,dx = \int (b\log(cx)+a)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*x))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x) + a)^p*x, x)
```

Sympy [F]

$$\int x(a+b\log(cx))^p\,dx = \int x(a+b\log{(cx)})^p\;dx$$

```
[In] integrate(x*(a+b*ln(c*x))**p,x)
[Out] Integral(x*(a + b*log(c*x))**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int x(a+b\log(cx))^p\,dx = -\frac{(b\log{(cx)}+a)^{p+1}e^{\left(-\frac{2a}{b}\right)}E_{-p}\left(-\frac{2\,(b\log(cx)+a)}{b}\right)}{bc^2}$$

```
[In] integrate(x*(a+b*log(c*x))^p,x, algorithm="maxima")
[Out] -(b*log(c*x) + a)^(p + 1)*e^(-2*a/b)*exp_integral_e(-p, -2*(b*log(c*x) + a)
/b)/(b*c^2)
```

Giac [F]

$$\int x(a+b\log(cx))^p\,dx = \int (b\log{(cx)}+a)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*x))^p,x, algorithm="giac")
[Out] integrate((b*log(c*x) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\log(cx))^p\,dx = \int x\,{\left(a+b\,\ln{\left(c\,x\right)}\right)}^p\,dx$$

```
[In] int(x*(a + b*log(c*x))^p,x)
[Out] int(x*(a + b*log(c*x))^p, x)
```

3.178 $\int (a+b\log(cx))^p\,dx$

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int (a+b\log(cx))^p\,dx=\frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log(cx)}{b}\right)(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}}{c}$$

```
[Out] GAMMA(p+1,(-a-b*ln(c*x))/b)*(a+b*ln(c*x))^p/c/exp(a/b)/(((-a-b*ln(c*x))/b)^
p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2336, 2212}

$$\int (a+b\log(cx))^p\,dx=\frac{e^{-\frac{a}{b}}(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(cx)}{b}\right)}{c}$$

```
[In] Int[(a + b*Log[c*x])^p,x]
```

```
[Out] (Gamma[1 + p, -((a + b*Log[c*x])/b)]*(a + b*Log[c*x])^p)/(c*E^(a/b)*(-((a +
 b*Log[c*x])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log(cx)\right)}{c}\\ &= \frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log(cx)}{b}\right)(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}}{c}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (a+b\log(cx))^p\,dx = \frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log(cx)}{b}\right)(a+b\log(cx))^p\left(-\frac{a+b\log(cx)}{b}\right)^{-p}}{c}$$

```
[In] Integrate[(a + b*Log[c*x])^p,x]
[Out] (Gamma[1 + p, -((a + b*Log[c*x])/b)]*(a + b*Log[c*x])^p)/(c*E^(a/b)*(-((a +
 b*Log[c*x])/b))^p)
```

Maple [F]

$$\int (a+b\ln(xc))^p\,dx$$

```
[In] int((a+b*ln(x*c))^p,x)
[Out] int((a+b*ln(x*c))^p,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int (a+b\log(cx))^p\,dx = \frac{e^{\left(-\frac{bp\log\left(-\frac{1}{b}\right)+a}{b}\right)}\Gamma\left(p+1,-\frac{b\log(cx)+a}{b}\right)}{c}$$

```
[In] integrate((a+b*log(c*x))^p,x, algorithm="fricas")
[Out] e^(-(b*p*log(-1/b) + a)/b)*gamma(p + 1, -(b*log(c*x) + a)/b)/c
```

Sympy [F]

$$\int (a+b\log(cx))^p\,dx = \int (a+b\log{(cx)})^p\;dx$$

```
[In] integrate((a+b*ln(c*x))**p,x)
[Out] Integral((a + b*log(c*x))**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (a+b\log(cx))^p\,dx = -\frac{(b\log{(cx)}+a)^{p+1}e^{\left(-\frac{a}{b}\right)}E_{-p}\left(-\frac{b\log(cx)+a}{b}\right)}{bc}$$

```
[In] integrate((a+b*log(c*x))^p,x, algorithm="maxima")
[Out] -(b*log(c*x) + a)^(p + 1)*e^(-a/b)*exp_integral_e(-p, -(b*log(c*x) + a)/b)/
(b*c)
```

Giac [F]

$$\int (a+b\log(cx))^p\,dx = \int (b\log{(cx)}+a)^p\,dx$$

```
[In] integrate((a+b*log(c*x))^p,x, algorithm="giac")
[Out] integrate((b*log(c*x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\log(cx))^p\,dx = \int (a+b\,\ln{(c\,x)})^p\,dx$$

```
[In] int((a + b*log(c*x))^p,x)
[Out] int((a + b*log(c*x))^p, x)
```

3.179 $\int \frac{(a+b\log(cx))^p}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 21

$$\int \frac{(a+b\log(cx))^p}{x}\,dx = \frac{(a+b\log(cx))^{1+p}}{b(1+p)}$$

```
[Out] (a+b*ln(c*x))^(p+1)/b/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2339, 30}

$$\int \frac{(a+b\log(cx))^p}{x}\,dx = \frac{(a+b\log(cx))^{p+1}}{b(p+1)}$$

```
[In] Int[(a + b*Log[c*x])^p/x,x]
```

```
[Out] (a + b*Log[c*x])^(1 + p)/(b*(1 + p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^p\,dx, x, a+b\log(cx)\right)}{b}\\ &= \frac{(a+b\log(cx))^{1+p}}{b(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx))^p}{x}\,dx = \frac{(a+b\log(cx))^{1+p}}{b(1+p)}$$

```
[In] Integrate[(a + b*Log[c*x])^p/x,x]
[Out] (a + b*Log[c*x])^(1 + p)/(b*(1 + p))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{(a+b\ln(xc))^{p+1}}{b(p+1)}$	22
default	$\frac{(a+b\ln(xc))^{p+1}}{b(p+1)}$	22
parallelrisch	$\frac{\ln(xc)(a+b\ln(xc))^p b+a(a+b\ln(xc))^p}{b(p+1)}$	39
norman	$\frac{\ln(xc)\,\mathrm{e}^{p\ln(a+b\ln(xc))}}{p+1}+\frac{a\,\mathrm{e}^{p\ln(a+b\ln(xc))}}{b(p+1)}$	46

```
[In] int((a+b*ln(x*c))^p/x,x,method=_RETURNVERBOSE)
[Out] (a+b*ln(x*c))^(p+1)/b/(p+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\log(cx))^p}{x}\,dx = \frac{(b\log(cx)+a)(b\log(cx)+a)^p}{bp+b}$$

```
[In] integrate((a+b*log(c*x))^p/x,x, algorithm="fricas")
```

```
[Out] (b*log(c*x) + a)*(b*log(c*x) + a)^p/(b*p + b)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{(a+b\log(cx))^p}{x}\,dx = -\begin{cases} -a^p\log(x) & \text{for } b=0 \\ -\frac{\begin{cases} \frac{(a+b\log(cx))^{p+1}}{p+1} & \text{for } p\neq -1 \\ \log(a+b\log(cx)) & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln(c*x))**p/x,x)
```

```
[Out] -Piecewise((-a**p*log(x), Eq(b, 0)), (-Piecewise(((a + b*log(c*x))**(p + 1)
/(p + 1), Ne(p, -1)), (log(a + b*log(c*x)), True))/b, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx))^p}{x}\,dx = \frac{(b\log(cx)+a)^{p+1}}{b(p+1)}$$

```
[In] integrate((a+b*log(c*x))^p/x,x, algorithm="maxima")
```

```
[Out] (b*log(c*x) + a)^(p + 1)/(b*(p + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx))^p}{x}\,dx = \frac{(b\log(cx)+a)^{p+1}}{b(p+1)}$$

```
[In] integrate((a+b*log(c*x))^p/x,x, algorithm="giac")
[Out] (b*log(c*x) + a)^(p + 1)/(b*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx))^p}{x}\,dx = \frac{(a+b\,\ln(c\,x))^{p+1}}{b\,(p+1)}$$

```
[In] int((a + b*log(c*x))^p/x,x)
[Out] (a + b*log(c*x))^(p + 1)/(b*(p + 1))
```

3.180 $\int \frac{(a+b\log(cx))^p}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{(a+b\log(cx))^p}{x^2}\,dx = -ce^{a/b}\Gamma\left(1+p,\frac{a+b\log(cx)}{b}\right)(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}$$

```
[Out] -c*exp(a/b)*GAMMA(p+1,(a+b*ln(c*x))/b)*(a+b*ln(c*x))^p/(((a+b*ln(c*x))/b)^p
)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2346, 2212}

$$\int \frac{(a+b\log(cx))^p}{x^2}\,dx = -ce^{a/b}(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}\Gamma\left(p+1,\frac{a+b\log(cx)}{b}\right)$$

```
[In] Int[(a + b*Log[c*x])^p/x^2,x]
```

```
[Out] -((c*E^(a/b)*Gamma[1 + p, (a + b*Log[c*x])/b]*(a + b*Log[c*x])^p)/((a + b*L
og[c*x])/b)^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\text{Subst}\left(\int e^{-x}(a+bx)^p\,dx, x, \log(cx)\right)\\ &= -ce^{a/b}\Gamma\left(1+p,\frac{a+b\log(cx)}{b}\right)(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\log(cx))^p}{x^2}\,dx = -ce^{a/b}\Gamma\left(1+p,\frac{a}{b}+\log(cx)\right)\left(\frac{a}{b}+\log(cx)\right)^{-p}(a+b\log(cx))^p$$

```
[In] Integrate[(a + b*Log[c*x])^p/x^2,x]
[Out] -((c*E^(a/b)*Gamma[1 + p, a/b + Log[c*x]]*(a + b*Log[c*x])^p)/(a/b + Log[c*
x])^p)
```

Maple [F]

$$\int \frac{(a+b\ln(xc))^p}{x^2}dx$$

```
[In] int((a+b*ln(x*c))^p/x^2,x)
[Out] int((a+b*ln(x*c))^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx))^p}{x^2}\,dx = \int \frac{(b\log(cx)+a)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*x) + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx))^p}{x^2}\,dx = \int \frac{(a+b\log{(cx)})^p}{x^2}\,dx$$

```
[In] integrate((a+b*ln(c*x))**p/x**2,x)
[Out] Integral((a + b*log(c*x))**p/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{(a+b\log(cx))^p}{x^2}\,dx = -\frac{(b\log{(cx)}+a)^{p+1}ce^{\frac{a}{b}}E_{-p}\left(\frac{b\log(cx)+a}{b}\right)}{b}$$

```
[In] integrate((a+b*log(c*x))^p/x^2,x, algorithm="maxima")
[Out] -(b*log(c*x) + a)^(p + 1)*c*e^(a/b)*exp_integral_e(-p, (b*log(c*x) + a)/b)/
b
```

Giac [F]

$$\int \frac{(a+b\log(cx))^p}{x^2}\,dx = \int \frac{(b\log{(cx)}+a)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*x) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx))^p}{x^2}\,dx = \int \frac{(a+b\,\ln{(c\,x)})^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*x))^p/x^2,x)
[Out] int((a + b*log(c*x))^p/x^2, x)
```

3.181 $\int \frac{(a+b\log(cx))^p}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{(a+b\log(cx))^p}{x^3}\,dx = -2^{-1-p}c^2e^{\frac{2a}{b}}\Gamma\left(1+p,\frac{2(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}$$

```
[Out] -2^(-1-p)*c^2*exp(2*a/b)*GAMMA(p+1,2*(a+b*ln(c*x))/b)*(a+b*ln(c*x))^p/(((a+
b*ln(c*x))/b)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2346, 2212}

$$\int \frac{(a+b\log(cx))^p}{x^3}\,dx = c^2\left(-2^{-p-1}\right)e^{\frac{2a}{b}}(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(cx))}{b}\right)$$

```
[In] Int[(a + b*Log[c*x])^p/x^3,x]
```

```
[Out] -((2^(-1 - p)*c^2*E^((2*a)/b)*Gamma[1 + p, (2*(a + b*Log[c*x]))/b]*(a + b*L
og[c*x])^p)/((a + b*Log[c*x])/b)^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c^2\text{Subst}\left(\int e^{-2x}(a+bx)^p\,dx, x, \log(cx)\right)\\ &= -2^{-1-p}c^2e^{\frac{2a}{b}}\Gamma\left(1+p, \frac{2(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx))^p}{x^3}\,dx = -2^{-1-p}c^2e^{\frac{2a}{b}}\Gamma\left(1+p, \frac{2(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}$$

```
[In] Integrate[(a + b*Log[c*x])^p/x^3,x]
[Out] -((2^(-1 - p)*c^2*E^((2*a)/b)*Gamma[1 + p, (2*(a + b*Log[c*x]))/b]*(a + b*L
og[c*x])^p)/((a + b*Log[c*x])/b)^p)
```

Maple [F]

$$\int \frac{(a+b\ln(xc))^p}{x^3}dx$$

```
[In] int((a+b*ln(x*c))^p/x^3,x)
[Out] int((a+b*ln(x*c))^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx))^p}{x^3}\,dx = \int \frac{(b\log(cx)+a)^p}{x^3}\,dx$$

[In] integrate((a+b*log(c*x))^p/x^3,x, algorithm="fricas")

[Out] integral((b*log(c*x) + a)^p/x^3, x)

Sympy [F]

$$\int \frac{(a+b\log(cx))^p}{x^3}\,dx = \int \frac{(a+b\log(cx))^p}{x^3}\,dx$$

[In] integrate((a+b*ln(c*x))**p/x**3,x)

[Out] Integral((a + b*log(c*x))**p/x**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{(a+b\log(cx))^p}{x^3}\,dx = -\frac{(b\log(cx)+a)^{p+1}c^2e^{\left(\frac{2a}{b}\right)}E_{-p}\left(\frac{2(b\log(cx)+a)}{b}\right)}{b}$$

[In] integrate((a+b*log(c*x))^p/x^3,x, algorithm="maxima")

[Out] -(b*log(c*x) + a)^(p + 1)*c^2*e^(2*a/b)*exp_integral_e(-p, 2*(b*log(c*x) + a)/b)/b

Giac [F]

$$\int \frac{(a+b\log(cx))^p}{x^3}\,dx = \int \frac{(b\log(cx)+a)^p}{x^3}\,dx$$

[In] integrate((a+b*log(c*x))^p/x^3,x, algorithm="giac")

[Out] integrate((b*log(c*x) + a)^p/x^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx))^p}{x^3}\,dx = \int \frac{(a+b\,\ln(c\,x))^p}{x^3}\,dx$$

```
[In] int((a + b*log(c*x))^p/x^3,x)
[Out] int((a + b*log(c*x))^p/x^3, x)
```

3.182 $\int \frac{(a+b\log(cx))^p}{x^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{(a+b\log(cx))^p}{x^4}\,dx = -3^{-1-p}c^3e^{\frac{3a}{b}}\Gamma\left(1+p,\frac{3(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}$$

```
[Out] -3^(-1-p)*c^3*exp(3*a/b)*GAMMA(p+1,3*(a+b*ln(c*x))/b)*(a+b*ln(c*x))^p/(((a+
b*ln(c*x))/b)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2346, 2212}

$$\int \frac{(a+b\log(cx))^p}{x^4}\,dx = c^3\left(-3^{-p-1}\right)e^{\frac{3a}{b}}(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}\Gamma\left(p+1,\frac{3(a+b\log(cx))}{b}\right)$$

```
[In] Int[(a + b*Log[c*x])^p/x^4,x]
```

```
[Out] -((3^(-1 - p)*c^3*E^((3*a)/b)*Gamma[1 + p, (3*(a + b*Log[c*x]))/b]*(a + b*L
og[c*x])^p)/((a + b*Log[c*x])/b)^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2346

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c^3\text{Subst}\left(\int e^{-3x}(a+bx)^p\,dx, x, \log(cx)\right)\\ &= -3^{-1-p}c^3e^{\frac{3a}{b}}\Gamma\left(1+p, \frac{3(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\log(cx))^p}{x^4}\,dx = -3^{-1-p}c^3e^{\frac{3a}{b}}\Gamma\left(1+p, \frac{3(a+b\log(cx))}{b}\right)(a+b\log(cx))^p\left(\frac{a+b\log(cx)}{b}\right)^{-p}$$

```
[In] Integrate[(a + b*Log[c*x])^p/x^4,x]
[Out] -((3^(-1 - p)*c^3*E^((3*a)/b)*Gamma[1 + p, (3*(a + b*Log[c*x]))/b]*(a + b*L
og[c*x])^p)/((a + b*Log[c*x])/b)^p)
```

Maple [F]

$$\int \frac{(a+b\ln(xc))^p}{x^4}dx$$

```
[In] int((a+b*ln(x*c))^p/x^4,x)
[Out] int((a+b*ln(x*c))^p/x^4,x)
```

Fricas [F]

$$\int \frac{(a+b\log(cx))^p}{x^4}\,dx = \int \frac{(b\log(cx)+a)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*x))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log(c*x) + a)^p/x^4, x)
```

Sympy [F]

$$\int \frac{(a+b\log(cx))^p}{x^4}\,dx = \int \frac{(a+b\log(cx))^p}{x^4}\,dx$$

```
[In] integrate((a+b*ln(c*x))**p/x**4,x)
[Out] Integral((a + b*log(c*x))**p/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{(a+b\log(cx))^p}{x^4}\,dx = -\frac{(b\log(cx)+a)^{p+1}c^3e^{\left(\frac{3a}{b}\right)}E_{-p}\left(\frac{3(b\log(cx)+a)}{b}\right)}{b}$$

```
[In] integrate((a+b*log(c*x))^p/x^4,x, algorithm="maxima")
[Out] -(b*log(c*x) + a)^(p + 1)*c^3*e^(3*a/b)*exp_integral_e(-p, 3*(b*log(c*x) +
a)/b)/b
```

Giac [F]

$$\int \frac{(a+b\log(cx))^p}{x^4}\,dx = \int \frac{(b\log(cx)+a)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*x))^p/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*x) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\log(cx))^p}{x^4}\,dx = \int \frac{(a+b\,\ln(c\,x))^p}{x^4}\,dx$$

```
[In] int((a + b*log(c*x))^p/x^4,x)
[Out] int((a + b*log(c*x))^p/x^4, x)
```

3.183 $\int (dx)^m \left(a+b\log\left(c\sqrt{x}\right)\right)^p\, dx$

Optimal result

Integrand size = 20, antiderivative size = 107

$$\int (dx)^m \left(a+b\log\left(c\sqrt{x}\right)\right)^p\, dx = \frac{2^{-p}e^{-\frac{2a(1+m)}{b}}\left(c\sqrt{x}\right)^{-2(1+m)}(dx)^{1+m}\Gamma\left(1+p,-\frac{2(1+m)\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{(1+m)\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)}{d(1+m)}$$

```
[Out] (d*x)^(1+m)*GAMMA(p+1,-2*(1+m)*(a+b*ln(c*x^(1/2)))/b)*(a+b*ln(c*x^(1/2)))^p
/(2^p)/d/exp(2*a*(1+m)/b)/(1+m)/((-(1+m)*(a+b*ln(c*x^(1/2)))/b)^p)/((c*x^(1
/2))^(2+2*m))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2347, 2212}

$$\int (dx)^m \left(a+b\log\left(c\sqrt{x}\right)\right)^p\, dx = \frac{2^{-p}e^{-\frac{2a(m+1)}{b}}\left(c\sqrt{x}\right)^{-2(m+1)}(dx)^{m+1}\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{(m+1)\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2(m+1)\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)}{d(m+1)}$$

```
[In] Int[(d*x)^m*(a + b*Log[c*Sqrt[x]])^p,x]
```

```
[Out] ((d*x)^(1 + m)*Gamma[1 + p, (-2*(1 + m)*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*L
og[c*Sqrt[x]])^p)/(2^p*d*E^((2*a*(1 + m))/b)*(1 + m)*(c*Sqrt[x])^(2*(1 + m)
)*(-(((1 + m)*(a + b*Log[c*Sqrt[x]]))/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(2\left(c\sqrt{x}\right)^{-2(1+m)}(dx)^{1+m}\right)\text{Subst}\left(\int e^{2(1+m)x}(a+bx)^p\,dx, x, \log\left(c\sqrt{x}\right)\right)}{d}$$

$$= \frac{2^{-p}e^{-\frac{2a(1+m)}{b}}\left(c\sqrt{x}\right)^{-2(1+m)}(dx)^{1+m}\Gamma\left(1+p, -\frac{2(1+m)\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{(1+m)(a+b}{b}\right.}{d(1+m)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

$$\int (dx)^m\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx = \frac{2^{-p}e^{-\frac{2a(1+m)}{b}}\left(c\sqrt{x}\right)^{-2m}(dx)^m\Gamma\left(1+p, -\frac{2(1+m)\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{(1+m)\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)^{-p}}{c^2(1+m)}$$

```
[In] Integrate[(d*x)^m*(a + b*Log[c*Sqrt[x]])^p,x]
```

```
[Out] ((d*x)^m*Gamma[1 + p, (-2*(1 + m)*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*S
qrt[x]])^p)/(2^p*c^2*E^((2*a*(1 + m))/b)*(1 + m)*(c*Sqrt[x])^(2*m)*(-(((1 +
 m)*(a + b*Log[c*Sqrt[x]]))/b))^p)
```

Maple [F]

$$\int (dx)^m \left(a+b\ln\left(c\sqrt{x}\right)\right)^p dx$$

```
[In] int((d*x)^m*(a+b*ln(c*x^(1/2)))^p,x)
[Out] int((d*x)^m*(a+b*ln(c*x^(1/2)))^p,x)
```

Fricas [F]

$$\int (dx)^m \left(a+b\log\left(c\sqrt{x}\right)\right)^p\, dx = \int (dx)^m \left(b\log\left(c\sqrt{x}\right)+a\right)^p dx$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^(1/2)))^p,x, algorithm="fricas")
[Out] integral((d*x)^m*(b*log(c*sqrt(x)) + a)^p, x)
```

Sympy [F]

$$\int (dx)^m \left(a+b\log\left(c\sqrt{x}\right)\right)^p\, dx = \int (dx)^m \left(a+b\log\left(c\sqrt{x}\right)\right)^p\, dx$$

```
[In] integrate((d*x)**m*(a+b*ln(c*x**(1/2)))**p,x)
[Out] Integral((d*x)**m*(a + b*log(c*sqrt(x)))**p, x)
```

Maxima [F]

$$\int (dx)^m \left(a+b\log\left(c\sqrt{x}\right)\right)^p\, dx = \int (dx)^m \left(b\log\left(c\sqrt{x}\right)+a\right)^p dx$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^(1/2)))^p,x, algorithm="maxima")
[Out] integrate((d*x)^m*(b*log(c*sqrt(x)) + a)^p, x)
```

Giac [F]

$$\int (dx)^m \left(a + b\log\left(c\sqrt{x}\right)\right)^p \, dx = \int (dx)^m \left(b\log\left(c\sqrt{x}\right) + a\right)^p \, dx$$

```
[In] integrate((d*x)^m*(a+b*log(c*x^(1/2)))^p,x, algorithm="giac")
[Out] integrate((d*x)^m*(b*log(c*sqrt(x)) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a + b\log\left(c\sqrt{x}\right)\right)^p \, dx = \int (d\,x)^m \left(a + b\,\ln\left(c\,\sqrt{x}\right)\right)^p \, dx$$

```
[In] int((d*x)^m*(a + b*log(c*x^(1/2)))^p,x)
[Out] int((d*x)^m*(a + b*log(c*x^(1/2)))^p, x)
```

3.184 $\int x^2 \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int x^2 \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx = \frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}}{c^6}$$

```
[Out] 3^(-1-p)*GAMMA(p+1,-6*(a+b*ln(c*x^(1/2)))/b)*(a+b*ln(c*x^(1/2)))^p/(2^p)/c^
6/exp(6*a/b)/(((-a-b*ln(c*x^(1/2)))/b)^p)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2347, 2212}

$$\int x^2 \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx = \frac{2^{-p}3^{-p-1}e^{-\frac{6a}{b}}\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{6\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)}{c^6}$$

```
[In] Int[x^2*(a + b*Log[c*Sqrt[x]])^p,x]
```

```
[Out] (3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*Sqrt[x
]])^p)/(2^p*c^6*E^((6*a)/b)*(-((a + b*Log[c*Sqrt[x]])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int e^{6x}(a+bx)^p\,dx, x, \log\left(c\sqrt{x}\right)\right)}{c^6}\\ &= \frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}}{c^6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx = \frac{2^{-p}3^{-1-p}e^{-\frac{6a}{b}}\Gamma\left(1+p,-\frac{6\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}}{c^6}$$

```
[In] Integrate[x^2*(a + b*Log[c*Sqrt[x]])^p,x]
[Out] (3^(-1 - p)*Gamma[1 + p, (-6*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*Sqrt[x
]])^p)/(2^p*c^6*E^((6*a)/b)*(-((a + b*Log[c*Sqrt[x]])/b))^p)
```

Maple [F]

$$\int x^2\left(a+b\ln\left(c\sqrt{x}\right)\right)^p\,dx$$

```
[In] int(x^2*(a+b*ln(c*x^(1/2)))^p,x)
[Out] int(x^2*(a+b*ln(c*x^(1/2)))^p,x)
```

Fricas [F]

$$\int x^2 \left(a + b\log\left(c\sqrt{x}\right)\right)^p \, dx = \int \left(b\log\left(c\sqrt{x}\right) + a\right)^p x^2 \, dx$$

```
[In] integrate(x^2*(a+b*log(c*x^(1/2)))^p,x, algorithm="fricas")
[Out] integral((b*log(c*sqrt(x)) + a)^p*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a + b\log\left(c\sqrt{x}\right)\right)^p \, dx = \int x^2 \left(a + b\log\left(c\sqrt{x}\right)\right)^p \, dx$$

```
[In] integrate(x**2*(a+b*ln(c*x**(1/2)))**p,x)
[Out] Integral(x**2*(a + b*log(c*sqrt(x)))**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int x^2 \left(a + b\log\left(c\sqrt{x}\right)\right)^p \, dx = -\frac{2 \left(b\log\left(c\sqrt{x}\right) + a\right)^{p+1} e^{\left(-\frac{6a}{b}\right)} E_{-p}\left(-\frac{6 \left(b\log\left(c\sqrt{x}\right)+a\right)}{b}\right)}{bc^6}$$

```
[In] integrate(x^2*(a+b*log(c*x^(1/2)))^p,x, algorithm="maxima")
[Out] -2*(b*log(c*sqrt(x)) + a)^(p + 1)*e^(-6*a/b)*exp_integral_e(-p, -6*(b*log(c
*sqrt(x)) + a)/b)/(b*c^6)
```

Giac [F]

$$\int x^2 \left(a + b\log\left(c\sqrt{x}\right)\right)^p \, dx = \int \left(b\log\left(c\sqrt{x}\right) + a\right)^p x^2 \, dx$$

```
[In] integrate(x^2*(a+b*log(c*x^(1/2)))^p,x, algorithm="giac")
[Out] integrate((b*log(c*sqrt(x)) + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + b \log (c\sqrt{x}))^p \, dx = \int x^2 \left(a + b \ln \left(c \sqrt{x}\right)\right)^p \, dx$$

```
[In] int(x^2*(a + b*log(c*x^(1/2)))^p,x)
[Out] int(x^2*(a + b*log(c*x^(1/2)))^p, x)
```

3.185 $\int x\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int x\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx = \frac{2^{-1-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p,-\frac{4\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}}{c^4}$$

```
[Out] 2^(-1-2*p)*GAMMA(p+1,-4*(a+b*ln(c*x^(1/2)))/b)*(a+b*ln(c*x^(1/2)))^p/c^4/ex
p(4*a/b)/(((-a-b*ln(c*x^(1/2)))/b)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2347, 2212}

$$\int x\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx = \frac{2^{-2p-1}e^{-\frac{4a}{b}}\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{4\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)}{c^4}$$

```
[In] Int[x*(a + b*Log[c*Sqrt[x]])^p,x]
```

```
[Out] (2^(-1 - 2*p)*Gamma[1 + p, (-4*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*Sqrt
[x]])^p)/(c^4*E^((4*a)/b)*(-((a + b*Log[c*Sqrt[x]])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int e^{4x}(a+bx)^p\,dx, x, \log\left(c\sqrt{x}\right)\right)}{c^4}$$

$$= \frac{2^{-1-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p, -\frac{4\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}}{c^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx$$

$$= \frac{2^{-1-2p}e^{-\frac{4a}{b}}\Gamma\left(1+p, -\frac{4\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}}{c^4}$$

```
[In] Integrate[x*(a + b*Log[c*Sqrt[x]])^p,x]
[Out] (2^(-1 - 2*p)*Gamma[1 + p, (-4*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*Sqrt
[x]])^p)/(c^4*E^((4*a)/b)*(-((a + b*Log[c*Sqrt[x]])/b))^p)
```

Maple [F]

$$\int x\left(a+b\ln\left(c\sqrt{x}\right)\right)^p dx$$

```
[In] int(x*(a+b*ln(c*x^(1/2)))^p,x)
[Out] int(x*(a+b*ln(c*x^(1/2)))^p,x)
```

Fricas [F]

$$\int x\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx=\int\left(b\log\left(c\sqrt{x}\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*x^(1/2)))^p,x, algorithm="fricas")
[Out] integral((b*log(c*sqrt(x)) + a)^p*x, x)
```

Sympy [F]

$$\int x\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx=\int x\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx$$

```
[In] integrate(x*(a+b*ln(c*x**(1/2)))**p,x)
[Out] Integral(x*(a + b*log(c*sqrt(x)))**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int x\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx=-\frac{2\left(b\log\left(c\sqrt{x}\right)+a\right)^{p+1}e^{\left(-\frac{4a}{b}\right)}E_{-p}\left(-\frac{4\left(b\log\left(c\sqrt{x}\right)+a\right)}{b}\right)}{bc^4}$$

```
[In] integrate(x*(a+b*log(c*x^(1/2)))^p,x, algorithm="maxima")
[Out] -2*(b*log(c*sqrt(x)) + a)^(p + 1)*e^(-4*a/b)*exp_integral_e(-p, -4*(b*log(c
*sqrt(x)) + a)/b)/(b*c^4)
```

Giac [F]

$$\int x\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx=\int\left(b\log\left(c\sqrt{x}\right)+a\right)^p x\,dx$$

```
[In] integrate(x*(a+b*log(c*x^(1/2)))^p,x, algorithm="giac")
[Out] integrate((b*log(c*sqrt(x)) + a)^p*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx=\int x\,\left(a+b\,\ln\left(c\,\sqrt{x}\right)\right)^p\,dx$$

```
[In] int(x*(a + b*log(c*x^(1/2)))^p,x)
[Out] int(x*(a + b*log(c*x^(1/2)))^p, x)
```

3.186 $\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx$

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx = \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}}{c^2}$$

```
[Out] GAMMA(p+1,-2*(a+b*ln(c*x^(1/2)))/b)*(a+b*ln(c*x^(1/2)))^p/(2^p)/c^2/exp(2*a
/b)/(((-a-b*ln(c*x^(1/2)))/b)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2336, 2212}

$$\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx = \frac{2^{-p}e^{-\frac{2a}{b}}\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{2\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)}{c^2}$$

```
[In] Int[(a + b*Log[c*Sqrt[x]])^p,x]
```

```
[Out] (Gamma[1 + p, (-2*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*Sqrt[x]])^p)/(2^p
*c^2*E^((2*a)/b)*(-((a + b*Log[c*Sqrt[x]])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2336

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int e^{2x}(a+bx)^p\,dx, x, \log\left(c\sqrt{x}\right)\right)}{c^2}\\ &= \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2(a+b\log(c\sqrt{x}))}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log(c\sqrt{x})}{b}\right)^{-p}}{c^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx = \frac{2^{-p}e^{-\frac{2a}{b}}\Gamma\left(1+p,-\frac{2(a+b\log(c\sqrt{x}))}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(-\frac{a+b\log(c\sqrt{x})}{b}\right)^{-p}}{c^2}$$

```
[In] Integrate[(a + b*Log[c*Sqrt[x]])^p,x]
[Out] (Gamma[1 + p, (-2*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*Sqrt[x]])^p)/(2^p
*c^2*E^((2*a)/b)*(-((a + b*Log[c*Sqrt[x]])/b))^p)
```

Maple [F]

$$\int \left(a+b\ln\left(c\sqrt{x}\right)\right)^p\,dx$$

```
[In] int((a+b*ln(c*x^(1/2)))^p,x)
[Out] int((a+b*ln(c*x^(1/2)))^p,x)
```

Fricas [F]

$$\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx=\int \left(b\log\left(c\sqrt{x}\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p,x, algorithm="fricas")
[Out] integral((b*log(c*sqrt(x)) + a)^p, x)
```

Sympy [F]

$$\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx=\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx$$

```
[In] integrate((a+b*ln(c*x**(1/2)))**p,x)
[Out] Integral((a + b*log(c*sqrt(x)))**p, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx=-\frac{2\left(b\log\left(c\sqrt{x}\right)+a\right)^{p+1}e^{\left(-\frac{2a}{b}\right)}E_{-p}\left(-\frac{2\left(b\log\left(c\sqrt{x}\right)+a\right)}{b}\right)}{bc^2}$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p,x, algorithm="maxima")
[Out] -2*(b*log(c*sqrt(x)) + a)^(p + 1)*e^(-2*a/b)*exp_integral_e(-p, -2*(b*log(c
*sqrt(x)) + a)/b)/(b*c^2)
```

Giac [F]

$$\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx=\int \left(b\log\left(c\sqrt{x}\right)+a\right)^p\,dx$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p,x, algorithm="giac")
[Out] integrate((b*log(c*sqrt(x)) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\log\left(c\sqrt{x}\right)\right)^p\,dx=\int \left(a+b\,\ln\left(c\,\sqrt{x}\right)\right)^p\,dx$$

```
[In] int((a + b*log(c*x^(1/2)))^p,x)
[Out] int((a + b*log(c*x^(1/2)))^p, x)
```

3.187 $\int \frac{(a+b\log(c\sqrt{x}))^p}{x} dx$

Optimal result

Integrand size = 18, antiderivative size = 26

$$\int \frac{(a+b\log(c\sqrt{x}))^p}{x} dx = \frac{2(a+b\log(c\sqrt{x}))^{1+p}}{b(1+p)}$$

```
[Out] 2*(a+b*ln(c*x^(1/2)))^(p+1)/b/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2339, 30}

$$\int \frac{(a+b\log(c\sqrt{x}))^p}{x} dx = \frac{2(a+b\log(c\sqrt{x}))^{p+1}}{b(p+1)}$$

```
[In] Int[(a + b*Log[c*Sqrt[x]])^p/x,x]
```

```
[Out] (2*(a + b*Log[c*Sqrt[x]])^(1 + p))/(b*(1 + p))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2339

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{Subst}\left(\int x^p\,dx, x, a+b\log\left(c\sqrt{x}\right)\right)}{b}\\ &= \frac{2\left(a+b\log\left(c\sqrt{x}\right)\right)^{1+p}}{b(1+p)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x}\,dx = \frac{2\left(a+b\log\left(c\sqrt{x}\right)\right)^{1+p}}{b(1+p)}$$

```
[In] Integrate[(a + b*Log[c*Sqrt[x]])^p/x,x]
[Out] (2*(a + b*Log[c*Sqrt[x]])^(1 + p))/(b*(1 + p))
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{2\left(a+b\ln\left(c\sqrt{x}\right)\right)^{p+1}}{b(p+1)}$	25
default	$\frac{2\left(a+b\ln\left(c\sqrt{x}\right)\right)^{p+1}}{b(p+1)}$	25

```
[In] int((a+b*ln(c*x^(1/2)))^p/x,x,method=_RETURNVERBOSE)
[Out] 2*(a+b*ln(c*x^(1/2)))^(p+1)/b/(p+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x}\,dx = \frac{2\left(b\log\left(c\sqrt{x}\right)+a\right)\left(b\log\left(c\sqrt{x}\right)+a\right)^p}{bp+b}$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x,x, algorithm="fricas")
[Out] 2*(b*log(c*sqrt(x)) + a)*(b*log(c*sqrt(x)) + a)^p/(b*p + b)
```

Sympy [A] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x}\,dx = -\begin{cases} -a^p\log(x) & \text{for } b=0 \\ -\frac{2\left(\begin{cases}\frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^{p+1}}{p+1} & \text{for } p\neq -1 \\ \log\left(a+b\log\left(c\sqrt{x}\right)\right) & \text{otherwise}\end{cases}\right)}{b} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*ln(c*x**(1/2)))**p/x,x)
```

```
[Out] -Piecewise((-a**p*log(x), Eq(b, 0)), (-2*Piecewise(((a + b*log(c*sqrt(x)))*
*(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*log(c*sqrt(x))), True))/b, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x}\,dx = \frac{2\left(b\log\left(c\sqrt{x}\right)+a\right)^{p+1}}{b(p+1)}$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x,x, algorithm="maxima")
```

```
[Out] 2*(b*log(c*sqrt(x)) + a)^(p + 1)/(b*(p + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x}\,dx = \frac{2\left(b\log(c)+\frac{1}{2}\,b\log(x)+a\right)^{p+1}}{b(p+1)}$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x,x, algorithm="giac")
```

```
[Out] 2*(b*log(c) + 1/2*b*log(x) + a)^(p + 1)/(b*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x}\,dx = \frac{2\left(a+b\,\ln\left(c\,\sqrt{x}\right)\right)^{p+1}}{b\,\left(p+1\right)}$$

```
[In] int((a + b*log(c*x^(1/2)))^p/x,x)
[Out] (2*(a + b*log(c*x^(1/2)))^(p + 1))/(b*(p + 1))
```

3.188 $\int \frac{(a+b\log(c\sqrt{x}))^p}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 73

$$\int \frac{(a+b\log(c\sqrt{x}))^p}{x^2}\,dx = -2^{-p}c^2e^{\frac{2a}{b}}\Gamma\left(1+p,\frac{2(a+b\log(c\sqrt{x}))}{b}\right)(a+b\log(c\sqrt{x}))^p\left(\frac{a+b\log(c\sqrt{x})}{b}\right)^{-p}$$

```
[Out] -c^2*exp(2*a/b)*GAMMA(p+1,2*(a+b*ln(c*x^(1/2)))/b)*(a+b*ln(c*x^(1/2)))^p/(2
^p)/(((a+b*ln(c*x^(1/2)))/b)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2347, 2212}

$$\int \frac{(a+b\log(c\sqrt{x}))^p}{x^2}\,dx = c^2\left(-2^{-p}\right)e^{\frac{2a}{b}}(a+b\log(c\sqrt{x}))^p\left(\frac{a+b\log(c\sqrt{x})}{b}\right)^{-p}\Gamma\left(p+1,\frac{2(a+b\log(c\sqrt{x}))}{b}\right)$$

```
[In] Int[(a + b*Log[c*Sqrt[x]])^p/x^2,x]
```

```
[Out] -((c^2*E^((2*a)/b)*Gamma[1 + p, (2*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*
Sqrt[x]])^p)/(2^p*((a + b*Log[c*Sqrt[x]])/b)^p))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (2c^2)\,\text{Subst}\left(\int e^{-2x}(a+bx)^p\,dx, x, \log\left(c\sqrt{x}\right)\right)\\ &= -2^{-p}c^2e^{\frac{2a}{b}}\Gamma\left(1+p, \frac{2\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^2}\,dx = -2^{-p}c^2e^{\frac{2a}{b}}\Gamma\left(1+p, \frac{2\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}$$

```
[In] Integrate[(a + b*Log[c*Sqrt[x]])^p/x^2,x]
[Out] -((c^2*E^((2*a)/b)*Gamma[1 + p, (2*(a + b*Log[c*Sqrt[x]]))/b]*(a + b*Log[c*
Sqrt[x]])^p)/(2^p*((a + b*Log[c*Sqrt[x]])/b)^p))
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\sqrt{x}\right)\right)^p}{x^2}dx$$

```
[In] int((a+b*ln(c*x^(1/2)))^p/x^2,x)
[Out] int((a+b*ln(c*x^(1/2)))^p/x^2,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\sqrt{x}\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x^2,x, algorithm="fricas")
[Out] integral((b*log(c*sqrt(x)) + a)^p/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*ln(c*x**(1/2)))**p/x**2,x)
[Out] Integral((a + b*log(c*sqrt(x)))**p/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^2}\,dx=-\frac{2\left(b\log\left(c\sqrt{x}\right)+a\right)^{p+1}c^2e^{\left(\frac{2a}{b}\right)}E_{-p}\left(\frac{2\left(b\log\left(c\sqrt{x}\right)+a\right)}{b}\right)}{b}$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x^2,x, algorithm="maxima")
[Out] -2*(b*log(c*sqrt(x)) + a)^(p + 1)*c^2*e^(2*a/b)*exp_integral_e(-p, 2*(b*log
(c*sqrt(x)) + a)/b)/b
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^2}\,dx=\int \frac{\left(b\log\left(c\sqrt{x}\right)+a\right)^p}{x^2}\,dx$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x^2,x, algorithm="giac")
[Out] integrate((b*log(c*sqrt(x)) + a)^p/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^2}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\sqrt{x}\right)\right)^p}{x^2}\,dx$$

```
[In] int((a + b*log(c*x^(1/2)))^p/x^2,x)
[Out] int((a + b*log(c*x^(1/2)))^p/x^2, x)
```

3.189 $\int \frac{(a+b\log(c\sqrt{x}))^p}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 75

$$\int \frac{(a+b\log(c\sqrt{x}))^p}{x^3} dx = -2^{-1-2p}c^4e^{\frac{4a}{b}}\Gamma\left(1+p,\frac{4(a+b\log(c\sqrt{x}))}{b}\right)(a+b\log(c\sqrt{x}))^p\left(\frac{a+b\log(c\sqrt{x})}{b}\right)^{-p}$$

```
[Out] -2^(-1-2*p)*c^4*exp(4*a/b)*GAMMA(p+1,4*(a+b*ln(c*x^(1/2)))/b)*(a+b*ln(c*x^(
1/2)))^p/(((a+b*ln(c*x^(1/2)))/b)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2347, 2212}

$$\int \frac{(a+b\log(c\sqrt{x}))^p}{x^3} dx = c^4\left(-2^{-2p-1}\right)e^{\frac{4a}{b}}(a+b\log(c\sqrt{x}))^p\left(\frac{a+b\log(c\sqrt{x})}{b}\right)^{-p}\Gamma\left(p+1,\frac{4(a+b\log(c\sqrt{x}))}{b}\right)$$

```
[In] Int[(a + b*Log[c*Sqrt[x]])^p/x^3,x]
```

```
[Out] -((2^(-1 - 2*p)*c^4*E^((4*a)/b)*Gamma[1 + p, (4*(a + b*Log[c*Sqrt[x]]))/b]*
(a + b*Log[c*Sqrt[x]])^p)/((a + b*Log[c*Sqrt[x]])/b)^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = (2c^4)\,\text{Subst}\left(\int e^{-4x}(a+bx)^p\,dx, x, \log\left(c\sqrt{x}\right)\right)$$

$$= -2^{-1-2p}c^4 e^{\frac{4a}{b}}\Gamma\left(1+p, \frac{4\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^3}\,dx = -2^{-1-2p}c^4 e^{\frac{4a}{b}}\Gamma\left(1+p, \frac{4\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}$$

```
[In] Integrate[(a + b*Log[c*Sqrt[x]])^p/x^3,x]
[Out] -((2^(-1 - 2*p)*c^4*E^((4*a)/b)*Gamma[1 + p, (4*(a + b*Log[c*Sqrt[x]]))/b]*
(a + b*Log[c*Sqrt[x]])^p)/((a + b*Log[c*Sqrt[x]])/b)^p)
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\sqrt{x}\right)\right)^p}{x^3}dx$$

```
[In] int((a+b*ln(c*x^(1/2)))^p/x^3,x)
[Out] int((a+b*ln(c*x^(1/2)))^p/x^3,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^3}\,dx=\int \frac{\left(b\log\left(c\sqrt{x}\right)+a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x^3,x, algorithm="fricas")
[Out] integral((b*log(c*sqrt(x)) + a)^p/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^3}\,dx=\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*ln(c*x**(1/2)))**p/x**3,x)
[Out] Integral((a + b*log(c*sqrt(x)))**p/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^3}\,dx=-\frac{2\left(b\log\left(c\sqrt{x}\right)+a\right)^{p+1}c^4e^{\left(\frac{4a}{b}\right)}E_{-p}\left(\frac{4\left(b\log\left(c\sqrt{x}\right)+a\right)}{b}\right)}{b}$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x^3,x, algorithm="maxima")
[Out] -2*(b*log(c*sqrt(x)) + a)^(p + 1)*c^4*e^(4*a/b)*exp_integral_e(-p, 4*(b*log
(c*sqrt(x)) + a)/b)/b
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^3}\,dx=\int \frac{\left(b\log\left(c\sqrt{x}\right)+a\right)^p}{x^3}\,dx$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x^3,x, algorithm="giac")
[Out] integrate((b*log(c*sqrt(x)) + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^3}\,dx = \int \frac{\left(a+b\,\ln\left(c\,\sqrt{x}\right)\right)^p}{x^3}\,dx$$

```
[In] int((a + b*log(c*x^(1/2)))^p/x^3,x)
[Out] int((a + b*log(c*x^(1/2)))^p/x^3, x)
```

3.190 $\int \frac{(a+b\log(c\sqrt{x}))^p}{x^4}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{(a+b\log(c\sqrt{x}))^p}{x^4}\,dx = -2^{-p}3^{-1-p}c^6e^{\frac{6a}{b}}\Gamma\left(1+p,\frac{6(a+b\log(c\sqrt{x}))}{b}\right)(a+b\log(c\sqrt{x}))^p\left(\frac{a+b\log(c\sqrt{x})}{b}\right)^{-p}$$

```
[Out] -3^(-1-p)*c^6*exp(6*a/b)*GAMMA(p+1,6*(a+b*ln(c*x^(1/2)))/b)*(a+b*ln(c*x^(1/
2)))^p/(2^p)/(((a+b*ln(c*x^(1/2)))/b)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2347, 2212}

$$\int \frac{(a+b\log(c\sqrt{x}))^p}{x^4}\,dx = c^6\left(-2^{-p}\right)3^{-p-1}e^{\frac{6a}{b}}(a+b\log(c\sqrt{x}))^p\left(\frac{a+b\log(c\sqrt{x})}{b}\right)^{-p}\Gamma\left(p+1,\frac{6(a+b\log(c\sqrt{x}))}{b}\right)$$

```
[In] Int[(a + b*Log[c*Sqrt[x]])^p/x^4,x]
```

```
[Out] -((3^(-1 - p)*c^6*E^((6*a)/b)*Gamma[1 + p, (6*(a + b*Log[c*Sqrt[x]]))/b]*(a
 + b*Log[c*Sqrt[x]])^p)/(2^p*((a + b*Log[c*Sqrt[x]])/b)^p))
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= (2c^6)\,\text{Subst}\left(\int e^{-6x}(a+bx)^p\,dx, x, \log\left(c\sqrt{x}\right)\right)\\ &= -2^{-p}3^{-1-p}c^6e^{\frac{6a}{b}}\Gamma\left(1+p, \frac{6\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^4}\,dx = -2^{-p}3^{-1-p}c^6e^{\frac{6a}{b}}\Gamma\left(1+p, \frac{6\left(a+b\log\left(c\sqrt{x}\right)\right)}{b}\right)\left(a+b\log\left(c\sqrt{x}\right)\right)^p\left(\frac{a+b\log\left(c\sqrt{x}\right)}{b}\right)^{-p}$$

```
[In] Integrate[(a + b*Log[c*Sqrt[x]])^p/x^4,x]
[Out] -((3^(-1 - p)*c^6*E^((6*a)/b)*Gamma[1 + p, (6*(a + b*Log[c*Sqrt[x]]))/b]*(a
 + b*Log[c*Sqrt[x]])^p)/(2^p*((a + b*Log[c*Sqrt[x]])/b)^p))
```

Maple [F]

$$\int \frac{\left(a+b\ln\left(c\sqrt{x}\right)\right)^p}{x^4}dx$$

```
[In] int((a+b*ln(c*x^(1/2)))^p/x^4,x)
[Out] int((a+b*ln(c*x^(1/2)))^p/x^4,x)
```

Fricas [F]

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^4}\,dx = \int \frac{\left(b\log\left(c\sqrt{x}\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x^4,x, algorithm="fricas")
[Out] integral((b*log(c*sqrt(x)) + a)^p/x^4, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*ln(c*x**(1/2)))**p/x**4,x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^4}\,dx = -\frac{2\left(b\log\left(c\sqrt{x}\right)+a\right)^{p+1}c^6e^{\left(\frac{6a}{b}\right)}E_{-p}\left(\frac{6\left(b\log\left(c\sqrt{x}\right)+a\right)}{b}\right)}{b}$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x^4,x, algorithm="maxima")
[Out] -2*(b*log(c*sqrt(x)) + a)^(p + 1)*c^6*e^(6*a/b)*exp_integral_e(-p, 6*(b*log
(c*sqrt(x)) + a)/b)/b
```

Giac [F]

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^4}\,dx = \int \frac{\left(b\log\left(c\sqrt{x}\right)+a\right)^p}{x^4}\,dx$$

```
[In] integrate((a+b*log(c*x^(1/2)))^p/x^4,x, algorithm="giac")
[Out] integrate((b*log(c*sqrt(x)) + a)^p/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\log\left(c\sqrt{x}\right)\right)^p}{x^4}\,dx=\int \frac{\left(a+b\,\ln\left(c\,\sqrt{x}\right)\right)^p}{x^4}\,dx$$

```
[In] int((a + b*log(c*x^(1/2)))^p/x^4,x)
[Out] int((a + b*log(c*x^(1/2)))^p/x^4, x)
```

3.191 $\int x^{-1+n}(a+b\log(cx^n))^p\,dx$

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int x^{-1+n}(a+b\log(cx^n))^p\,dx=\frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{b}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{b}\right)^{-p}}{cn}$$

```
[Out] GAMMA(p+1,(-a-b*ln(c*x^n))/b)*(a+b*ln(c*x^n))^p/c/exp(a/b)/n/(((-a-b*ln(c*x
^n))/b)^p)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2347, 2212}

$$\int x^{-1+n}(a+b\log(cx^n))^p\,dx=\frac{e^{-\frac{a}{b}}(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{b}\right)^{-p}\Gamma\left(p+1,-\frac{a+b\log(cx^n)}{b}\right)}{cn}$$

```
[In] Int[x^(-1 + n)*(a + b*Log[c*x^n])^p,x]
```

```
[Out] (Gamma[1 + p, -((a + b*Log[c*x^n])/b)]*(a + b*Log[c*x^n])^p)/(c*E^(a/b)*n*(
-((a + b*Log[c*x^n])/b))^p)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int e^x(a+bx)^p\,dx, x, \log(cx^n)\right)}{cn}\\ &= \frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{b}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{b}\right)^{-p}}{cn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x^{-1+n}(a+b\log(cx^n))^p\,dx = \frac{e^{-\frac{a}{b}}\Gamma\left(1+p,-\frac{a+b\log(cx^n)}{b}\right)(a+b\log(cx^n))^p\left(-\frac{a+b\log(cx^n)}{b}\right)^{-p}}{cn}$$

```
[In] Integrate[x^(-1 + n)*(a + b*Log[c*x^n])^p,x]
[Out] (Gamma[1 + p, -((a + b*Log[c*x^n])/b)]*(a + b*Log[c*x^n])^p)/(c*E^(a/b)*n*(
-((a + b*Log[c*x^n])/b))^p)
```

Maple [F]

$$\int x^{n-1}(a+b\ln(c\,x^n))^p dx$$

```
[In] int(x^(n-1)*(a+b*ln(c*x^n))^p,x)
[Out] int(x^(n-1)*(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int x^{-1+n}(a+b\log(cx^n))^p\,dx = \int (b\log(cx^n)+a)^p x^{n-1}\,dx$$

```
[In] integrate(x^(-1+n)*(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral((b*log(c*x^n) + a)^p*x^(n - 1), x)
```

Sympy [F]

$$\int x^{-1+n}(a+b\log(cx^n))^p\,dx=\int x^{n-1}(a+b\log(cx^n))^p\,dx$$

```
[In] integrate(x**(-1+n)*(a+b*ln(c*x**n))**p,x)
[Out] Integral(x**(n - 1)*(a + b*log(c*x**n))**p, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{-1+n}(a+b\log(cx^n))^p\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^(-1+n)*(a+b*log(c*x^n))^p,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: In function CAR, the value of t
he first argument is  0which is not of the expected type LIST
```

Giac [F]

$$\int x^{-1+n}(a+b\log(cx^n))^p\,dx=\int (b\log(cx^n)+a)^p x^{n-1}\,dx$$

```
[In] integrate(x^(-1+n)*(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate((b*log(c*x^n) + a)^p*x^(n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+n}(a+b\log(cx^n))^p\,dx=\int x^{n-1}\,(a+b\,\ln(c\,x^n))^p\,dx$$

```
[In] int(x^(n - 1)*(a + b*log(c*x^n))^p,x)
[Out] int(x^(n - 1)*(a + b*log(c*x^n))^p, x)
```

3.192 $\int (dx^q)^m (a+b\log (cx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 114

$$\int (dx^q)^m (a+b\log (cx^n))^p \, dx = \frac{e^{-\frac{a+amq}{bn}}x(cx^n)^{-\frac{1+mq}{n}}(dx^q)^m\Gamma\left(1+p,-\frac{(1+mq)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(1+mq)(a+b\log(cx^n))}{bn}\right)^{-p}}{1+mq}$$

```
[Out] x*(d*x^q)^m*GAMMA(p+1,-(m*q+1)*(a+b*ln(c*x^n))/b/n)*(a+b*ln(c*x^n))^p/exp((
a*m*q+a)/b/n)/(m*q+1)/((c*x^n)^((m*q+1)/n))/((-(m*q+1)*(a+b*ln(c*x^n))/b/n)
^p)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 2347, 2212}

$$\int (dx^q)^m (a+b\log (cx^n))^p \, dx = \frac{x(dx^q)^m e^{-\frac{amq+a}{bn}}(cx^n)^{-\frac{mq+1}{n}}(a+b\log(cx^n))^p\left(-\frac{(mq+1)(a+b\log(cx^n))}{bn}\right)^{-p}\Gamma\left(p+1,-\frac{(mq+1)(a+b\log(cx^n))}{bn}\right)}{mq+1}$$

```
[In] Int[(d*x^q)^m*(a + b*Log[c*x^n])^p,x]
```

```
[Out] (x*(d*x^q)^m*Gamma[1 + p, -(((1 + m*q)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*L
og[c*x^n])^p)/(E^((a + a*m*q)/(b*n))*(1 + m*q)*(c*x^n)^((1 + m*q)/n)*(-(((1
 + m*q)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \left(x^{-mq}(dx^q)^m\right)\int x^{mq}(a+b\log(cx^n))^p\,dx$$
$$= \frac{\left(x(cx^n)^{-\frac{1+mq}{n}}(dx^q)^m\right)\text{Subst}\left(\int e^{\frac{(1+mq)x}{n}}(a+bx)^p\,dx, x, \log(cx^n)\right)}{n}$$
$$= \frac{e^{-\frac{a+amq}{bn}}x(cx^n)^{-\frac{1+mq}{n}}(dx^q)^m\,\Gamma\left(1+p,-\frac{(1+mq)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(1+mq)(a+b\log(cx^n))}{bn}\right)^{-}}{1+mq}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int (dx^q)^m\,(a+b\log(cx^n))^p\,dx$$
$$= \frac{e^{-\frac{(1+mq)(a+b(-n\log(x)+\log(cx^n)))}{bn}}x^{-mq}(dx^q)^m\,\Gamma\left(1+p,-\frac{(1+mq)(a+b\log(cx^n))}{bn}\right)(a+b\log(cx^n))^p\left(-\frac{(1+mq)(a+b\log(cx^n))}{bn}\right.}{1+mq}$$

```
[In] Integrate[(d*x^q)^m*(a + b*Log[c*x^n])^p,x]
[Out] ((d*x^q)^m*Gamma[1 + p, -(((1 + m*q)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log
[c*x^n])^p)/(E^(((1 + m*q)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 +
m*q)*x^(m*q)*(-(((1 + m*q)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Maple [F]

$$\int \left(d\,x^{q}\right)^{m} \left(a+b\ln\left(c\,x^{n}\right)\right)^{p}dx$$

```
[In] int((d*x^q)^m*(a+b*ln(c*x^n))^p,x)
[Out] int((d*x^q)^m*(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int (dx^q)^m (a + b\log(cx^n))^p\, dx = \int (dx^q)^m (b\log(cx^n) + a)^p\, dx$$

```
[In] integrate((d*x^q)^m*(a+b*log(c*x^n))^p,x, algorithm="fricas")
[Out] integral((d*x^q)^m*(b*log(c*x^n) + a)^p, x)
```

Sympy [F]

$$\int (dx^q)^m (a + b\log(cx^n))^p\, dx = \int (dx^q)^m (a + b\log(cx^n))^p\, dx$$

```
[In] integrate((d*x**q)**m*(a+b*ln(c*x**n))**p,x)
[Out] Integral((d*x**q)**m*(a + b*log(c*x**n))**p, x)
```

Maxima [F]

$$\int (dx^q)^m (a + b\log(cx^n))^p\, dx = \int (dx^q)^m (b\log(cx^n) + a)^p\, dx$$

```
[In] integrate((d*x^q)^m*(a+b*log(c*x^n))^p,x, algorithm="maxima")
[Out] integrate((d*x^q)^m*(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int (dx^q)^m (a + b \log (cx^n))^p \, dx = \int (dx^q)^m (b \log (cx^n) + a)^p \, dx$$

```
[In] integrate((d*x^q)^m*(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate((d*x^q)^m*(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx^q)^m (a + b \log (cx^n))^p \, dx = \int (d\,x^q)^m \,(a + b\,\ln (c\,x^n))^p \, dx$$

```
[In] int((d*x^q)^m*(a + b*log(c*x^n))^p,x)
[Out] int((d*x^q)^m*(a + b*log(c*x^n))^p, x)
```

3.193 $\int (\mathbf{d1}x^{\mathbf{q1}})^{\mathbf{m1}} (\mathbf{d2}x^{\mathbf{q2}})^{\mathbf{m2}} (a+b\log(cx^n))^p \, dx$

Optimal result

Integrand size = 27, antiderivative size = 136

$$\int (\mathrm{d1}x^{\mathrm{q1}})^{\mathrm{m1}} (\mathrm{d2}x^{\mathrm{q2}})^{\mathrm{m2}} (a+b\log(cx^n))^p \, dx = \frac{e^{-\frac{a(1+\mathrm{m1q1}+\mathrm{m2q2})}{bn}} x(cx^n)^{-\frac{1+\mathrm{m1q1}+\mathrm{m2q2}}{n}} (\mathrm{d1}x^{\mathrm{q1}})^{\mathrm{m1}} (\mathrm{d2}x^{\mathrm{q2}})^{\mathrm{m2}} \Gamma\left(1+p,-\frac{(1+\mathrm{m1q1}+\mathrm{m2q2})(a+b\log(cx^n))}{bn}\right)(a+b\,\mathrm{lo}}{1+\mathrm{m1q1}+\mathrm{m2q2}}$$

```
[Out] x*(d1*x^q1)^m1*(d2*x^q2)^m2*GAMMA(p+1,-(m1*q1+m2*q2+1)*(a+b*ln(c*x^n))/b/n)
*(a+b*ln(c*x^n))^p/exp(a*(m1*q1+m2*q2+1)/b/n)/(m1*q1+m2*q2+1)/((c*x^n)^((m1
*q1+m2*q2+1)/n))/((-(m1*q1+m2*q2+1)*(a+b*ln(c*x^n))/b/n)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 2347, 2212}

$$\int (\mathrm{d1}x^{\mathrm{q1}})^{\mathrm{m1}} (\mathrm{d2}x^{\mathrm{q2}})^{\mathrm{m2}} (a+b\log(cx^n))^p \, dx = \frac{x(\mathrm{d1}x^{\mathrm{q1}})^{\mathrm{m1}} (\mathrm{d2}x^{\mathrm{q2}})^{\mathrm{m2}} (a+b\log(cx^n))^p e^{-\frac{a(\mathrm{m1q1}+\mathrm{m2q2}+1)}{bn}} (cx^n)^{-\frac{\mathrm{m1q1}+\mathrm{m2q2}+1}{n}} \left(-\frac{(\mathrm{m1q1}+\mathrm{m2q2}+1)(a+b\log(cx^n))}{bn}\right.}{\mathrm{m1q1}+\mathrm{m2q2}+1}$$

```
[In] Int[(d1*x^q1)^m1*(d2*x^q2)^m2*(a + b*Log[c*x^n])^p,x]
```

```
[Out] (x*(d1*x^q1)^m1*(d2*x^q2)^m2*Gamma[1 + p, -(((1 + m1*q1 + m2*q2)*(a + b*Log
[c*x^n]))/(b*n))]*(a + b*Log[c*x^n])^p)/(E^((a*(1 + m1*q1 + m2*q2))/(b*n))*
(1 + m1*q1 + m2*q2)*(c*x^n)^((1 + m1*q1 + m2*q2)/n)*(-(((1 + m1*q1 + m2*q2)
*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 2347

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Dist[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n)), Subst[Int[E^(((m + 1)/n)
*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \left(x^{-\text{m1q1}}\left(\text{d1}x^{\text{q1}}\right)^{\text{m1}}\right)\int x^{\text{m1q1}}\left(\text{d2}x^{\text{q2}}\right)^{\text{m2}}(a+b\log(cx^n))^p\,dx$$

$$=\left(x^{-\text{m1q1}-\text{m2q2}}\left(\text{d1}x^{\text{q1}}\right)^{\text{m1}}\left(\text{d2}x^{\text{q2}}\right)^{\text{m2}}\right)\int x^{\text{m1q1}+\text{m2q2}}(a+b\log(cx^n))^p\,dx$$

$$=\frac{\left(x(cx^n)^{-\frac{1+\text{m1q1}+\text{m2q2}}{n}}\left(\text{d1}x^{\text{q1}}\right)^{\text{m1}}\left(\text{d2}x^{\text{q2}}\right)^{\text{m2}}\right)\text{Subst}\left(\int e^{\frac{(1+\text{m1q1}+\text{m2q2})x}{n}}(a+bx)^p\,dx,x,\log(cx^n)\right)}{n}$$

$$=\frac{e^{-\frac{a(1+\text{m1q1}+\text{m2q2})}{bn}}x(cx^n)^{-\frac{1+\text{m1q1}+\text{m2q2}}{n}}\left(\text{d1}x^{\text{q1}}\right)^{\text{m1}}\left(\text{d2}x^{\text{q2}}\right)^{\text{m2}}\Gamma\left(1+p,-\frac{(1+\text{m1q1}+\text{m2q2})(a+b\log(cx^n))}{bn}\right)(a}{1+\text{m1q1}+\text{m2q2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04

$$\int\left(\text{d1}x^{\text{q1}}\right)^{\text{m1}}\left(\text{d2}x^{\text{q2}}\right)^{\text{m2}}(a+b\log(cx^n))^p\,dx$$
$$=\frac{e^{-\frac{(1+\text{m1q1}+\text{m2q2})(a+b(-n\log(x)+\log(cx^n)))}{bn}}x^{-\text{m1q1}-\text{m2q2}}\left(\text{d1}x^{\text{q1}}\right)^{\text{m1}}\left(\text{d2}x^{\text{q2}}\right)^{\text{m2}}\Gamma\left(1+p,-\frac{(1+\text{m1q1}+\text{m2q2})(a+b\log(cx^n))}{bn}\right)}{1+\text{m1q1}+\text{m2q2}}$$

```
[In] Integrate[(d1*x^q1)^m1*(d2*x^q2)^m2*(a + b*Log[c*x^n])^p,x]
```

```
[Out] (x^(-(m1*q1) - m2*q2)*(d1*x^q1)^m1*(d2*x^q2)^m2*Gamma[1 + p, -(((1 + m1*q1
+ m2*q2)*(a + b*Log[c*x^n]))/(b*n))]*(a + b*Log[c*x^n])^p)/(E^(((1 + m1*q1
+ m2*q2)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m1*q1 + m2*q2)*(-(
((1 + m1*q1 + m2*q2)*(a + b*Log[c*x^n]))/(b*n)))^p)
```

Maple [F]

$$\int \left(\mathrm{d1}\ x^{\mathrm{q1}}\right)^{\mathrm{m1}} \left(\mathrm{d2}\ x^{\mathrm{q2}}\right)^{\mathrm{m2}} \left(a+b\ln\left(c\,x^{n}\right)\right)^{p}dx$$

```
[In] int((d1*x^q1)^m1*(d2*x^q2)^m2*(a+b*ln(c*x^n))^p,x)
[Out] int((d1*x^q1)^m1*(d2*x^q2)^m2*(a+b*ln(c*x^n))^p,x)
```

Fricas [F]

$$\int \left(\mathrm{d1}x^{\mathrm{q1}}\right)^{\mathrm{m1}} \left(\mathrm{d2}x^{\mathrm{q2}}\right)^{\mathrm{m2}} \left(a+b\log\left(cx^{n}\right)\right)^{p}\,dx = \int \left(d_{1}x^{q_{1}}\right)^{m_{1}} \left(d_{2}x^{q_{2}}\right)^{m_{2}} \left(b\log\left(cx^{n}\right)+a\right)^{p}dx$$

```
[In] integrate((d1*x^q1)^m1*(d2*x^q2)^m2*(a+b*log(c*x^n))^p,x, algorithm="fricas
")
[Out] integral((d1*x^q1)^m1*(d2*x^q2)^m2*(b*log(c*x^n) + a)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(\mathrm{d1}x^{\mathrm{q1}}\right)^{\mathrm{m1}} \left(\mathrm{d2}x^{\mathrm{q2}}\right)^{\mathrm{m2}} \left(a+b\log\left(cx^{n}\right)\right)^{p}\,dx = \text{Timed out}$$

```
[In] integrate((d1*x**q1)**m1*(d2*x**q2)**m2*(a+b*ln(c*x**n))**p,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(\mathrm{d1}x^{\mathrm{q1}}\right)^{\mathrm{m1}} \left(\mathrm{d2}x^{\mathrm{q2}}\right)^{\mathrm{m2}} \left(a+b\log\left(cx^{n}\right)\right)^{p}\,dx = \int \left(d_{1}x^{q_{1}}\right)^{m_{1}} \left(d_{2}x^{q_{2}}\right)^{m_{2}} \left(b\log\left(cx^{n}\right)+a\right)^{p}dx$$

```
[In] integrate((d1*x^q1)^m1*(d2*x^q2)^m2*(a+b*log(c*x^n))^p,x, algorithm="maxima
")
[Out] integrate((d1*x^q1)^m1*(d2*x^q2)^m2*(b*log(c*x^n) + a)^p, x)
```

Giac [F]

$$\int \left(\mathrm{d}1x^{\mathrm{q}1}\right)^{\mathrm{m}1}\left(\mathrm{d}2x^{\mathrm{q}2}\right)^{\mathrm{m}2}(a+b\log\left(cx^n\right))^p\,dx=\int \left(d_1x^{q_1}\right)^{m_1}\left(d_2x^{q_2}\right)^{m_2}\left(b\log\left(cx^n\right)+a\right)^p\,dx$$

```
[In] integrate((d1*x^q1)^m1*(d2*x^q2)^m2*(a+b*log(c*x^n))^p,x, algorithm="giac")
[Out] integrate((d1*x^q1)^m1*(d2*x^q2)^m2*(b*log(c*x^n) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\mathrm{d}1x^{\mathrm{q}1}\right)^{\mathrm{m}1}\left(\mathrm{d}2x^{\mathrm{q}2}\right)^{\mathrm{m}2}(a+b\log\left(cx^n\right))^p\,dx=\int \left(d_1\,x^{q_1}\right)^{m_1}\left(d_2\,x^{q_2}\right)^{m_2}\left(a+b\,\ln\left(c\,x^n\right)\right)^p\,dx$$

```
[In] int((d1*x^q1)^m1*(d2*x^q2)^m2*(a + b*log(c*x^n))^p,x)
[Out] int((d1*x^q1)^m1*(d2*x^q2)^m2*(a + b*log(c*x^n))^p, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                              convert(ExpnType_result,string)," vs. order ",
                              convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
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            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
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```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
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